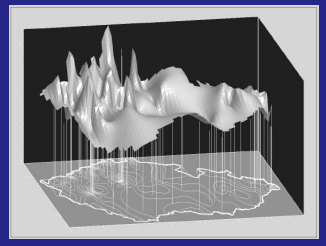


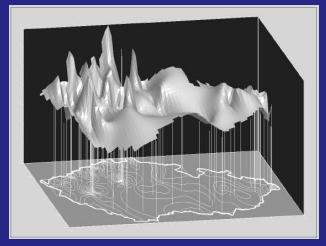
COMPLEX WAVELET TRANSFORM IN BIOMEDICAL IMAGE DENOISING

Eva Hošťálková & Aleš Procházka

Institute of Chemical Technology in Prague
Dept of Computing and Control Engineering
http://dsp.vscht.cz/

ICT PRAGUE

Technical Computing Prague 2007



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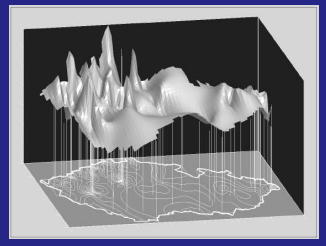

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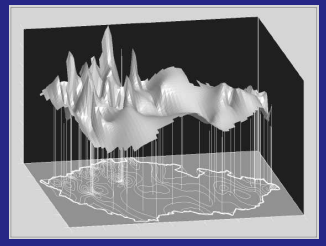

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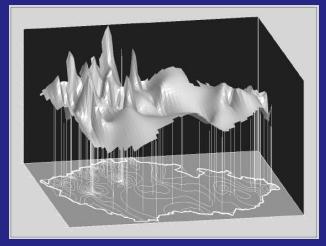

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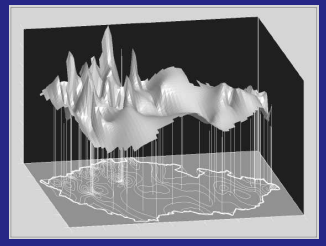


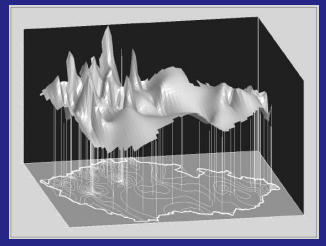
Introduction

Applications of the Wavelet Transform in Image processing

- Noise reduction
- Image compression and coding
- Edge detection
- Feature extraction $\Rightarrow$ segmentation & retrieval
- Restoration of missing or corrupted components

Limitations of the Discrete Wavelet Transform (DWT)

- Zero crossings of the coefficients at a singularity
- Strong shift dependence
- Aliasing $\Leftarrow$ downsampling and non-ideal filters
- Lack of directional selectivity - unable to distinguish between $+45^\circ$ and -45°



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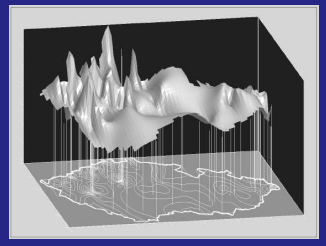


Introduction

Undecimated DWT

- DWT without downsampling.

ADVANTAGES

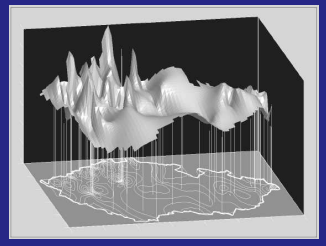
- Shift independence

DISADVANTAGES

- Poor directional selectivity
- Great computation cost

Complex Wavelet Transform (CWT)

- Employs analytic complex wavelets
- $\Rightarrow$ Magnitude-phase representation
  - Large magnitude $\Rightarrow$ presence of a singularity
  - Phase: its position within the support of the wavelet
- $\Rightarrow$ Shift invariance & no aliasing
- In this work: Dual-Tree CWT (DTCWT) by Kingsbury, Selesnick



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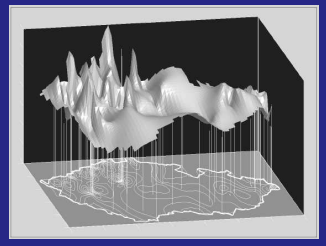

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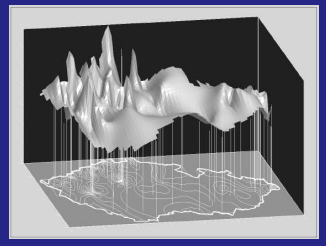


Introduction to DTCWT

Dual Tree Complex Wavelet Transform

- Dual tree (two DWT trees) of real filters $\Rightarrow$ real and imaginary parts of each complex coefficient
- Perfect reconstruction (PR)
- Approx. analytic filters $\Rightarrow$ approx. shift invariance
- Directional selectivity in 2D:

