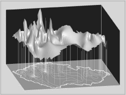
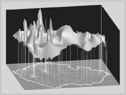


WAVELET DE-NOISING AND GRADIENT ENHANCEMENT IN BIOMEDICAL IMAGE PROCESSING

Aleš Procházka, Eva Hošťálková, and Oldřich Vyšata

Institute of Chemical Technology, Prague
Dept of Computing and Control Engineering

VIIP 2008
Mallorca



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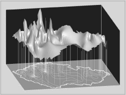


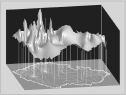
Introduction

Image Edges

- Most important for image perception
- Abrupt changes of intensity (high frequencies)
- Problems: blurring & noise

Applications of Edge Detection

- Image enhancement
- Image segmentation (indexing of objects)
- Image recognition (databases)



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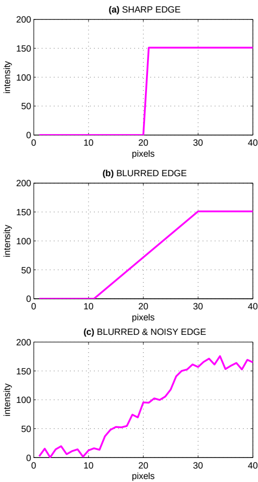

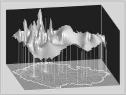

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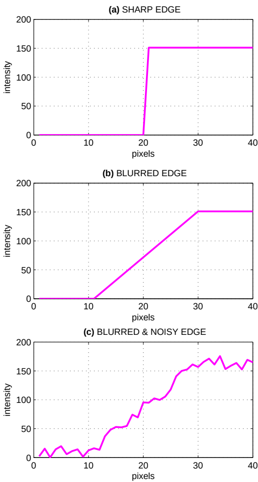
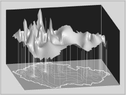
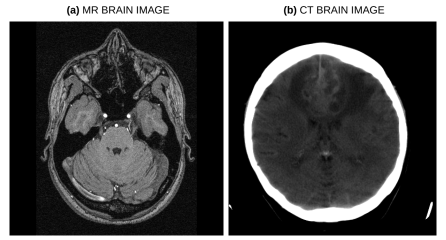




Introduction

Biomedical Image Data

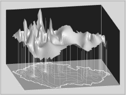
- Magnetic Resonance (MR) images
- Computed Tomography (CT) images



(a) MR BRAIN IMAGE **(b)** CT BRAIN IMAGE

Prior to Edge Detection

- Noise reduction by wavelet coefficients shrinkage



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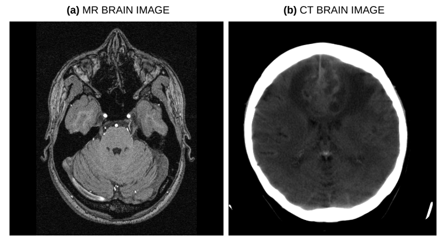

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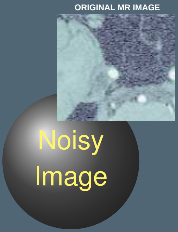
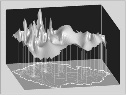
Image De-Noising

Wavelet Shrinkage Algorithm



ORIGINAL MR IMAGE

Noisy Image



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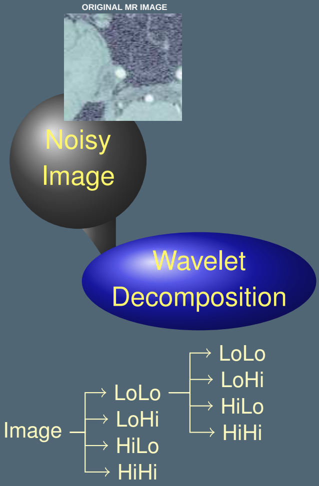


ORIGINAL MR IMAGE

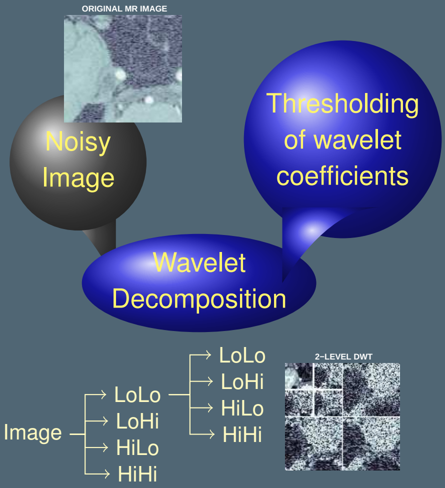
Noisy Image

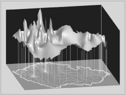
Wavelet Decomposition

- Image
  - LoLo
    - LoLo
    - LoHi
    - HiLo
    - HiHi
  - LoHi
  - HiLo
  - HiHi

