WAVELET-BASED HIDDEN MARKOV TREES FOR IMAGE NOISE REDUCTION

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In the field of signal processing, the Discrete Wavelet Transform (DWT) has proved a very useful tool for recovering signals from additive Gaussian noise by the means of wavelet thresholding. During this procedure, wavelet coefficients with small magnitudes are set to zero, however, usually without taking into account their mutual dependencies. The Hidden Markov Models (HMM) are designed to capture such dependencies through modelling the statistical properties of the coefficients. Apart from noise reduction, the HMM are successfully used in edge detection, texture recognition, and other applications.

Markovian dependencies tie together the hidden states assigned to the coefficients rather than their values, which are thus treated as independent of all variables given the hidden state. Modelling of the full joint probability distribution is computationally expensive and decreases robustness of the model. Thanks to the approximate de-correlating ability of the DWT, the most significant correlations are the parent-child dependencies running across scale. This rationale is implemented by the Hidden Markov Trees (HMT) reflecting sparsity and persistence of the coefficients.

The *sparsity* property is entailed by the shape of the marginal probability distribution of each wavelet coefficient value, which is in real images peaky and heavy tailed with relatively few large coefficients and many small ones. Hence the marginal distribution may be modeled as a 2-component mixture of Gaussian conditional distributions with a small and a large variance. Each of the two distributions is associated with one of the two hidden states.

The *persistence* property denotes strong parent-child relations in the wavelet decomposition hierarchy. The relative size of the coefficients propagates through their children across scale. To describe such dependencies, the 2-state model uses the state transition probabilities relating the hidden state of a parent and that of its children.

In our paper, we process simulated intensity images with additional Gaussian noise. The DWT wavelet coefficients are modeled using three independent HMT models. In this way, we *tie* together all trees belonging to each of the three detail subbands. The models are adjusted to the noisy data using the iterative expectation-maximization (EM) training algorithm. The computation results are the conditional mean estimates of the original image coefficients given the noisy observation coefficients. We then reconstruct the image from these estimated wavelet coefficients and the unchanged scaling coefficients.

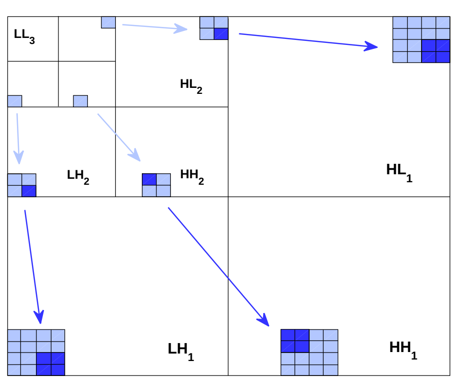

Figure 1: The persistence property of wavelet coefficients. In the 2-dimensional decomposition hierarchy, each parent coefficient has four children.