WAVELET SIGNAL AND IMAGE DENOISING

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Abstract

The paper deals with the use of wavelet transform for signal and image de-noising employing a selected method of thresholding of appropriate decomposition coefficients. The proposed technique is based upon the analysis of wavelet transform and it includes description of global modification of its values. The whole method is verified for simulated signals and applied to processing of biomedical signals representing EEG signals and MR images corrupted by additional random noise.

Introduction

The *wavelet transform* (WT), and particularly its discrete version *DWT*, is a powerful tool of signal processing for its multiresolutional possibilities.Unlike the Fourier transform, the WT is suitable for application to non-stationary signals with transitory phenomena, whose frequency response varies in time [2].

The wavelet coefficients:

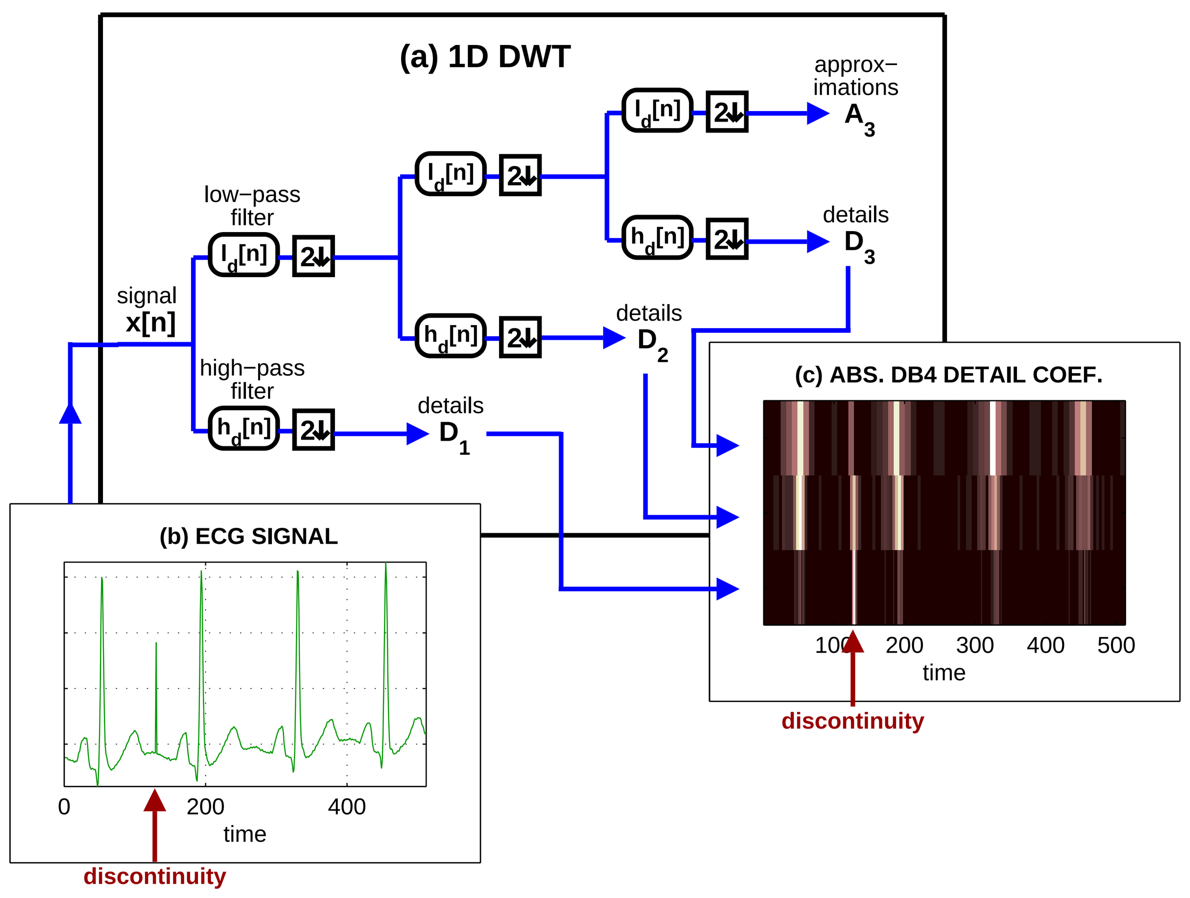
- A measure of similarity in the frequency content between a signal and a chosen wavelet function [2], i.e. a convolution of the signal and the scaled wavelet function (a dilated band-pass filter) [5].
- In practise, computed by *subband coding* algorithm [3]: passing a signal successively through a high-pass and a low-pass filter.
  - The high-pass filter h_d (the wavelet function) → the *approximations* A.
  - The complementary low-pass filter l_d (the scaling function) → the *details* D.

