# WAVELET SIGNAL AND IMAGE DENOISING

*E. Hošťálková, A. Procházka*

Institute of Chemical Technology
Department of Computing and Control Engineering

**Abstract**

**The paper deals with the use of wavelet transform for signal and image de-noising employing a selected method of thresholding of appropriate decomposition coefficients. The proposed technique is based upon the analysis of wavelet transform and it includes description of global modification of its values. The whole method is verified for simulated signals and applied to processing of biomedical signals representing EEG signals and MR images corrupted by additional random noise.**

## 1 Introduction

The *wavelet transform* (WT) is a powerful tool of signal processing for its multiresolutional possibilities. Unlike the Fourier transform, the WT is suitable for application to non-stationary signals with transitory phenomena, whose frequency response varies in time [2].

The wavelet coefficients represent a measure of similarity in the frequency content between a signal and a chosen wavelet function [2]. These coefficients are computed as a convolution of the signal and the scaled wavelet function, which can be interpreted as a dilated band-pass filter because of its band-pass like spectrum [5].

The *scale* is inversely proportional to radian frequency. Consequently, low frequencies correspond to high scales and a dilated wavelet function. By wavelet analysis at high scales, we extract global information from a signal called *approximations*. Whereas at low scales, we extract fine information from a signal called *details*.

Signals are usually band-limited, which is equivalent to having finite energy, and therefore we need to use just a constrained interval of scales. However, the continuous wavelet transform provides us with lots of redundant information.

The *discrete wavelet transform* (DWT) requires less space utilising the *space-saving coding* based on the fact that wavelet families are orthogonal or biorthogonal bases, and thus do not produce redundant analysis. The DWT corresponds to its continuous version sampled usually on a *dyadic* grid, which means that the scales and translations are powers of two [5].

In practise, the DWT is computed by passing a signal successively through a high-pass and a low-pass filter. For each decomposition level, the high-pass filter $h_d$ forming the wavelet function produces the *approximations* $A$. The complementary low-pass filter $l_d$ representing the scaling function produces the *details* $D$ [3]. This computational algorithm shown in Fig. 1a is called the *subband coding.*

The resolution is altered by the filtering process, and the scale is changed by either *upsampling* or *downsampling* by 2. This is described by the following two equations [4]

$$D_1[n] = \sum_{k=-\infty}^{\infty} h_d[k]\, x[2n-k] \tag{1}$$

$$A_1[n] = \sum_{k=-\infty}^{\infty} l_d[k]\, x[2n-k] \tag{2}$$

where $n$ and $k$ denote discrete time coefficients, $x$ the decomposed signal.