DTCWT

- 6 directional subbands
- $\pm 15^\circ$, $\pm 45^\circ$ and $\pm 75^\circ$

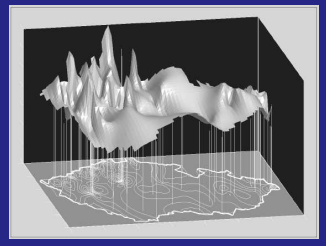
DWT

- 3 directional subbands
- 0°, 45° and 90°

- Limited redundancy 2^d in d-dimensional space

Q-Shift DTCWT

- By Prof. Kingsbury, used in this work
- Q-shift . . . quarter of a sample period shift



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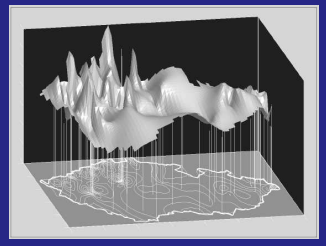
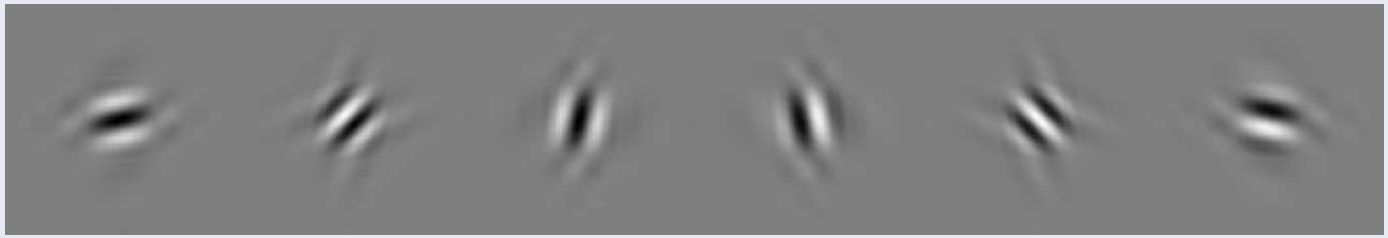


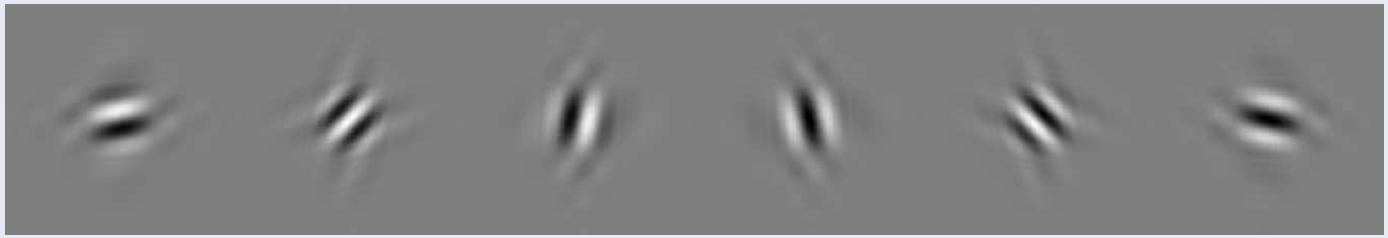
Directional Selectivity of 2D Wavelets

(a) REAL PARTS OF 2D Q−SHIFT COMPLEX WAVELETS



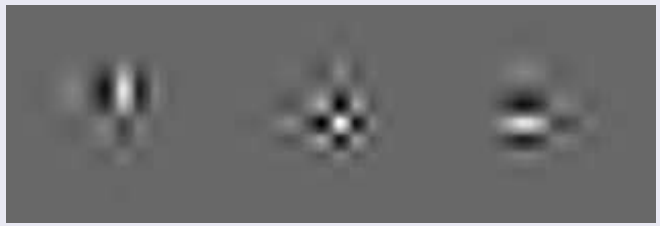
$+15°$ $+45°$ $+75°$ $-75°$ $-45°$ $-15°$