Image De-Noising

Wavelet Shrinkage Algorithm





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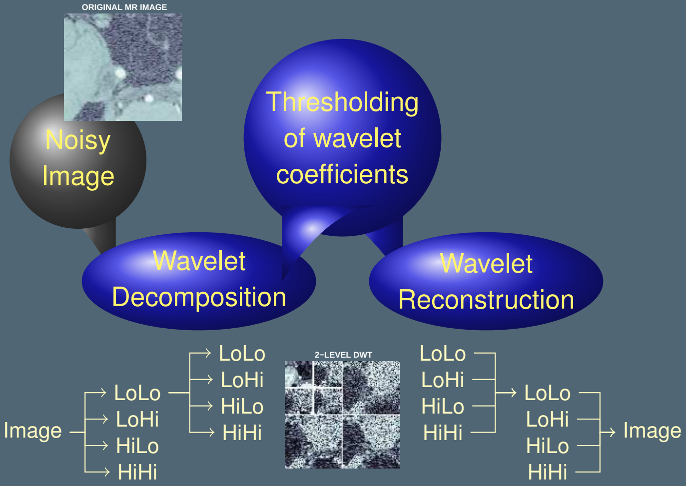


Noisy Image

Thresholding of wavelet coefficients

Wavelet Decomposition

Wavelet Reconstruction

Image → LoLo, LoHi, HiLo, HiHi

LoLo → LoLo, LoHi, HiLo, HiHi

LoLo, LoHi, HiLo, HiHi → LoLo

LoLo, LoHi, HiLo, HiHi → Image

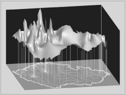
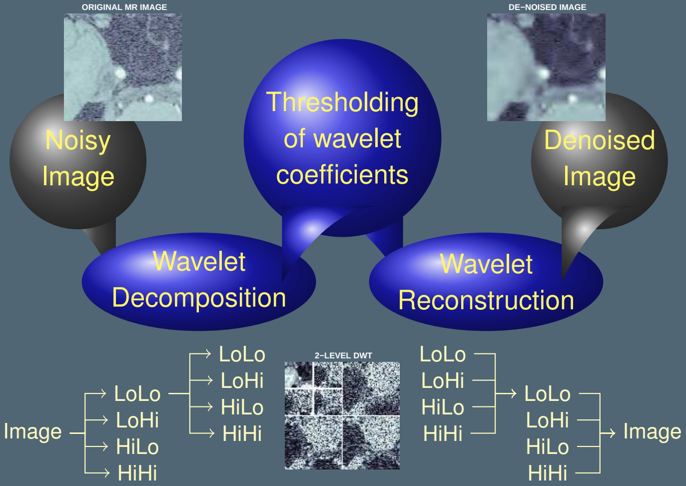


Image De-Noising

Wavelet Shrinkage Algorithm



ORIGINAL MR IMAGE

DE–NOISED IMAGE

Noisy Image

Thresholding of wavelet coefficients

Denoised Image

Wavelet Decomposition

Wavelet Reconstruction

2–LEVEL DWT

Image → LoLo, LoHi, HiLo, HiHi

LoLo → LoLo, LoHi, HiLo, HiHi

LoLo, LoHi, HiLo, HiHi → LoLo

LoLo, LoHi, HiLo, HiHi → Image

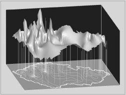
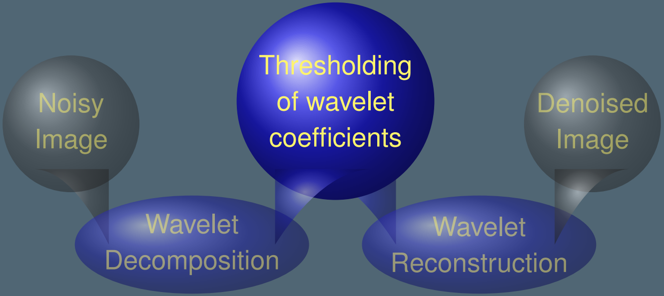


Image De-Noising

Wavelet Shrinkage Algorithm

How to threshold the wavelet coefficients?



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Wavelet Decomposition

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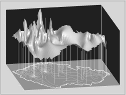

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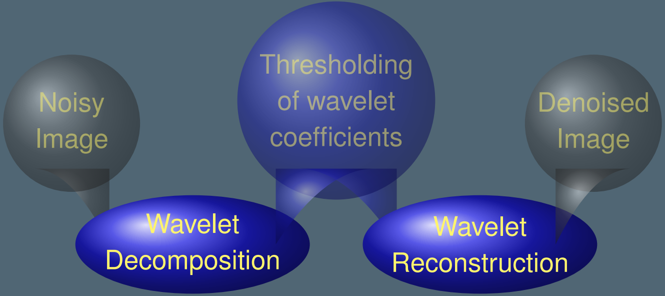
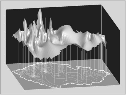
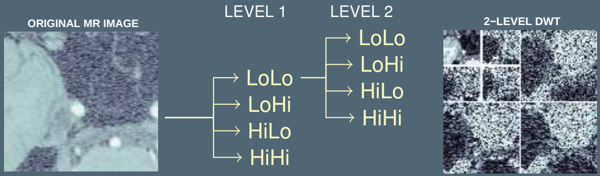


Which wavelet transform to use?



