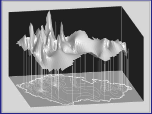


EDGE DETECTION IN BIOMEDICAL IMAGES

Eva Hošťálková & Aleš Procházka

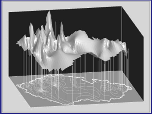
Institute of Chemical Technology in Prague

Dept of Computing and Control Engineering

http://dsp.vscht.cz/

ICT PRAGUE

Process Control 2008, Kouty nad Desnou



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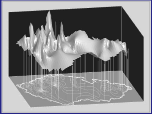

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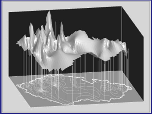

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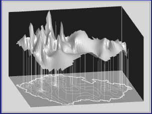

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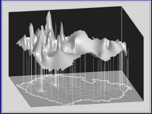

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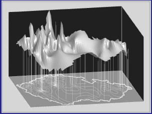
Image Edges

- Most important for image perception
- Abrupt changes of intensity
  - High frequencies

Methods Used

- Short gradient filters:
  - Insufficient for blurred or noisy images
- Canny detector:
  - More robust against noise
  - Operating at various scales
- Hidden Markov Models:
  - In our future work



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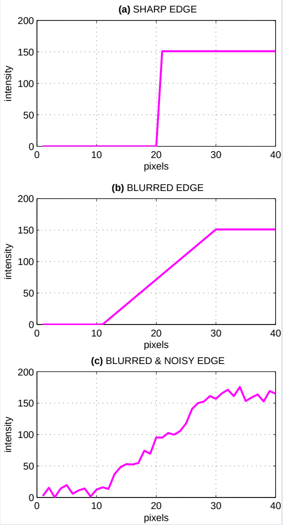

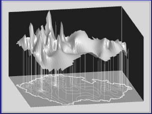

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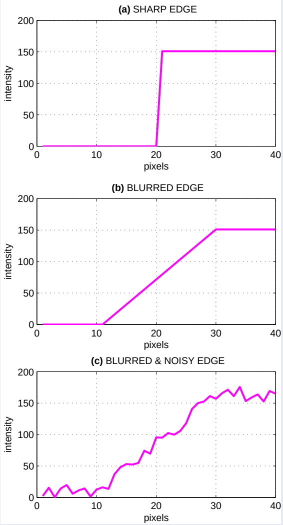

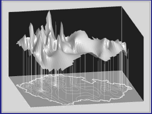

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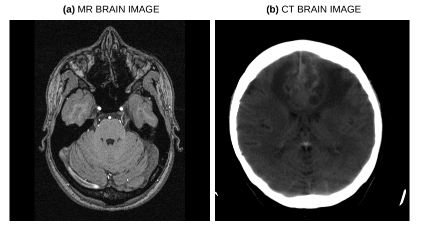
The Data

- Magnetic Resonance (MR) images
- Computed Tomography (CT) images



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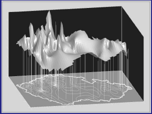
- Noise reduction prior to edge detection
- By wavelet coefficients shrinkage



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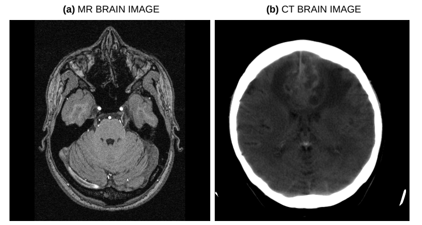

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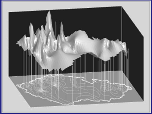

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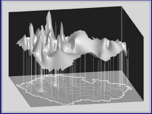
Denoising Prior to Edge Detection

Wavelet Shrinkage Algorithm:

1. Wavelet decomposition
2. Thresholding of wavelet coefficients
3. Reconstruction using the altered coefficients

Alternatives of the Wavelet Transform

- Discrete Wavelet Transform (DWT)
- Dual-Tree Complex Wavelet Transform (DTCWT) by Prof. Kingsbury and Prof. Selesnick



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E. Hošťálková, A. Procházka

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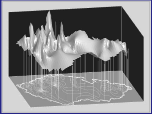

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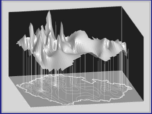