(b) IMAGINARY PARTS OF 2D Q−SHIFT COMPLEX WAVELETS

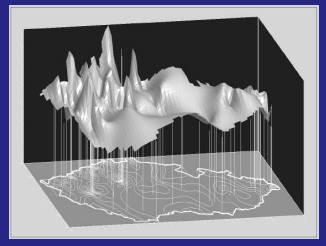


$+15°$ $+45°$ $+75°$ $-75°$ $-45°$ $-15°$

(c) 2D DB4 REAL WAVELETS



$90°$ $(LoHi)$ $45°$ $(HiHi)$ $0°$ $(HiLo)$



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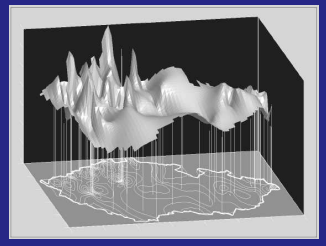
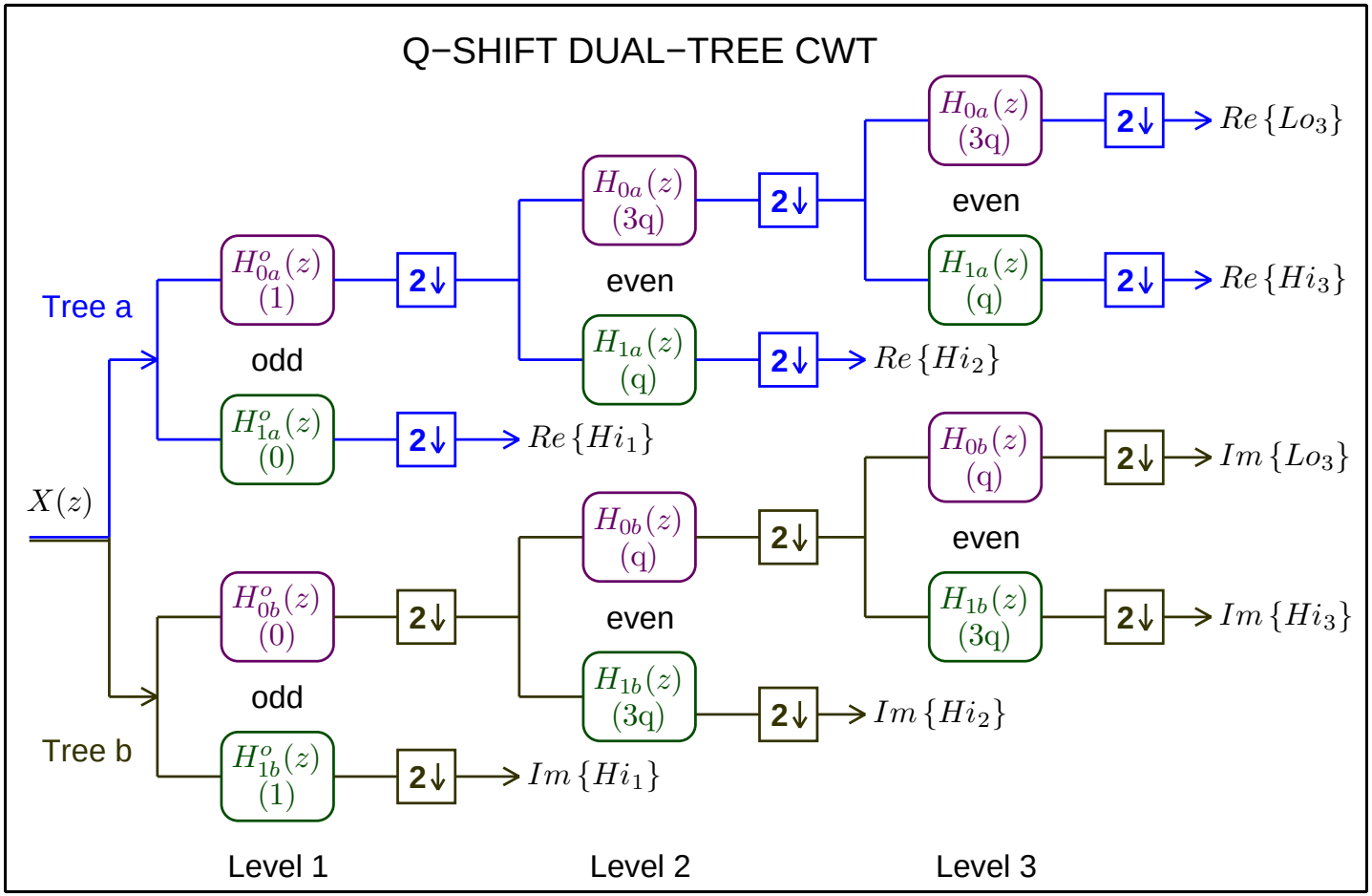
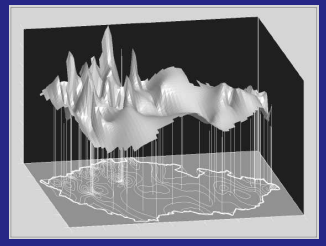


Q-Shift Dual Tree CWT

Q-Shift DTCWT 3-level analysis scheme



Red - lowpass filters, green - highpass filters, $2\downarrow$ - downsampling by 2



Q-Shift Dual Tree CWT

Q-Shift DTCWT

- Level 1: any orthogonal/biorthogonal set of filters
- Beyond level 1: even-tap Q-shift filters
- Both trees - same frequency response
- Conjugate symmetry $\Rightarrow$ linear phase
- Individual asymmetry $\Rightarrow$ orthonormal PR

Orthonormal Set of Q-Shift Filters

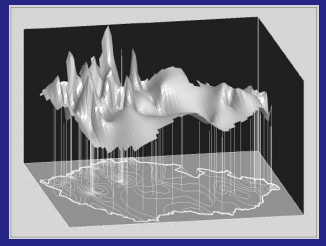
- Filters in tree b - reverse of the filters in tree a

$$h_{0b}(n) = h_{0a}(N-1-n)$$

- Synthesis filters - reverse of the analysis filters

$$g_{0a}(n) = h_{0a}(N-1-n)$$

where $n = 0, \ldots, N-1$ and N is the filter length



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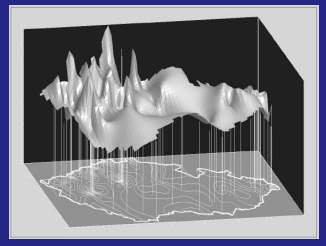