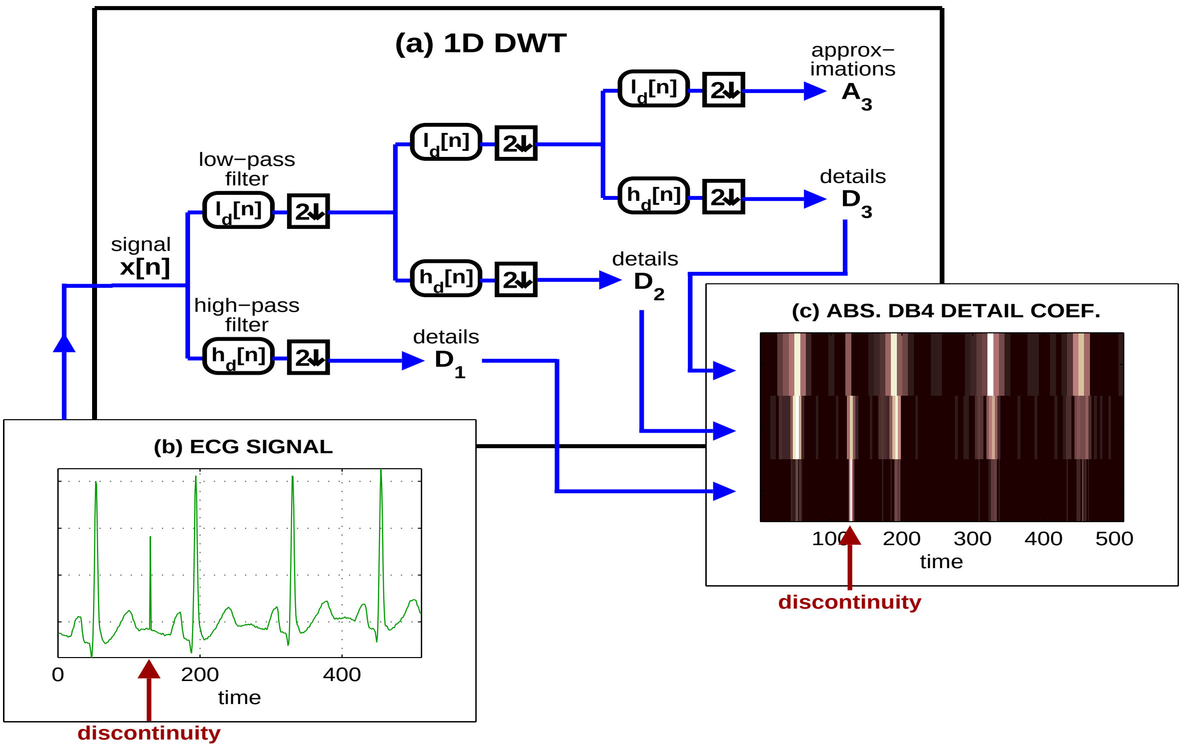


Figure 1: Discontinuity detection in ECG signal applying wavelet analysis. **(a)** one-dimensional wavelet subband coding scheme, **(b)** ECG signal with a discontinuity, and **(c)** absolute detail decomposition coefficients using the *db*4 wavelet function

Half-band filters form orthonormal bases, and therefore make the reconstruction easy. The synthesis consists of upsampling by 2 and filtering [4]:

$$x[n] = \sum_{k=-\infty}^{\infty} \left( D_1[k]\, h_r[2k-n] + A_1[k]\, l_r[2k-n] \right) \tag{3}$$

The *reconstruction* filters $l_r$ and $h_r$ and identical with the decomposition filters $l_d$ and $h_d$, respectively, except the reverse time course. These filters attain to produce perfect signal reconstruction from the DWT coefficients provided that the signal is of finite energy, and that the wavelet satisfies the admissibility condition [1]. Both these conditions are satisfied with natural signals and usual wavelets [2, 5].

The practical use of the DWT is to be discussed in later sections. We employ here two types of wavelet functions, which are the Daubeschies wavelet *db*4 and the symlet wavelet (*sym*4). Both functions are given by 8 coefficients and have similar properties [2].

## 2 Signal Analysis

In signal processing, wavelets are used for many purposes [2]. Such as denoising, detecting trends, breakdown points, discontinuities in higher derivatives and self-similarity in signals. At first, we focus on *discontinuity detection*.

For the *Electrocardiogram* (ECG) signal, Fig. 1 demonstrates the use of the *db*4 wavelet for impulse detection, i.e. detection of a discontinuity in frequency. The impulse is generated artificially for our purposes. The *db*4 wavelet is chosen because of its good performance in this case. The decomposition runs up to level 3, which is enough to make the discontinuity apparent.

# 3 Signal Denoising

This section describes signal denoising with the application on the ECG signal.

## 3.1 Soft and Hard Thresholding

*Signal denoising* using the DWT consists of the three successive procedures, namely, signal *decomposition*, *thresholding* of the DWT coefficients, and signal *reconstruction*. Firstly, we carry out the wavelet analysis of a noisy signal up to a chosen level $N$. Secondly, we perform thresholding of the detail coefficients from level 1 to $N$. Lastly, we synthesize the signal using the altered detail coefficients from level 1 to $N$ and approximation coefficients of level $N$ [2]. However, it is generally impossible to remove all the noise without corrupting the signal.

As for *thresholding*, we can settle either a *level-dependent* threshold vector of length $N$ or a *global* threshold of a constant value for all levels. According to *D. Donoho's method*, the threshold estimate $\delta$ for denoising with an orthonormal basis is given by [1]

$$\delta = \sigma \sqrt{2 \log L} \tag{4}$$

where the noise is Gaussian with standard deviation $\sigma$ of the DWT coefficients and $L$ is the number of samples or pixels of the processed signal or image. This estimation concept is used by Matlab.

From another point of view, thresholding can be either *soft* or *hard* [1]. Hard thresholding zeroes out all the signal values smaller than $\delta$. Soft thresholding does the same thing, and apart form that, subtracts $\delta$ from the values larger than $\delta$. In contrast to hard thresholding, soft thresholding causes no discontinuities in the resulting signal. In Matlab, by default, soft thresholding is used for denoising and hard thresholding for compression [2].