The resolution is altered by the filtering process, and the scale is changed by either *upsampling* or *downsampling* by 2. This is described by the following two equations [4]

$$D_1[n] = \sum_{k=-\infty}^{\infty} h_d[k]\, x[2n-k] \quad (1)$$

$$A_1[n] = \sum_{k=-\infty}^{\infty} l_d[k]\, x[2n-k] \quad (2)$$

where n and k denote discrete time coefficients, x the decomposed signal.



Discontinuity detection in ECG signal applying wavelet analysis displaying **(a)** one-dimensional wavelet subband coding scheme, **(b)** ECG signal with a discontinuity, and **(c)** absolute detail decomposition coefficients using the $db4$ wavelet function

Signal Analysis

Applications of wavelets in signal processing [2]:

- Signal denoising.
- Detecting trends, breakdown points, self-similarities and discontinuities in higher derivatives in signals.

Discontinuity detection in the *Electrocardiogram* (ECG) signal is displayed in the figure above. Detection of a discontinuity in frequency domain corresponds to impulse detection in time domain. The analysis have the following features:

- An artificially generated impulse.
- The $db4$ wavelet (good performance in this case).
- Decomposition up to level 3.

Thresholding

Three steps of *signal denoising* using the DWT [2]:

1. Signal *decomposition* (the wavelet analysis of a noisy signal up to a chosen level N).
2. *Thresholding* of the DWT coefficients (thresholding of the detail coefficients from level 1 to N).
3. Signal *reconstruction* (using the altered detail coefficients from level 1 to N and approximation coefficients of level N).

Donoho's threshold estimate also employed in Matlab δ for denoising with an orthonormal basis is given by [1]

$$\delta = \sigma \sqrt{2 \log L} \quad (3)$$

where the noise is Gaussian with standard deviation σ of the DWT coefficients and L is the number of samples or pixels of the processed signal or image.

Thresholding [1]:

- Level-dependent (a vector of threshold values of length N).
- Global (threshold of a constant value for all decomposition levels).