Q-Shift Dual Tree CWT

Analytic Wavelets

- Complex wavelet $\psi_c(t) = \psi_r(t) + j \cdot \psi_i(t)$ is analytic when

$$\psi_i(t) = HT\{\psi_r(t)\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\psi_r(t)}{t-\tau} d\tau = \psi_r(t) \frac{1}{\pi\, t}$$

where t,τ is continuous time

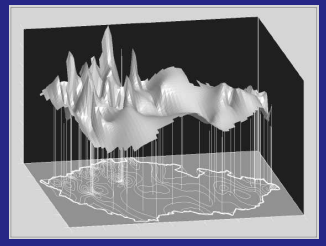
- Fourier transform of a Hilbert transform pair

$$H_i(\omega) = FT\{HT\{\psi_r(t)\}\} = -j \cdot sgn(\omega) H_r(\omega)$$

where ω denotes frequency and j the complex unit

Implications

- Single sided spectrum $\Rightarrow$ no aliasing $\Rightarrow$ shift invariance
- Impossible with compact support! $\Rightarrow$ only approximately analytic



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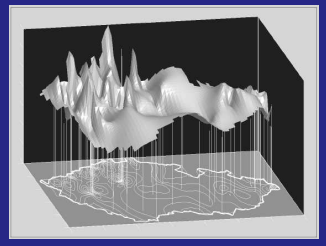

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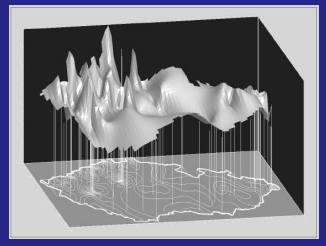

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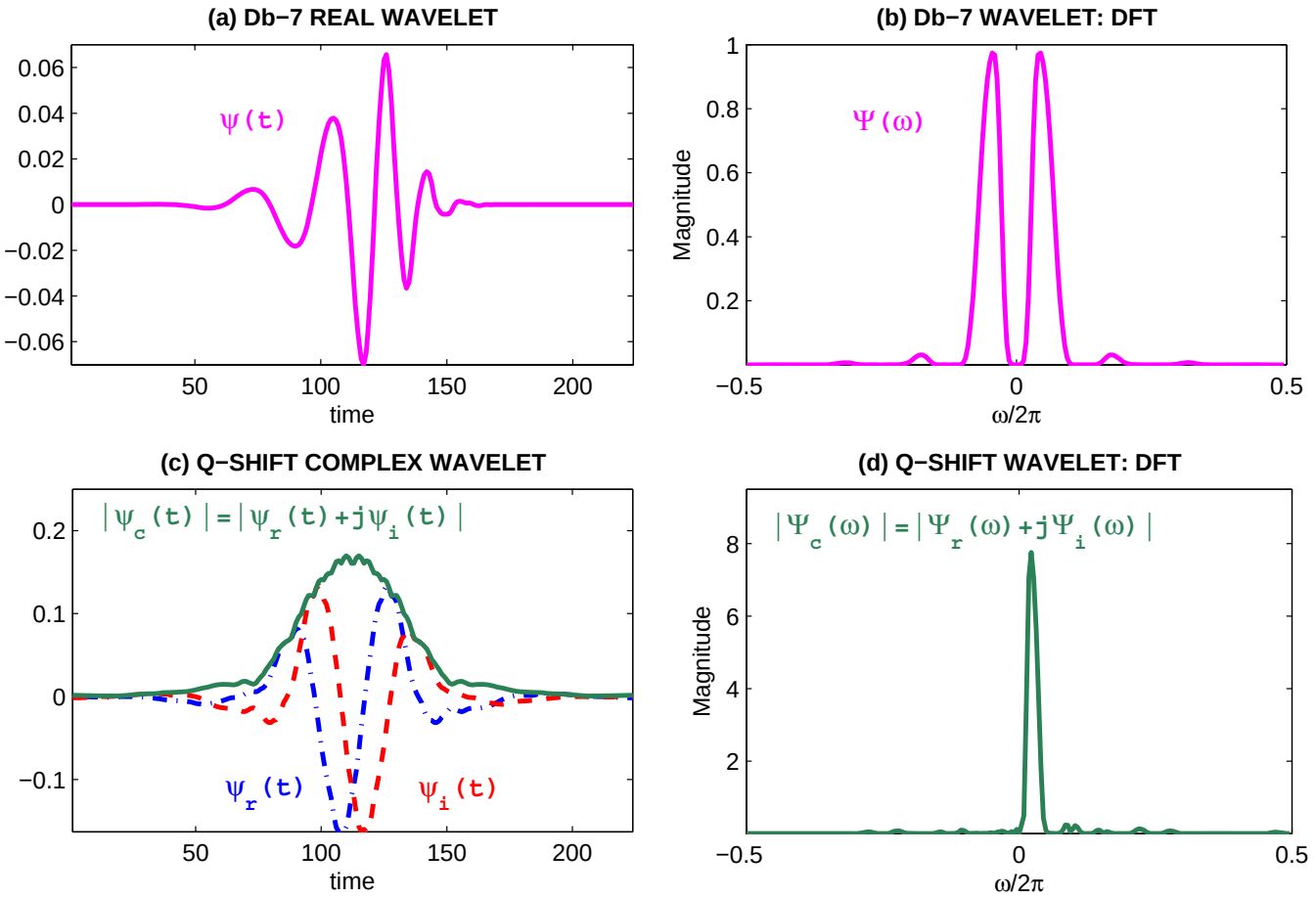
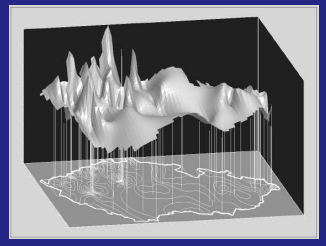
Frequency Spectra of a Real and an Analytic Wavelet