## 3.2 Applications to ECG Signals

Denoising of the (ECG) signal is displayed in Fig. 2. The removing of artificially added random noise is carried out by thresholding of the DWT coefficients up to level 3. As a wavelet function, we choose the *sym*4, since in this application it performs better than the *db*4. As a thresholding method, we use a soft global threshold $\delta$ of an estimated value given by Eq. (4). The results are left to visual examination. The Matlab code is also enclosed.

```
%% ECG SIGNAL DENOISING BY THRESHOLDING ITS DWT COEFFICIENTS
% ORIGINAL ECG SIGNAL
ecg=load('ECG01.TXT');
ecg=detrend(ecg); % Remove a linear trend
ecg=ecg(100:611); L=length(ecg)
% NOISY ECG SIGNAL
ecgN=ecg+160*randn(L,1);
%% WAVELET DECOMPOSITION
% Default values for denoising using a wavelet
[THR,SORH,KEEPAPP]=ddencmp('den','wv',ecgN)
level=3 % decomposition up to level 3
% De-noising by thresholding SYM4 detail coefficients
[ecgC,CecgC,LecgC,PERF0,PERFL2]=wdencmp('gbl',ecgN,...
'sym4',level,THR,SORH,KEEPAPP); % obtaining clean ECG signal ecgN
```

