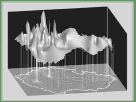

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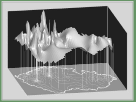
WAVELET-BASED HIDDEN MARKOV TREES FOR IMAGE NOISE REDUCTION

Eva Hošťálková & Aleš Procházka

Institute of Chemical Technology, Prague
Dept of Computing and Control Engineering

Technical Computing Prague 2008



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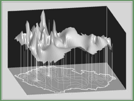

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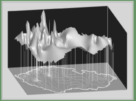

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- Image enhancement
- Preprocessing step to other techniques (e.g. segmentation, edge detection)

Noise Reduction via Wavelet Shrinkage

+ Recovering signals from additive Gaussian noise
− Thresholding w. coefficients without considering their dependencies ⇒ artifacts, blurred edges

Noise Reduction via Wavelet-Based HMTs

- Hidden Markov Tree (HMT) models
- Aiming to capture these dependencies

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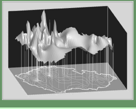

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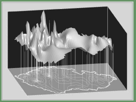

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Other Applications of Wavelet-Based HMTs

- Image segmentation (texture features)
- Edge detection, signal prediction etc.

Image Data

- The mandrill image, a 240×240 cut out, corruption by iid Gaussian noise

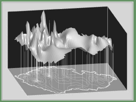

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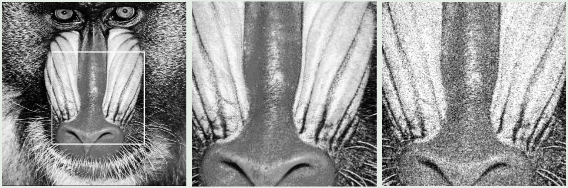
Image Data

(a) MANDRILL IMAGE (b) CUT OUT (c) NOISY IMAGE



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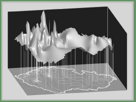

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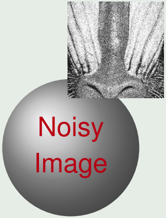

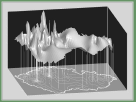

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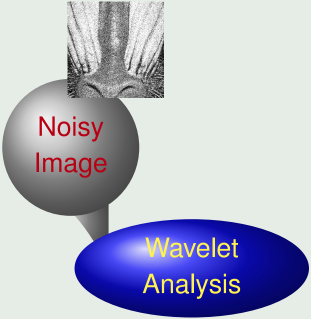
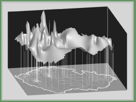


- Image
  - $LoLo_1$
    - $LoLo_2$
    - $LoHi_2$
    - $HiLo_2$
    - $HiHi_2$
  - $LoHi_1$
  - $HiLo_1$
  - $HiHi_1$



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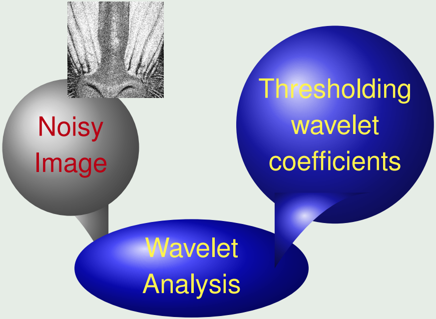
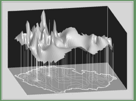


Image → $LoLo_1$ → $LoLo_2$, $LoHi_2$, $HiLo_2$, $HiHi_2$

Image → $LoHi_1$

Image → $HiLo_1$

Image → $HiHi_1$



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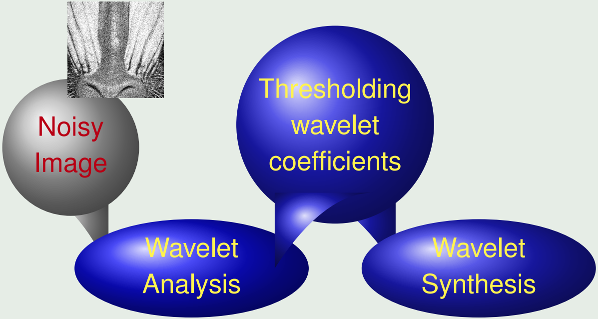
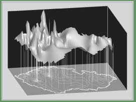


Image → *LoLo*$_1$ → *LoLo*$_2$, *LoHi*$_2$, *HiLo*$_2$, *HiHi*$_2$; *LoHi*$_1$; *HiLo*$_1$; *HiHi*$_1$