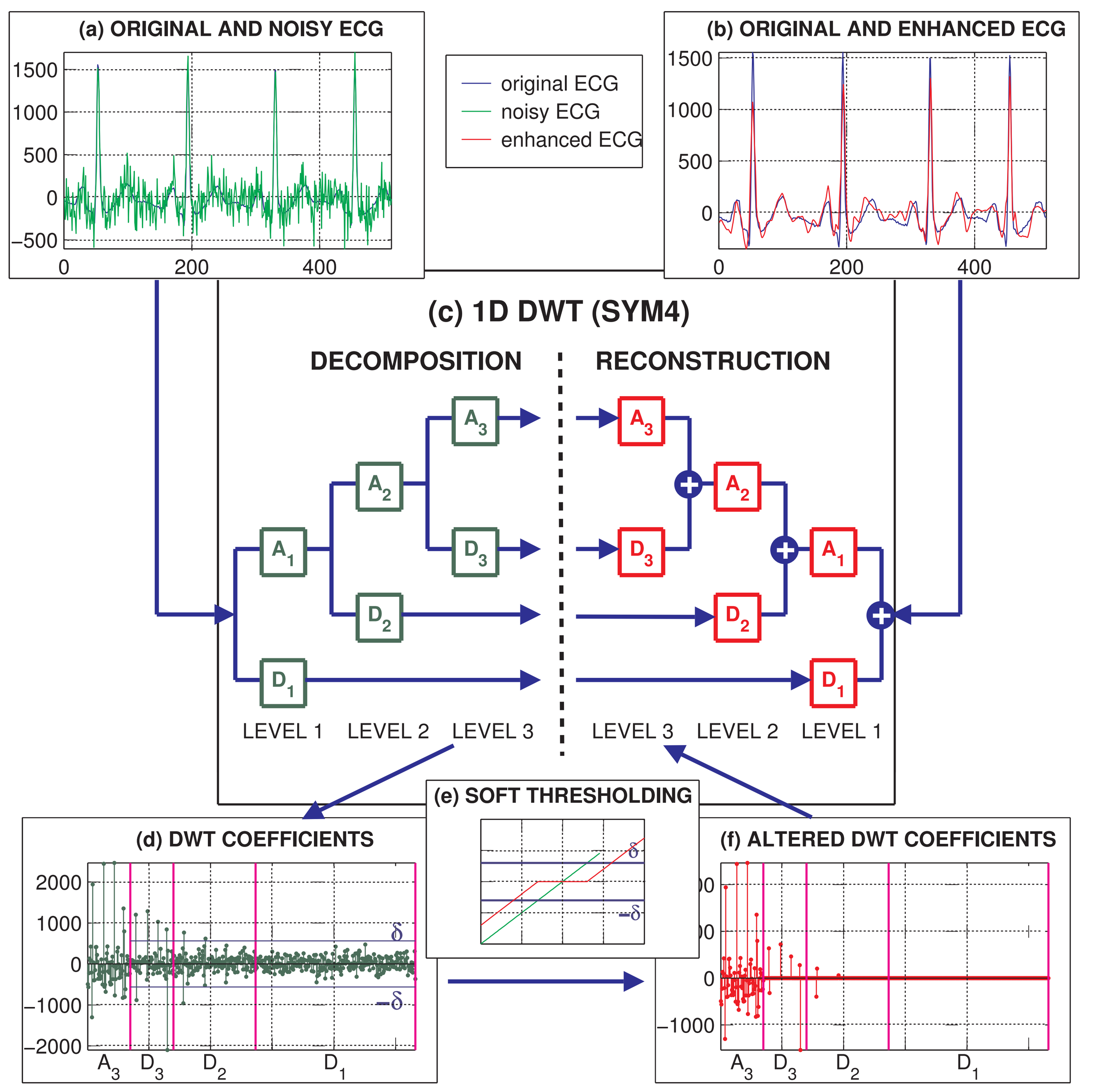
Thresholding [1]:

- Hard:
  - Zeroes out all the signal values smaller than δ.
  - In Matlab by default, used for compression [2].
- Soft:
  - Zeroes out all the signal values smaller than δ and subtracts δ from the values larger than δ.
  - Causes no discontinuities in the resulting signal.
  - In Matlab by default, used for denoising [2].

Denoising of ECG Signals

In this application:

- Artificially added random noise.
- Thresholding of the DWT detail coefficients up to level 3.
- The $sym4$ wavelet (performs better than the $db4$ in this application).
- A soft global threshold δ of an estimated value given by Eq. (3).



ECG signal de-noising by thresholding of wavelet detail coefficients up to the third level presenting **(a)** original and noisy ECG signal, **(b)** original and enhanced ECG signal, **(c)** decomposition and reconstruction up to the third level using the $sym4$ wavelet function, **(d)** wavelet coefficients of the noisy signal and the estimated threshold level δ, **(e)** principles of soft thresholding, and **(f)** altered wavelet coefficients for signal reconstruction

Image Denoising

Applications of wavelets in image processing [2]:

- Detecting edges and textures.
- Watermarking.
- Compression.
- Denoising by thresholding of the DWT coefficients (applying 2D DWT).
- Coding of interesting features for subsequent classification.

To compute 2D DWT, we decompose the approximations at level j to obtain four matrixes of coefficients at level $j+1$, i.e. the *approximations* and the *horizontal* , *vertical* and *diagonal details* of level j+1.