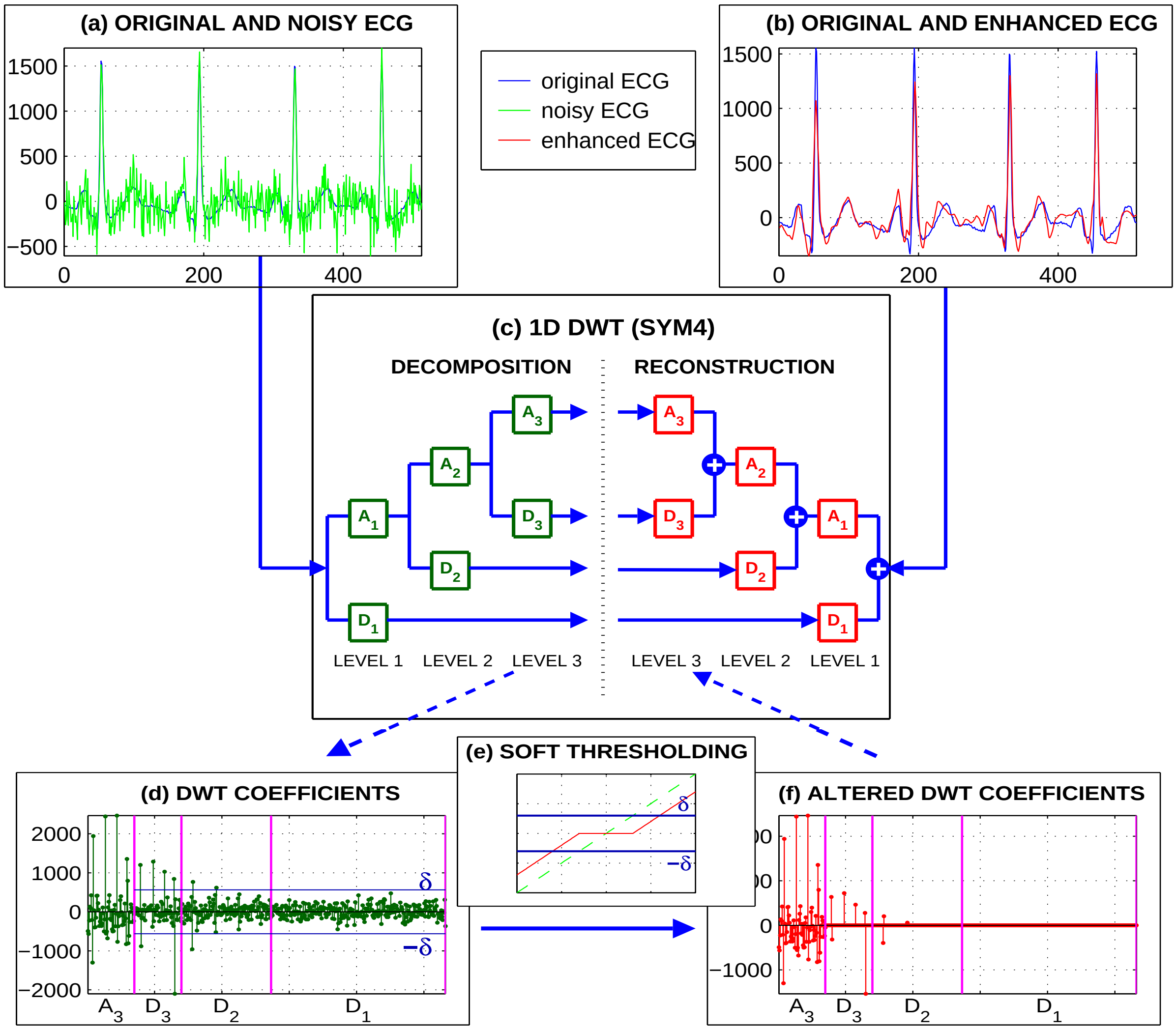


Figure 2: ECG signal de-noising by thresholding of wavelet detail coefficients up to the third level presenting **(a)** original and noisy ECG signal, **(b)** original and enhanced ECG signal, **(c)** decomposition and reconstruction up to the third level using the *sym*4 wavelet function, **(d)** wavelet coefficients of the noisy signal and the estimated threshold level $\delta$, **(e)** principles of soft thresholding, and **(f)** altered wavelet coefficients for signal reconstruction

# 4 Image Denoising

In image processing, wavelets are used for instance for edges detection, watermarking, texture detection, compression, denoising, and coding of interesting features for subsequent classification [2]. *Image denoising* by thresholding of the DWT coefficients is discussed in the following subsections.

## 4.1 Principles

The principles of image denoising using the DWT are analogous to that for signals described above. For images, we need to extend our work to two dimensions.

To compute the two-dimensional DWT of an image, we decompose the approximations at level $j$ to obtain four matrixes of coefficients at level $j+1$. These four matrixes for single level decomposition using *db*4 displayed in Fig. 3c are, clockwise, the *approximations* and the *horizontal* , *vertical* and *diagonal details* of level 1.