Wavelet Transform Alternatives

DWT = Discrete Wavelet Transform



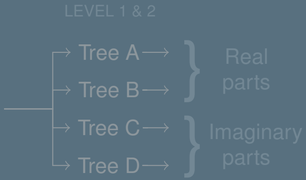
ORIGINAL MR IMAGE

LEVEL 1
LoLo
LoHi
HiLo
HiHi

LEVEL 2
LoLo
LoHi
HiLo
HiHi

2–LEVEL DWT

DTCWT = Dual-Tree Complex Wavelet Transform

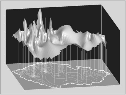


LEVEL 1 & 2

Tree A
Tree B
Real parts

Tree C
Tree D
Imaginary parts

- Employs 2^d DWT trees of real-tap filters in d-dimensions



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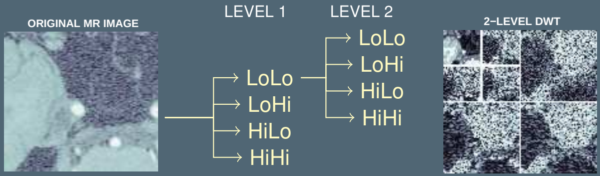

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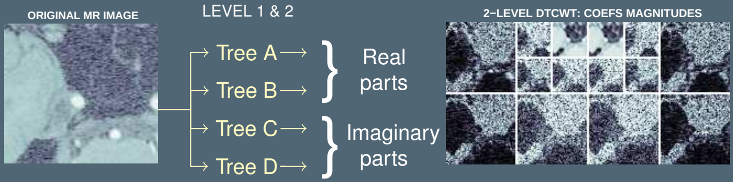

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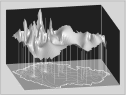


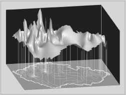
Wavelet Transform Alternatives

DTCWT

- Analytic complex wavelets $\psi_c(t) = \psi_r(t) + j \cdot \psi_i(t)$
  - $\Rightarrow$ Correct magnitude-phase representation
  - $\Rightarrow$ Shift invariance & no aliasing
- Impossible for wavelets of compact support $\Rightarrow$ only approximately analytic

Directional Selectivity of 2D Wavelets

- DTCWT: 6 subbands
- DWT: 3 subbands



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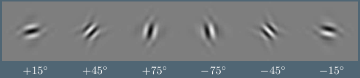
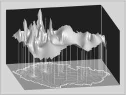
REAL PARTS OF Q−SHIFT WAVELETS



IMAGINARY PARTS OF Q−SHIFT WAVELETS

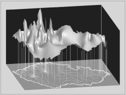
DB4 REAL WAVELETS



Wavelet Transform Alternatives

DWT versus DTCWT

DWT	DTCWT
− Zero-crossings at a singularity	+ Large magnitudes at a singularity
− Strongly shift dependent	+ Approx. shift independent
− Aliasing	+ Approx. no aliasing
− Lack of directional selectivity (±45°)	+ Improved directional selectivity
+ Critically decimated	− Moderately redundant



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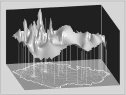

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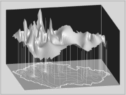

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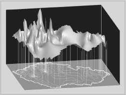

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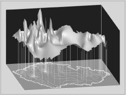

Image De-Noising

Wavelet Shrinkage Algorithm

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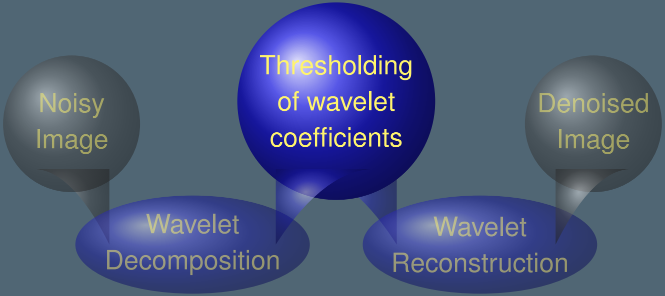
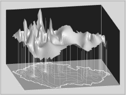




Wavelet Shrinkage

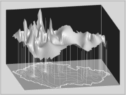
Wavelet Shrinkage Principles

- Suppressing lower energy wavelet coefficients (noise)
- Wavelet coefficients: thresholded (their magnitudes)
- Scaling coefficients: left unchanged