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DTCWT

Ideal Complex Wavelet Transform

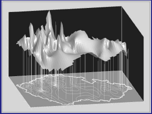
- Employs analytic complex wavelets
- $\Rightarrow$ Magnitude-phase representation
  - Large magnitude $\Rightarrow$ presence of a singularity
  - Phase: its position within the support of the wavelet
- $\Rightarrow$ Shift invariance & no aliasing

Analytic Wavelets

A complex wavelet $\psi_c(t) = \psi_r(t) + j \cdot \psi_i(t)$ is analytic when its real and imaginary part form a Hilbert transform (HT) pair

$$\psi_i(t) = HT\{\psi_r(t)\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\psi_r(t)}{t-\tau} d\tau = \psi_r(t) \frac{1}{\pi \, t} \tag{1}$$

t, τ ... continuous time



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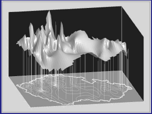


Analytic Wavelets

Fourier Transform of a HT Pair

$$\Psi_i(\omega) = FT\{\psi_i(t)\} = FT\{HT\{\psi_r(t)\}\} = -j \cdot sgn(\omega)\Psi_r(\omega) \quad (2)$$

$\omega \ldots$ frequency; $j \ldots$ the complex unit

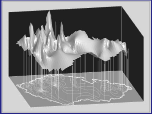
Single-Sided Spectrum as a Consequence

$$\Psi_c(\omega) = \Psi_r(\omega) + sgn(\omega)\Psi_r(\omega) \quad (3)$$

$$\Psi_c(\omega) = \begin{cases} 0 & \text{for } \omega < 0 \\ \Psi_r(\omega) & \text{for } \omega = 0 \\ 2\,\Psi_r(\omega) & otherwise \end{cases} \quad (4)$$

Implications

- No aliasing $\Rightarrow$ shift invariance
- Impossible for wavelets of compact support $\Rightarrow$ only approximately analytic



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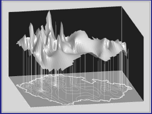

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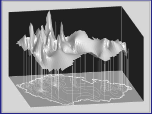
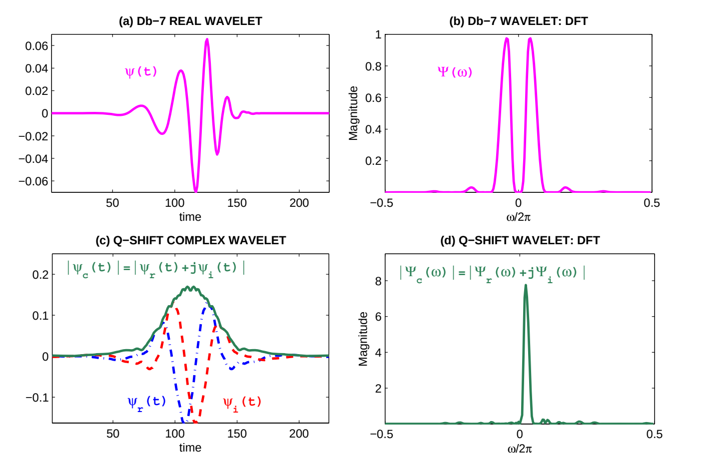
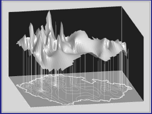


Analytic Wavelets

Frequency Spectra of a Real and an Analytic Wavelet



Level 4, 14-tap filters: Daubechies (for DWT) and q-shift (for DTCWT).



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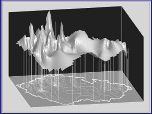


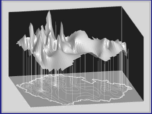
Directional Selectivity

Dual Tree Complex Wavelet Transform (DTCWT)

- Dual tree (two DWT trees) of real filters $\Rightarrow$ real and imaginary parts of each complex coefficient
- $\Rightarrow$ Directional selectivity in 2D:

DTCWT	DWT
6 directional subbands	3 directional subbands
$\pm 15^\circ$, $\pm 45^\circ$ and $\pm 75^\circ$	0°, 45° and 90°