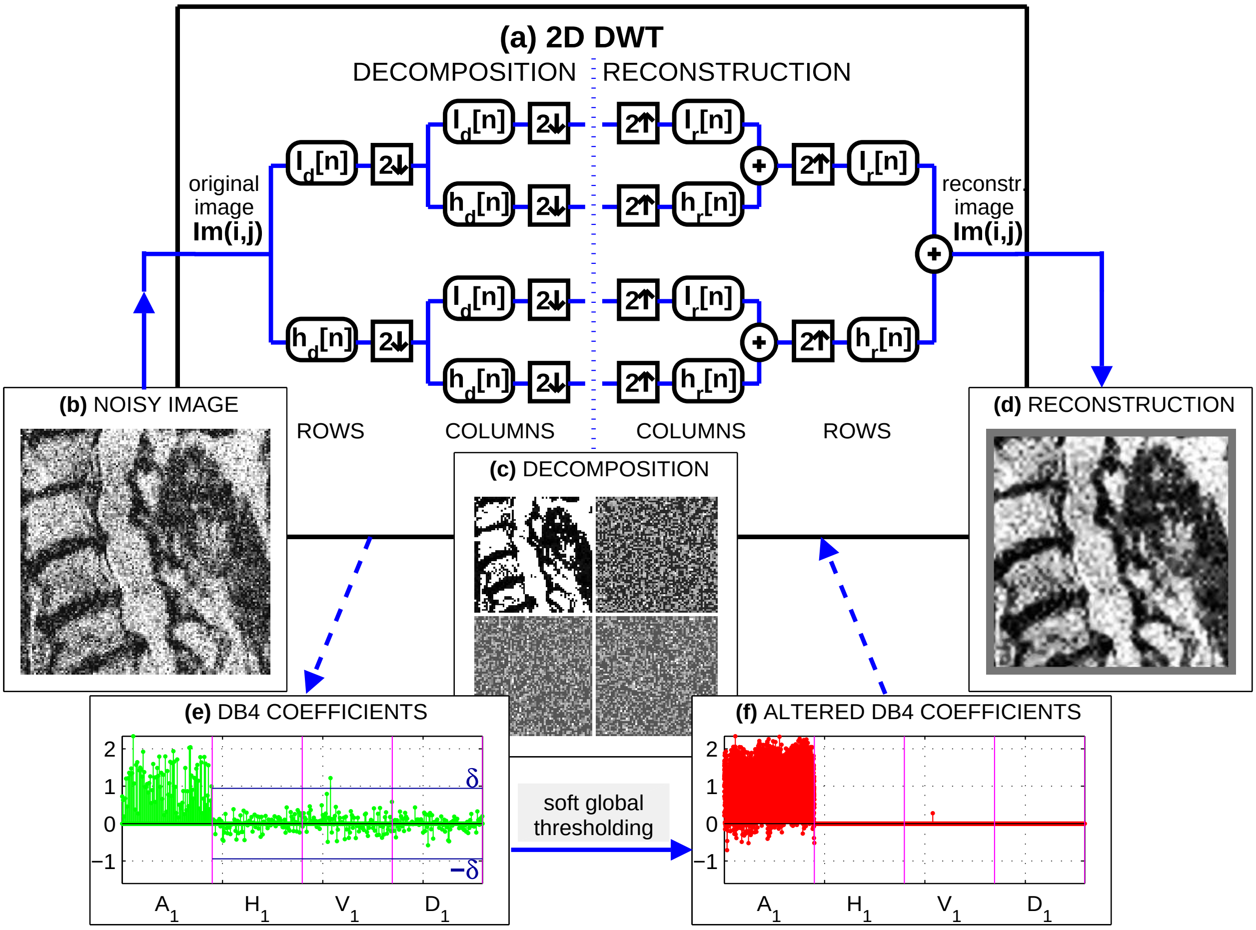


Figure 3: MR image enhancement by thresholding of the wavelet decomposition coefficients displaying **(a)** one-dimensional wavelet subband coding scheme, **(b)** noisy MR image, **(c)** first level decomposition using the *db*4 wavelet function, **(d)** reconstructed and interpolated image, **(e)** absolute *db*4 decomposition coefficients, and **(f)** thresholded decomposition coefficients used for image reconstruction

As shown in the scheme in Fig. 3a, first, we convolve the rows of the image, or generally the matrix of the approximations at level $j$, with a low-pass and a high-pass decomposition filter $l_d[n]$ and $h_d[n]$, respectively. Then we downsample both resulting matrixes by 2 keeping every even column. Second, we filter each of the matrixes by their columns using the previously mentioned filters. Then we downsample all four resulting matrixes by 2 keeping every even row to obtain four matrixes of one-level decomposition coefficients, or generally four matrixes of $(j+1)$-level coefficients [2]. We can also reconstruct the image by using these coefficients matrixes, upsampling by 2 and the reconstruction filters $l_r[n]$ and $h_r[n]$.

### 4.2 Applications to MR Images

*Magnetic Resonance* (MR) image denoising by thresholding of the wavelet detail coefficients is illustrated in Fig. 4. The programme code is also enclosed. The decomposition runs up to level 2 using the *db*4 wavelet function. The wavelet coefficient are altered with a soft global threshold $\delta$ estimated from Eq. (4). The reconstructed image is smoothed by cubic interpolation. The areas along the image boundaries are coloured with grey, hence these pixels would require different handling.

```
%% MR IMAGE DENOISING BY THRESHOLDING ITS DWT COEFFICIENTS
% GIVEN IMAGE DEFINITION
load('MRpater004.mat'), A=im2double(A);
A=A(64:191,64:191);  % Image cut
% RANDOM NOISE ADDITION
An=A+0.2*randn(size(A));   % Noisy image cut
% DB4 DECOMPOSITION TO LEVEL 2
level=2;
[c,s]=wavedec2(An,level,'db4'); % Decomposition vector c and the
          % corresponding bookkeeping matrix s
% Modify matriw s for generating indexes ind (below)
s2=s(2:level+1,1)'; s2=[s2; s2; s2]; s2=[s(1);s2(:)]; ss=s2.^2;
% THRESHOLDING OF DWT DETAIL COEFFICIENTS
THR = ddencmp('den','wv',An)   % Global threshold estimate
% Leave out approximations from thresholding
for i=1:7, if i==1 thr(i)=0; else thr(i)=THR; end, end
% Indexes dividing individual coefficients sets in vector c
ind(1)=0;
for i=1:7, ind(i+1)=sum(ss(1:i)); end
% Soft thresholding of dwt coefficients
for i=1:7
    k=find(abs(c(ind(i)+1:ind(i+1)))<=thr(i));
    k=k+ind(i); cd(k)=0;
    k=find(abs(c(ind(i)+1:ind(i+1)))>thr(i));
    k=k+ind(i); cd(k)=sign(c(k)).*(abs(c(k))-thr(i));
end;
% IMAGE RECONSTRUCTION
Z=waverec2(cd,s,'db4');
% Cubic interpolation
Zinterp=interp2(Z,'*cubic');
% Display grey image boundaries
[m,n]=size(Z); Zmean=mean2(Z); Z([1:4,m-3:m],:)=Zmean;
Z(:,[1:4,n-3:n])=Zmean; [m,n]=size(Zinterp);
Zinterp([1:8,m-7:m],:)=Zmean; Zinterp(:,[1:8,n-7:n])=Zmean;
```