LoLo$_1$, *LoHi*$_1$, *HiLo*$_1$, *HiHi*$_1$ → *LoLo*$_2$; *LoLo*$_2$, *LoHi*$_2$, *HiLo*$_2$, *HiHi*$_2$ → Image



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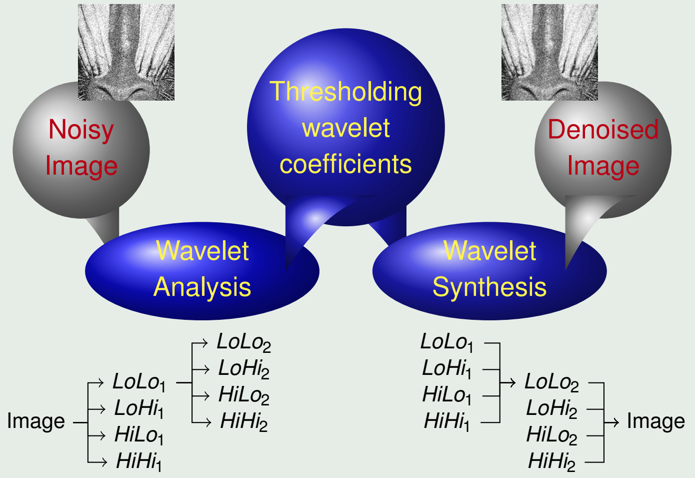

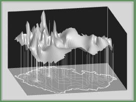

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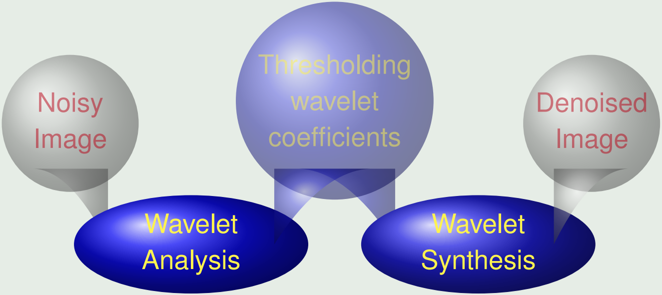

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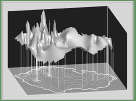

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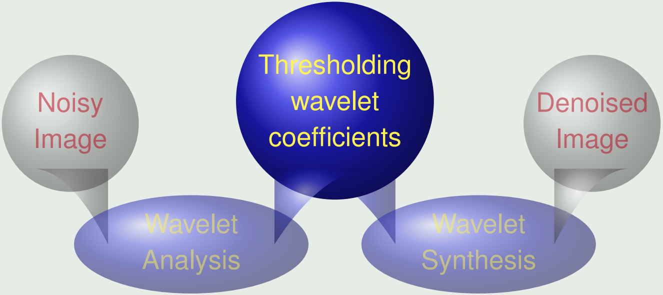

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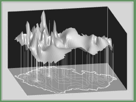

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NormalShrink

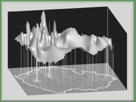
- Subband-adaptive threshold computation

- MAD estimate of iid Gaussian noise std. deviation

$$\hat{\sigma}_n{}^{mad} = median\{|w_1^{hh1}|, |w_2^{hh1}|, \ldots, |w_{N/4}^{hh1}|\}/0.6745$$

  - $\mathbf{w}^{\mathbf{hh1}}$ ... HiHi wavelet coefficient of level 1 (noise dominated), N ... image size
  - Robust against large deviations of noise variance

- Soft shrinkage function



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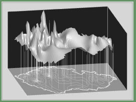

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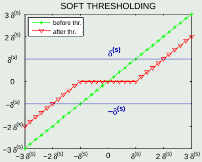

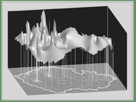

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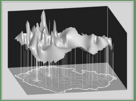

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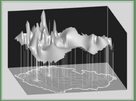
Interdependencies of the DWT Coefficients

- Shrinkage methods assume the DWT to de-correlate signals thoroughly (incorrect)
- DWT coefficients reveal clustering and persistence

Persistence & Clustering Properties

- Clustering within scale
  - We expect large (small) coefficients in the vicinity of a large (small) coef.
- Persistence across scale
  - Parent-child relations
  - We expect a large (small) parent coef. to have large (small) children
- Both captured by the HMT models