- $\Rightarrow$ 2^d redundancy in d-dimensional space



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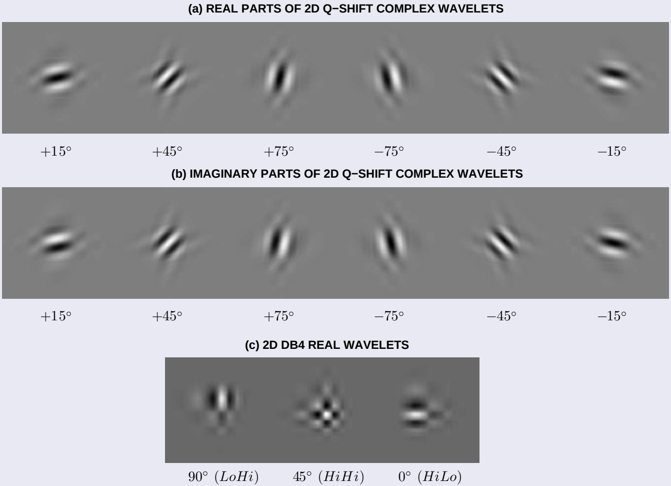
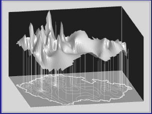
Directional Selectivity

Directional Selectivity of 2D Wavelets





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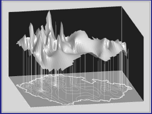

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Properties of DWT and DTCWT

DWT versus DTCWT

DWT

- Zero crossings at a singularity
- Strong shift dependence
- Aliasing
- Lack of directional selectivity ($\pm 45°$)
- Critically decimated
- Perfect reconstruction

DTCWT

- Large magnitudes at a singularity
- Approx. shift independence
- Approx. no aliasing
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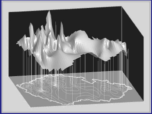

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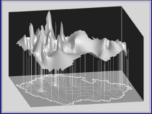

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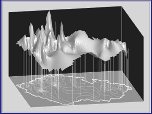

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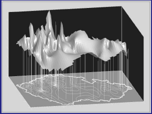

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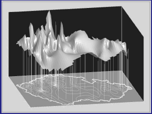

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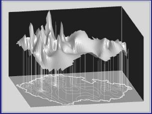

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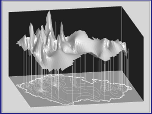

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Denoising Technique

Wavelet Shrinkage

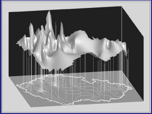
- Suppressing lower energy wavelet coefficients (noise)
- Thresholding magnitudes of complex w. coefficients
  - Vary slowly
  - Not distorted by aliasing

Soft Universal Thresholding

$$\overline{c}_s(k) = \begin{cases} sgn(c(k))\,(|\,c(k)\,| - \delta^{(s)}) & \text{for } |\,c(k)\,| > \delta^{(s)} \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

$\{c(k)\}_{k=0}^{M-1}$ . . . w. coefficients of all levels

δ . . . threshold level



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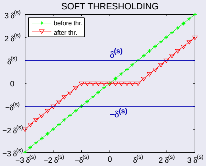

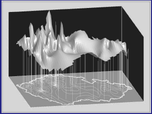

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Denoising Technique

Donoho Soft Threshold Estimate

$$\delta^{(s)} = \sqrt{2\ \hat{\sigma}_n^2\ log(N)} \qquad (6)$$

$\hat{\sigma}_n \ldots$ noise std. deviation estimate; $N \ldots$ no. w. coefficients

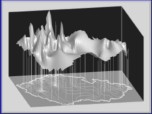
Median Absolute Deviation (MAD) Estimator

$$\hat{\sigma}_{mad} = \frac{median\{\ |c_1^{hh}(0)|, |c_1^{hh}(1)|, \ldots, |c_1^{hh}(N/4-1)|\ \}}{0.6745} \qquad (7)$$

$\{cc_1^{hh}(n)\}_{n=0}^{N/4-1} \ldots$ HiHi w. coefficient of level 1; $N \ldots$ image size

MAD Estimator Assumptions

- Smallest scale HiHi coefficients - noise dominated
- For i.i.d. Gaussian noise
- Robust against large deviations of noise variance



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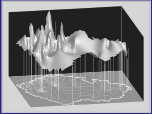