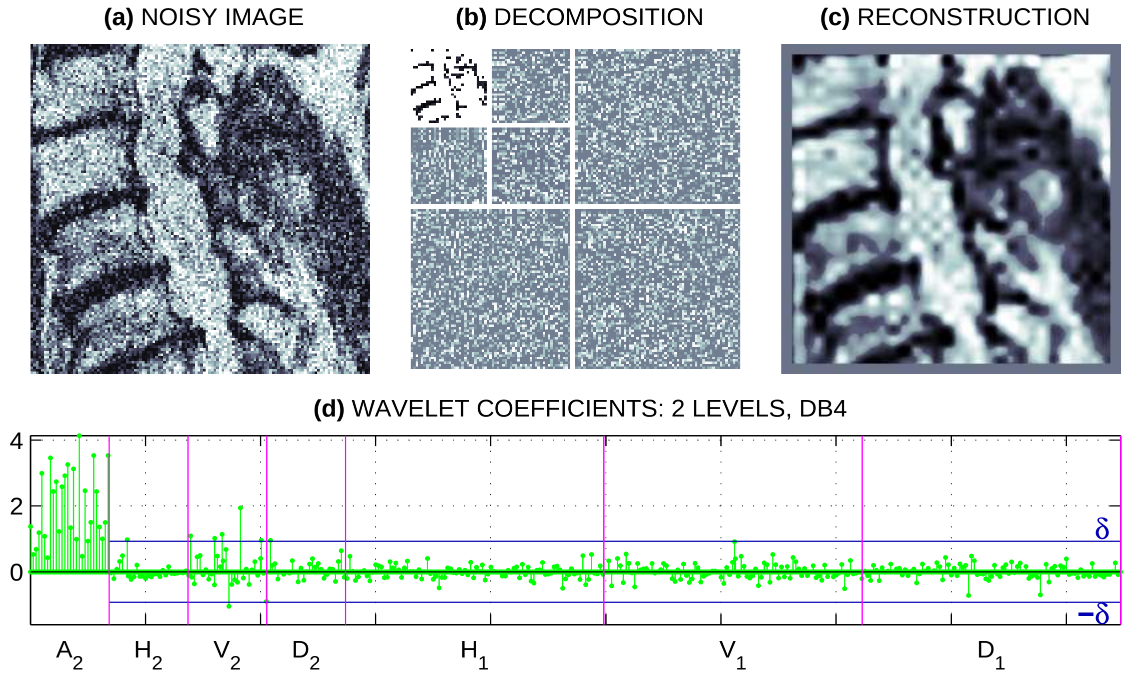


Figure 4: MR image de-noising by thresholding of the wavelet detail coefficients up to the second level presenting **(a)** MR image with the additional random noise, **(b)** decomposition up to the second level using the *db*4 wavelet function, **(c)** image reconstruction after the thresholding of wavelet coefficients, and **(d)** wavelet coefficients of the noisy image and the estimated threshold level $\delta$

# 5 Conclusions

This work provides practical examples of signal and image enhancement and components detection using the wavelet transform along with the enclosed Matlab code. The data we process are a real biomedical ECG signal and a spinal MR image. Detection of signal and image components can be utilised for their classification.

# 6 Acknowledgments

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# References

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Prof. Aleš Procházka, Ing. Eva Hošťálková
Institute of Chemical Technology, Prague
Department of Computing and Control Engineering
Technická 1905, 166 28 Prague 6
Phone: 00420-22435 4198, Fax: 00420-22435 5053
E-mail: A.Prochazka@ieee.org, Eva.Hostalkova@vscht.cz