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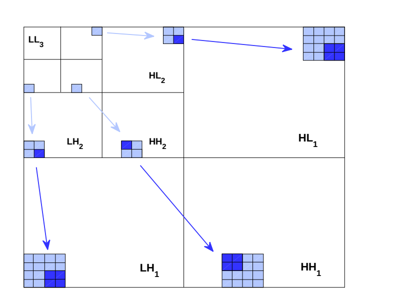
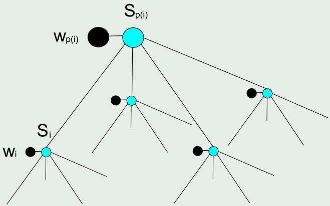
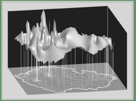




- 2D: each parent coefficient $p(i)$ has four children i
- HMT connects the hidden states S_i, $S_{p(i)}$ - not the actual coefficients values w_i, $w_{p(i)}$



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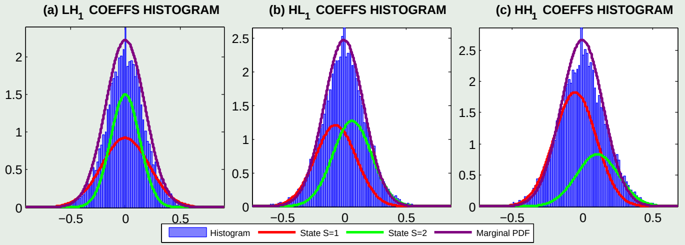
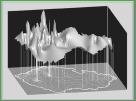


Probability Distribution of Wavelet Coefficients

- Non-Gaussian distribution (peaky and heavy tailed)
- M-component mixture of conditional G. distributions $G(\mu_{i,m}, \sigma^2_{i,m})$ associated with hidden states $S_i = m$
- For $M = 2$ (2-state models) $m = 1,2$



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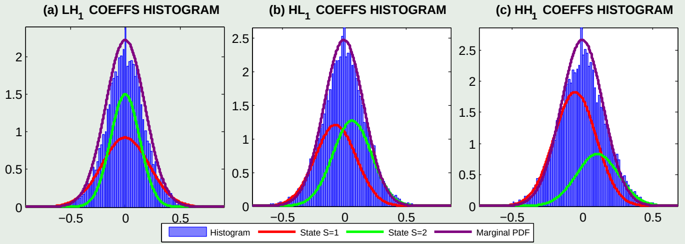

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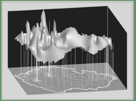

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The Overall PDF

$$f(w_i)=p(S_i=m)\, f(w_i \mid S_i=m)$$

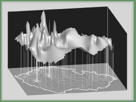
- $p(S_i=m)\dots$ PMF of the hidden states $\sum_{m=1}^{M} p(S_i=m) = 1$
- $f(w_i \mid S_i = m)\dots$ conditional probability of the coefficients value w_i given the state S_i corresponds to $G(\mu_{i,m}, \sigma^2_{i,m})$

Transition Probabilities

- Children hidden states S_i given the parent state $S_{p(i)}$

$$f(S_i=m \mid S_{p(i)}=n)=\begin{bmatrix} f(S_i=1 \mid S_{p(i)}=1) & f(S_i=1 \mid S_{p(i)}=2) \\ f(S_i=2 \mid S_{p(i)}=1) & f(S_i=2 \mid S_{p(i)}=2) \end{bmatrix}$$

- For $M=2$ (2-state model)
- Persistence $\Rightarrow f_{1,1} >> f_{2,1},\ f_{2,2} >> f_{1,2}$



Wavelet-Based HMT Models

The Overall PDF

$$f(w_i) = p(S_i = m)\, f(w_i \mid S_i = m)$$

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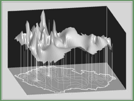
- For $M = 2$ (2-state model)
- Persistence $\Rightarrow f_{1,1} >> f_{2,1},\ f_{2,2} >> f_{1,2}$



Wavelet-Based HMT Models

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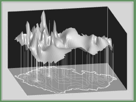
HMT Training

- Tying within subbands to prevent model overfitting $\Rightarrow$ 3 independent HMT trees
- Expectation Maximization (EM) training algorithm



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WAVELET HMT FOR NOISE REDUCTION