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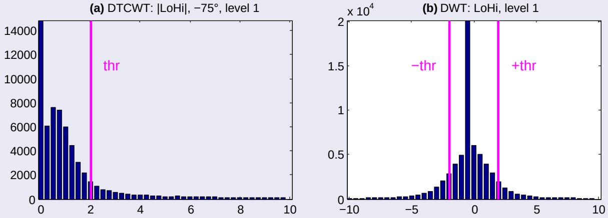
Denoising Technique

Histograms of Wavelet Coefficients



4 levels, 14-tap filters: Daubechies for DWT and q-shift for DTCWT

EDGE DETECTION IN BIOMEDICAL IMAGES

E. Hošťálková, A. Procházka

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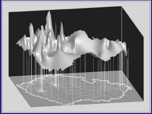

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Gradient Masks

Gradient Edge Detectors

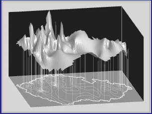
- Filters approximating the intensity gradient
- 2D convolution between the filter and the image
- Short filters: too sensitive to noise and blurring
- Longer filters:
  - More robust against noise
  - Blur the originally sharp edges

Sobel Filter

- Rotation: detection of $0°$, $\pm 45°$ and $90°$ edges

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

- For every root pixel - the rotation variant with the absolute maximum value of convolution



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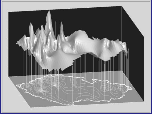

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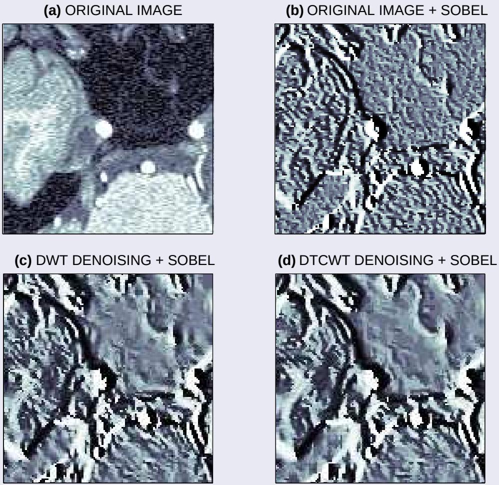
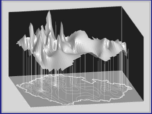
Sobel Filter for MR Brain Image After Denoising





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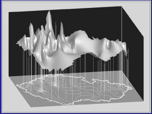

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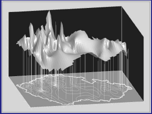
Canny Edge Detector

Canny Edge Detector

- Approximates the derivative of a 2D Gaussian in the direction of the gradient
- Robust against noise
  - $\Leftarrow$ Gaussian smoothing filter prior to edge detection
  - $\Leftarrow$ Weak edges pixels identification alorithm
- Adjustable value of the scale σ (the standard deviation in the Gaussian)



Canny Edge Detector

Canny Algorithm

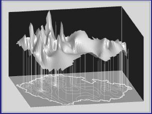
1. Convolution with 1D Gaussian masks in x and y-direction

$$G_{\sigma,0}(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{8}$$

2. Convolution with the derivatives of the 2D Gaussian in x-direction (and also in y-direction)

$$\frac{\partial G_{\sigma,0}(x,y)}{\partial x} = -\frac{x}{\sqrt{2\pi}\sigma^3} \cdot exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \tag{9}$$

3. Combining of these two matrices
4. Strong edges: pels value above the upper threshold
5. Weak edges:
   - Pels value above the lower threshold
   - The gradient $\equiv$ the direction of the strong edges in the neighborhood



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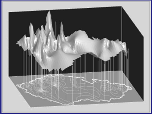

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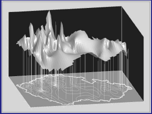

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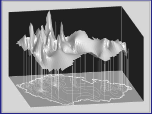

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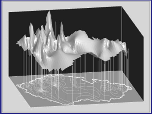

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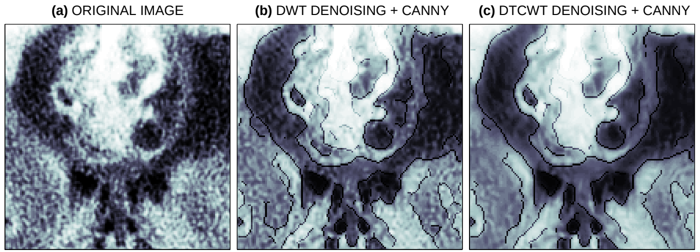
Canny Method for the CT Image After Denoising ($\sigma = 1.8$)