4 levels, 14-tap filters: Daubechies for DWT and q-shift for DTCWT.



Q-Shift Dual Tree CWT

Half-Sample Delay Condition

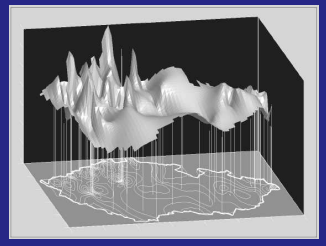
- Half of a sample period difference between filters in tree a and $b \Rightarrow$ analytic
- In the Fourier domain:

$$\text{MAGNITUDE} \quad |H_{0b}(e^{j\omega})| = |H_{0a}(e^{j\omega})|$$

$$\text{PHASE} \quad \angle H_{0b}(e^{j\omega}) = \angle H_{0a}(e^{j\omega}) - 0.5\omega$$

Q-Shift Filters Design

- Fulfill the phase condition only approximately
- $\Rightarrow$ Only approx. shift independent
- Group delays $\simeq \frac{1}{4}$ and $\frac{3}{4}$ of a sample period



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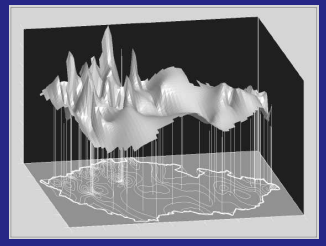

Q-Shift Dual Tree CWT

Approximate Shift Invariance

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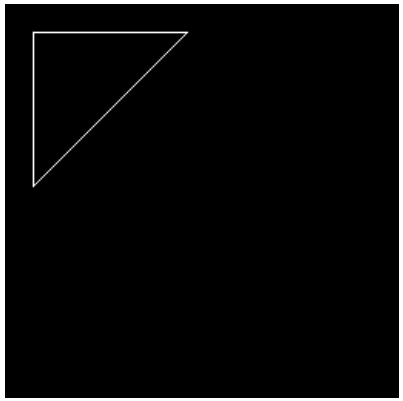
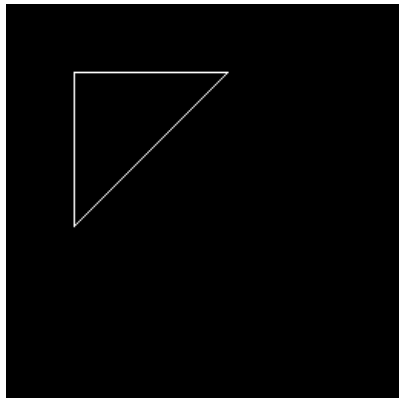


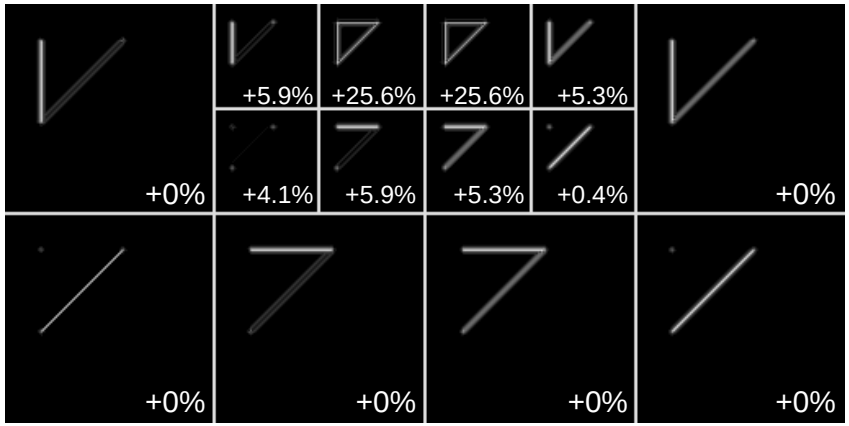
(b) AFTER THE SHIFT



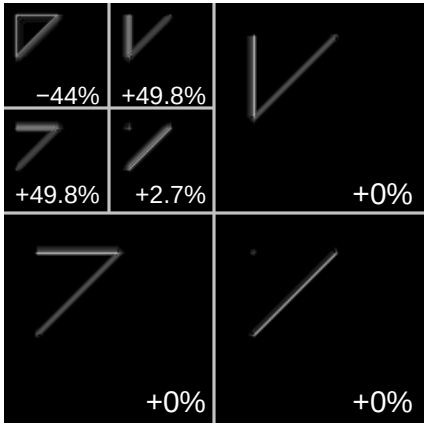
(c) CHANGE OF SUBBAND ENERGY

	DTCWT	DWT
level 1	0 %	0 %
level 2	4.5 %	34.1 %
level 3	5.7 %	118.8 %
level 4	6.2 %	77.6 %
level 4 (LoLo)	27.7 %	95.7 %