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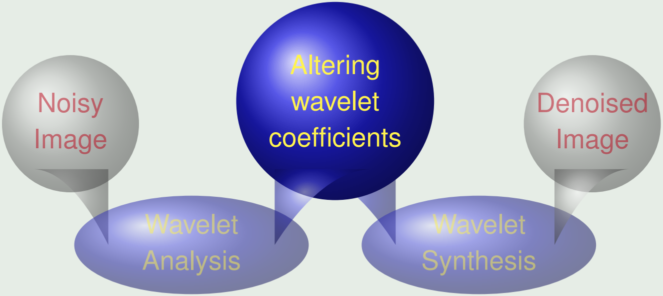
Via HMT models



The Haar wavelet transform

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Additional Noise Model

- Wavelet domain, a noisy w. coefficient observation w_i

$$w_i = y_i + n_i$$

$\mathbf{y}$ . . . desired noise-free signal, $\mathbf{n}$ . . . iid Gaussian noise

Noise Reduction

- Conditional mean estimate of y_i, given the observed coefficients $\mathbf{w}$ and the HMT model parameters θ

$$E[y_i \,|\, \mathbf{w}, \theta] = \sum_{m=1}^{M} p(S_i = m) \cdot \frac{\sigma_{i,m}^2}{\sigma_n^2 + \sigma_{i,m}^2} \cdot w_i \qquad (1)$$

  - $p(S_i = m)$, $\sigma_{i,m}$ . . . obtained from the HMT model
  - σ_n . . . noise std. deviation (MAD estimate)

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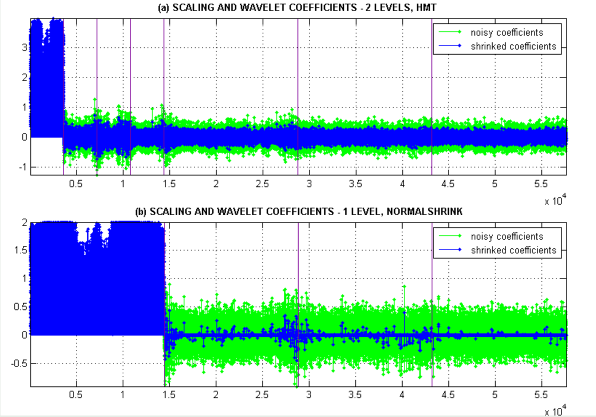
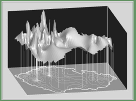
Altering Wavelet Coefficients



- Noise reduction via HMT and NormalShrink



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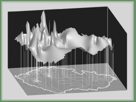

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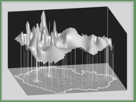
Residual Images Parameters in Our Experiments

Noise		*NormalShrink*		*HMT*	
μ_n $[10^{-2}]$	σ_n^2 $[10^{-2}]$	μ $[10^{-2}]$	σ^2 $[10^{-2}]$	μ $[10^{-2}]$	σ^2 $[10^{-2}]$
5.00	3.00	0.04	2.18	1.12	**0.60**
0.00	1.00	0.00	1.12	0.16	**0.32**
5.00	1.00	0.46	1.04	1.00	**0.32**

- Residual image - difference between the noise reduction result and the original image



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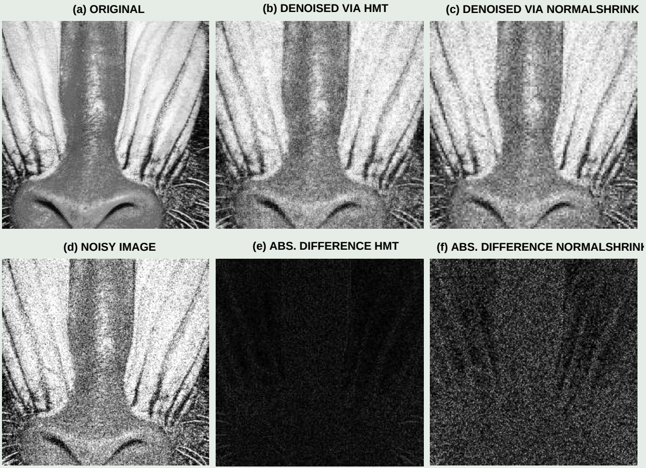
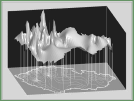
Noise Reduction Results





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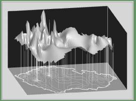
Noise Reduction Experiments

- HMT models outperform the NormalShrink method (at the expense of greater computation cost)
- NormalShrink causes artifacts and blurs edges

Future Work

- Our experiment have been very limited so far
- Next step: Carry out more experiments on biomedical image data



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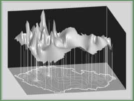

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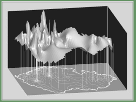

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