Denoising by wavelet shrinkage:

- DWT: 16-tap symlet filters, 4 levels
- DTCWT: 16-tap q-shift filters, 4 levels

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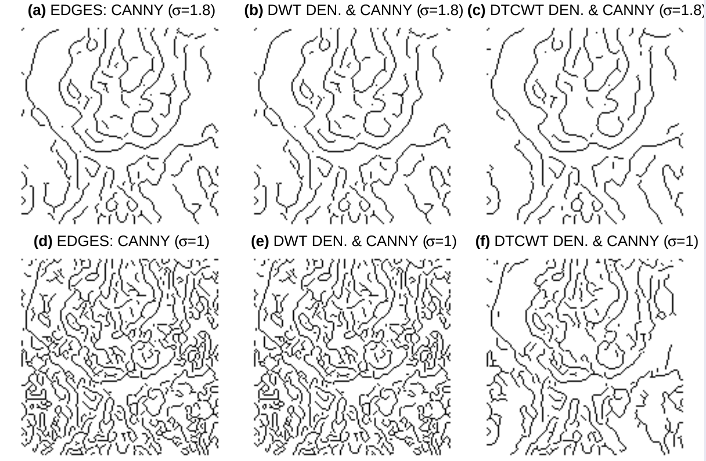
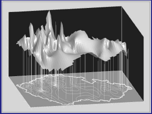
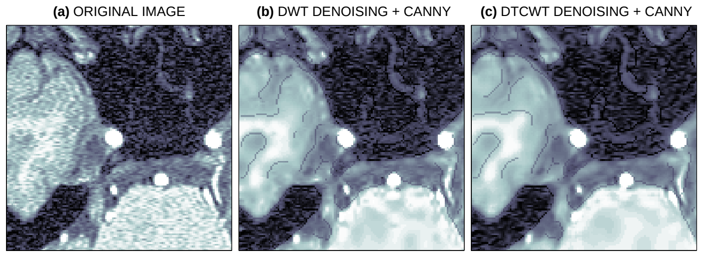
Edges of the CT Image After Denoising ($\sigma = 1.8,\ 1$)





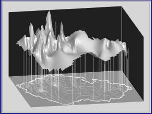
Canny Edge Detector

Canny Method for the MR Image After Denoising ($\sigma = 2.5$)



Denoising by wavelet shrinkage:

- DWT: 14-tap symlet filters, 3 levels
- DTCWT: 14-tap q-shift filters, 3 levels



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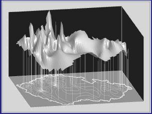


Hidden Markov Models (HMM)

Hidden Markov Models (HMM)

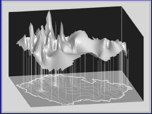
- The focus of our future work on edge detection
- Utilizing sparsity and persistence of the DTCWT coefficients (shift invariant)

Sparsity

- Many small coefficients from smooth regions
- Fewer large coefficients corresponding to singularities
- The marginal distribution of the coefficients within each scale - modeled as a 2-component mixture of distributions (2 values of variance)

Persistence

- Strong parent-child relations - the relative size of a coefficient propagates through its children across scale



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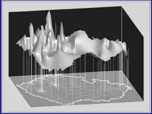

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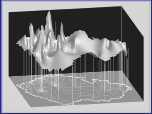

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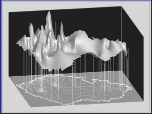
Conclusions

- DTCWT outperformes the DWT
  - Approximate shift invariance
  - Steady values of the magnitude across scale
  - Phase representation of edges orientation
  - Improved directional selectivity in higher dimensions
- Both transforms
  - For noise reduction in biomedical images
  - By soft wavelet shrinkage
- Edge detection for the resulting images:
  - Gradient approximating masks
  - Canny detector
  - Possible use of the DTCWT through hidden Markov models



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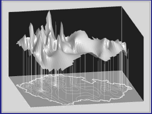

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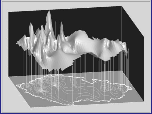
  - Canny detector
  - Possible use of the DTCWT through hidden Markov models



Further Reading

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