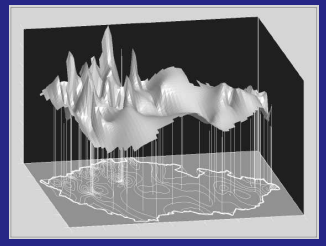
(d) 2−LEVEL DTCWT AFTER THE SHIFT



(e) DWT AFTER THE SHIFT



Percentual changes of subband energy.



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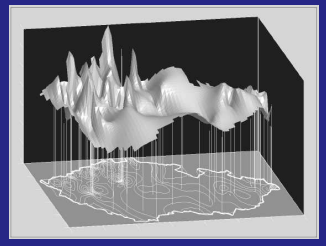
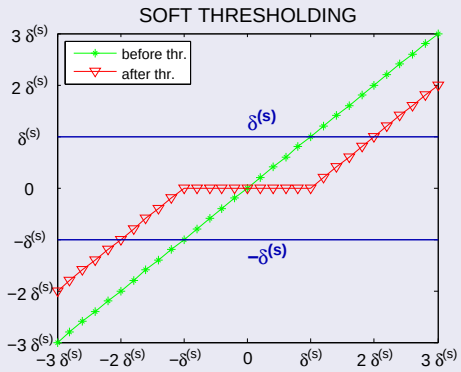


Denoising Technique

Denoising

- Computed Tomography (CT) images
- Suppressing lower energy wavelet coefficients (noise)
- Soft universal wavelet shrinkage

Soft Thresholding

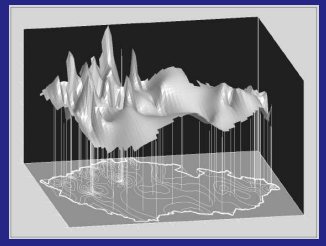


- Thresholding magnitudes of complex coefficients
  - Vary slowly
  - Not distorted by aliasing
- Signal to noise ratio [dB]

$$SNR = 20 \cdot log_{10} \frac{I_{max} - I_{min}}{\hat{\sigma}_n}$$

I_{max}, I_{min} . . . max. and min. pixel value, resp.

$\hat{\sigma}_n$ . . . noise standard deviation estimate (from areas - no image component)



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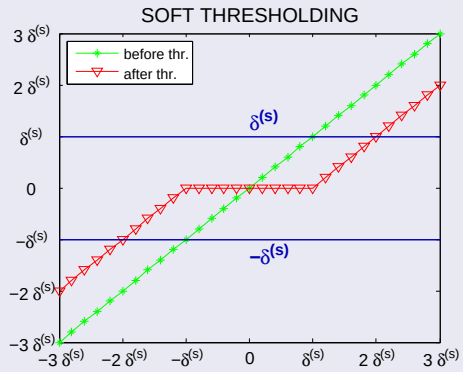

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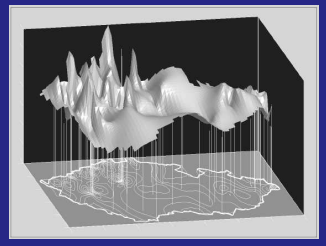


Denoising Technique

Median Absolute Deviation (MAD)

- Noise standard deviation estimator (for 1D signal)

$$\hat{\sigma}_{(mad)} = \frac{median\{|W_{1,0}|,|W_{1,1}|,\ldots,|W_{1,N/2-1}|\}}{0.6745}$$

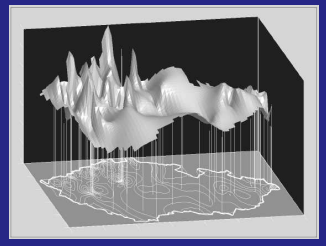
  where $W_{1,l} \ldots$ l-th wavelet coefficient of level 1
- Smallest scale w. coefficients - noise dominated
- For independent identically distributed Gaussian noise
- Robust against large deviations $\Rightarrow$ noise variance

Donoho Threshold

- Donoho soft universal threshold

$$\delta^{(s)} = \sqrt{2\,\hat{\sigma}^2_{(mad)}\,log(N)}$$

  where N is no. coefficients



Denoising Technique

Median Absolute Deviation (MAD)

- Noise standard deviation estimator (for 1D signal)

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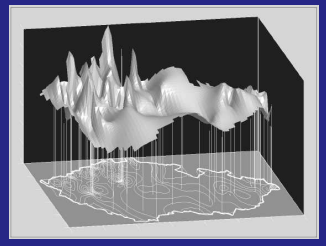