2D DWT decomposition [2]:

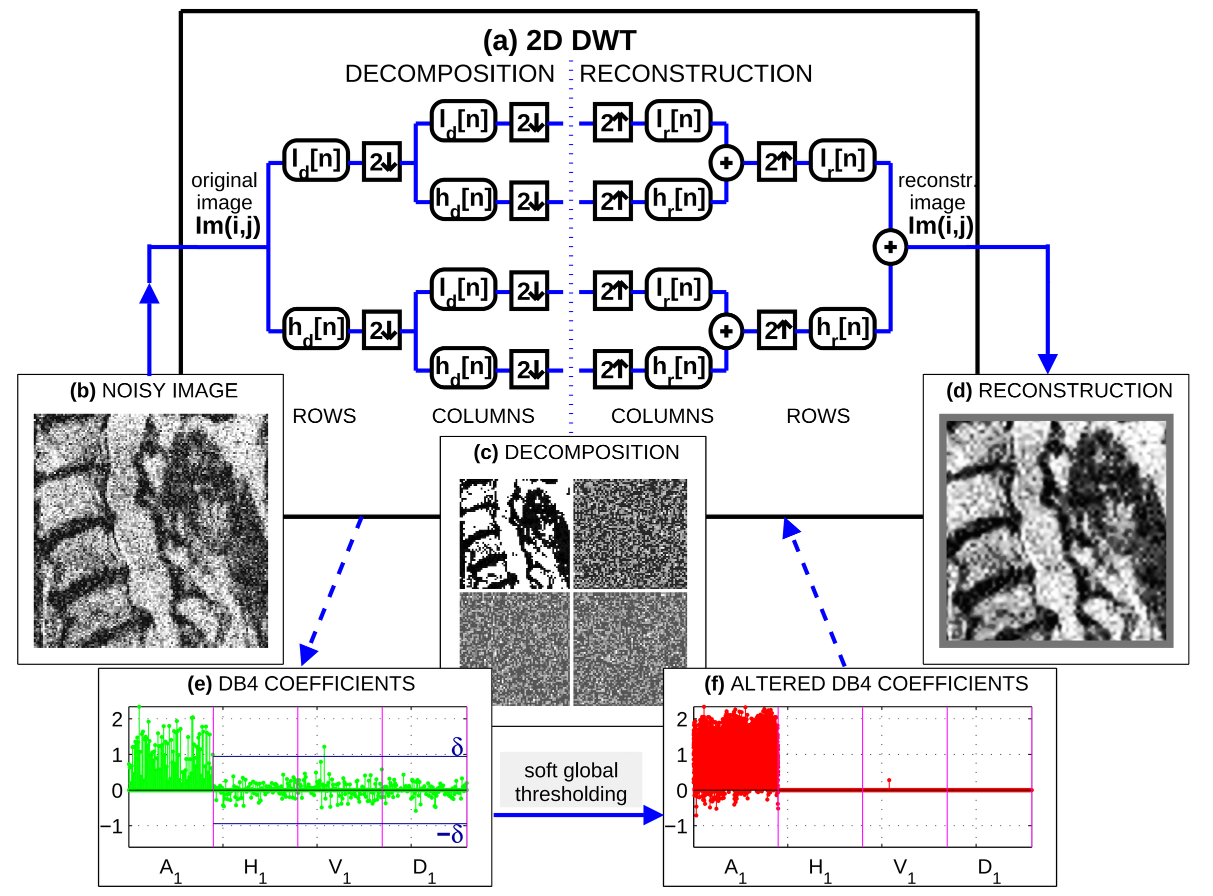
1. Convolvolution of the rows of the image with a low-pass and a high-pass decomposition filter $l_d[n]$ and $h_d[n]$, respectively.
2. Downsampling of the both resulting matrixes by 2 keeping every even column.
3. Filtering of each of the matrixes by their columns using $l_d[n]$ and $h_d[n]$.
4. Downsampling of all four resulting matrixes by 2 keeping every even row to obtain four matrixes of one-level decomposition coefficients.

We can also reconstruct the image by using these coefficients matrixes, upsampling by 2 and the reconstruction filters $l_r[n]$ and $h_r[n]$.