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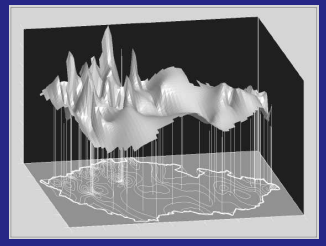
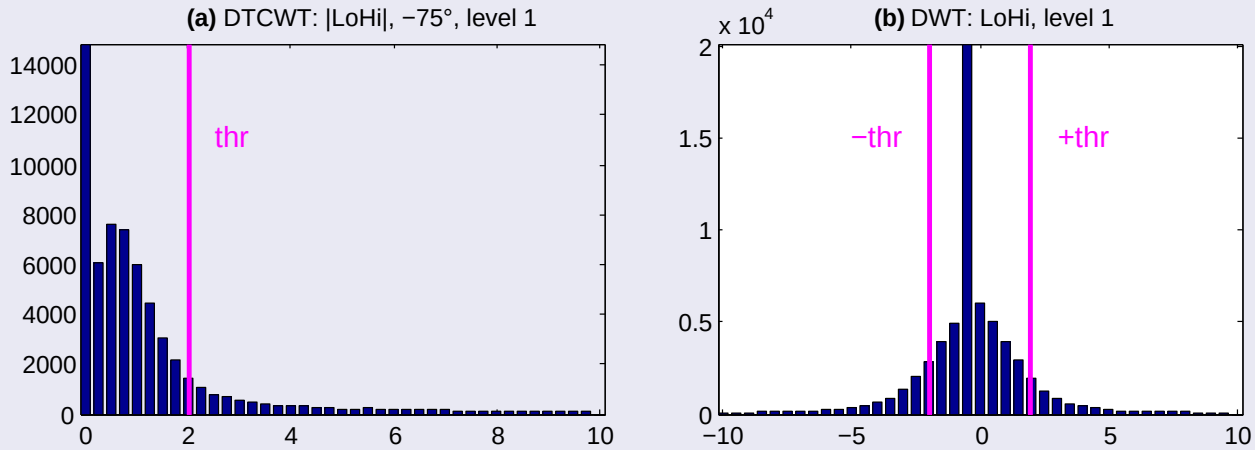
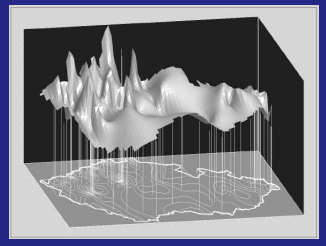


Denoising Results

Histograms of Wavelet Coefficients

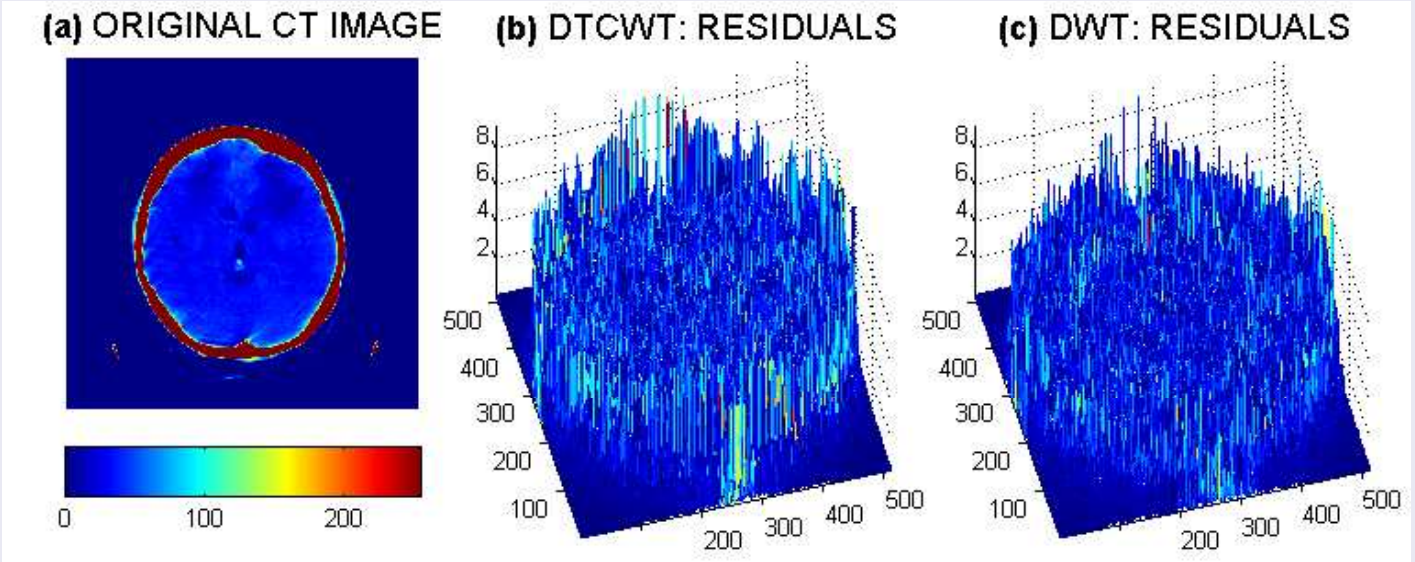


4 levels, 14-tap filters: Daubechies for DWT and q-shift for DTCWT

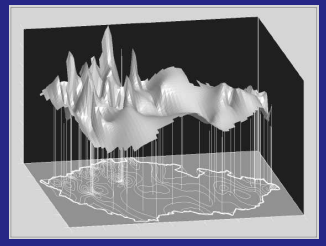


Denoising Results

Residuals After Denoising



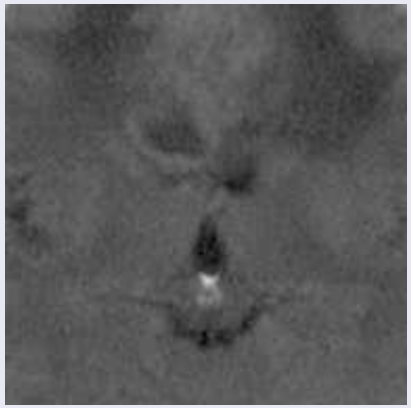
Axial brain CT image.



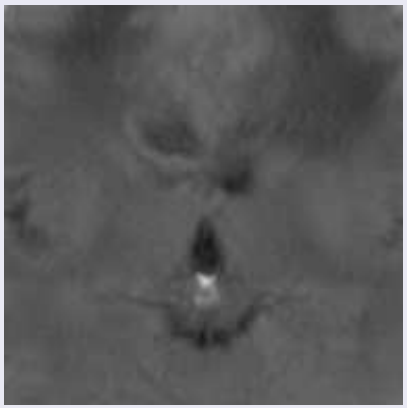
Denoising Results

CT Image Denoising (Cuts)

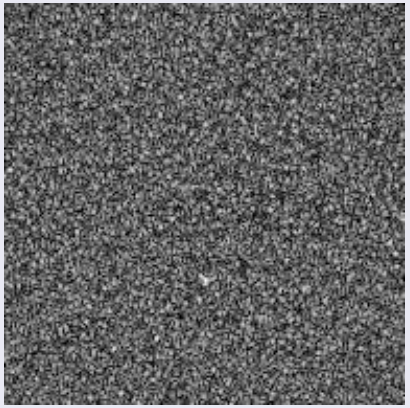
(a) ORIGINAL 44.15 dB



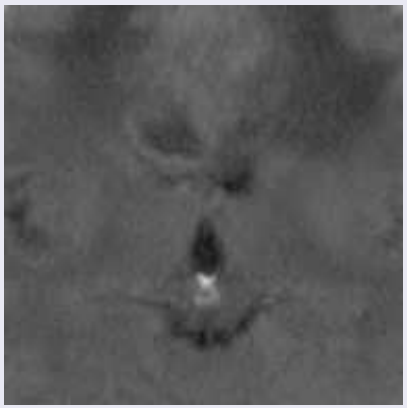
(b) DTCWT DEN. 46.38 dB



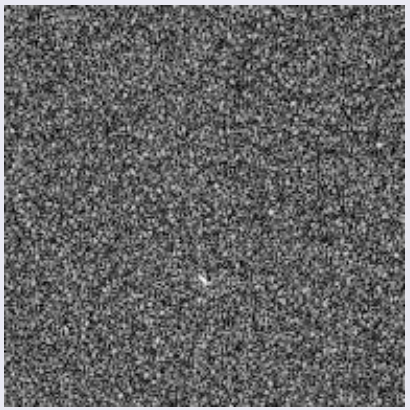
(c) DTCWT: RESIDUALS



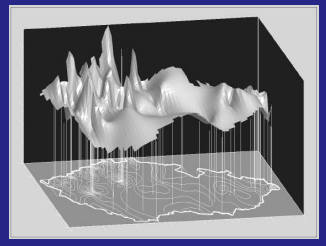
(d) DWT DEN. 46.1 dB



(e) DWT: RESIDUALS



Donoho universal soft threshold with MAD estimate.
Similar SNR results for 2- and 3-level decomposition.



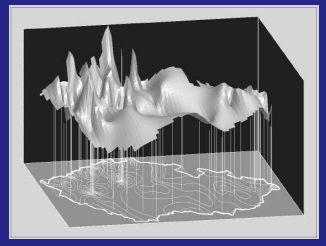
Conclusions

DTCWT surpasses DWT

- Shift dependence (reduced aliasing)
- Directional selectivity
- No zero-crossing of the coefficients at a singularity

Future Study

- DTCWT in biomedical image denoising and enhancement
- Probability distribution of noise and of its wavelet coefficients in these images
- Wavelet shrinkage techniques



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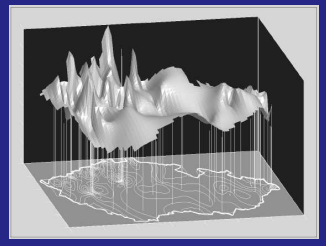

Further Reading

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