International Conference

TECHNICAL COMPUTING PRAGUE 2006

October 26, 2006

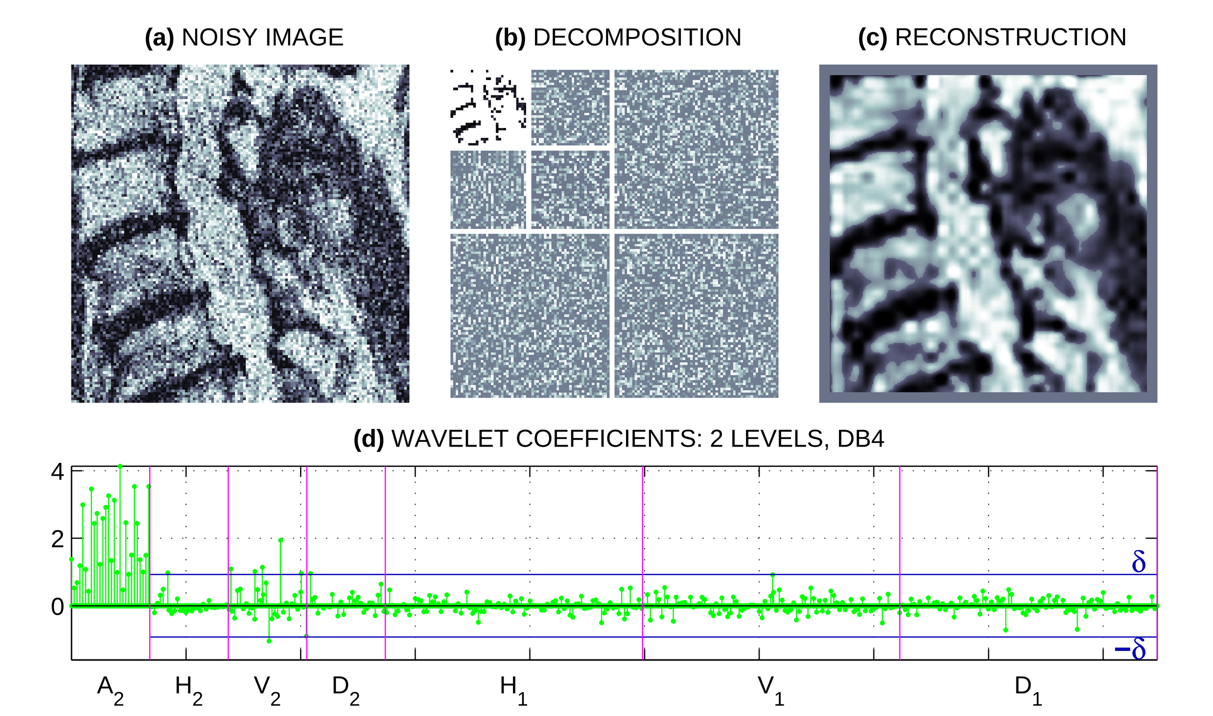


MR image enhancement by thresholding of the wavelet decomposition coefficients displaying **(a)** one-dimensional wavelet subband coding scheme, **(b)** noisy MR image, **(c)** first level decomposition using the $db4$ wavelet function, **(d)** reconstructed and interpolated image, **(e)** absolute $db4$ decomposition coefficients, and **(f)** thresholded decomposition coefficients used for image reconstruction

Denoising of MR Images

Magnetic Resonance (MR) image denoising by thresholding of the wavelet coefficients:

- Artificially added random noise.
- Thresholding of the detail coefficients up to level 2.
- The $db4$ wavelet.
- A soft global threshold δ of an estimated value given by Eq. (3).
- Cubic interpolation of the reconstructed image (The areas along the image boundaries are coloured with grey, hence these pixels would require different handling.).



MR image de-noising by thresholding of the wavelet detail coefficients up to the second level presenting **(a)** MR image with the additional random noise, **(b)** decomposition up to the second level using the $db4$ wavelet function, **(c)** image reconstruction after the thresholding of wavelet coefficients, and **(d)** wavelet coefficients of the noisy image and the estimated threshold level δ

Conclusions

This work provides practical examples of signal and image enhancement and components detection using the one-dimensional and two-dimensional discrete wavelet transform (DWT), respectively. The processed data are a real biomedical ECG signal and a spinal MR image, which have been kindly provided by Dr. Oldřich Vyšata from Neurocenter in Rychnov nad Kněžnou.
In this work, denoising is performed for the both signals contaminated with artificial random noise. Impulse detection is successfully carried out on the ECG signal. Detection of signal and image components can be utilised for their classification.

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