Soft Universal Shrinkage

- For wavelet coefficients $\{c_k\}_{k=0}^{M-1}$ of all levels:

$$c_k^{(s)} = \begin{cases} sgn(c_k)\cdot(|c_k| - \delta^{(s)}) & for\ |c_k| > \delta^{(s)} \\ 0 & otherwise \end{cases}$$



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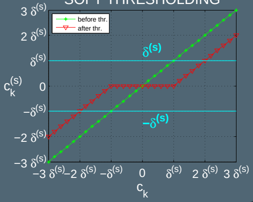
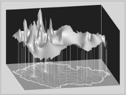
SOFT THRESHOLDING





Wavelet Shrinkage

Donoho Estimator

- Soft universal threshold value:

$$\delta^{(s)} = \sqrt{2\,\hat{\sigma}_n{}^2\ log(N)}$$

$\hat{\sigma}_n \ldots$ noise std. deviation estimate; $N \ldots$ image size

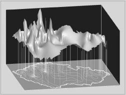
Median Absolute Deviation (MAD) Estimator

- Estimate of std. deviation for i.i.d. Gaussian noise:

$$\hat{\sigma}_n^{(MAD)} = \frac{median\{\,|c_1^{hh}(k)|\,\}_{k=0}^{N/4-1}}{0.6745}$$

$c_1^{hh} \ldots$ HiHi wavelet coefficient of level 1 (noise dominated)

- Robust against large deviations of noise variance



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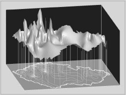
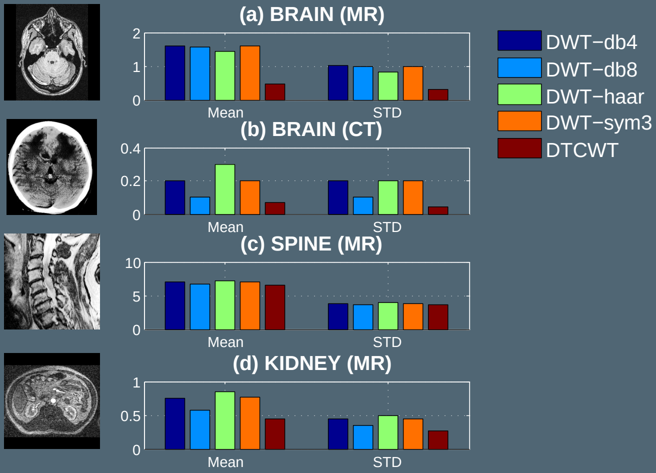
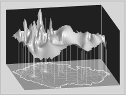


De-Noising Results

Statistics of High-Frequency Image Components





Gradient Masks

Gradient Edge Detectors

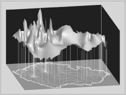
- Filters approximating the intensity gradient
- 2D convolution between the filter and the image

Sobel Filter

- By rotation: detection of $0^\circ, +45^\circ, +90^\circ, -45^\circ$ edges

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

- For every root pixel - choose the rotation variant with the absolute maximum value of convolution



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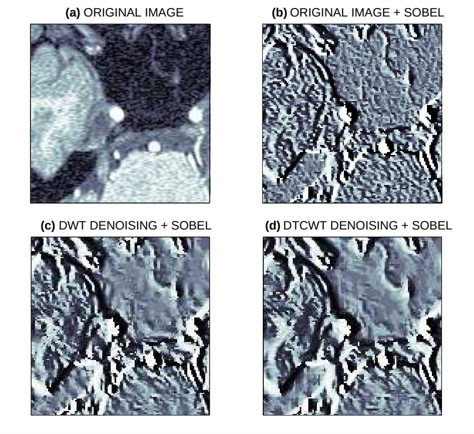
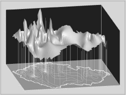
Gradient Masks

Application of the Sobel Filter



- MR brain image de-noised using the DWT and DTCWT



Gradient Masks

Drawbacks of Gradient Masks

Short filters:

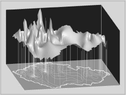
- Too sensitive to noise and blurring

Longer filters:

- More robust against noise
- Blur the originally sharp edges

Canny Edge Detector

- Robust against noise
- Operates at various scales



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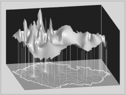
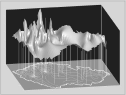


Canny Edge Detector

Canny Edge Detector

- Approximates the derivative of a 2D Gaussian in the direction of the gradient
- Robust against noise
  - ⇐ Gaussian smoothing filter prior to edge detection
  - ⇐ Weak edges pixels identification algorithm
- Adjustable value of the scale σ (the standard deviation in the Gaussian)



Canny Edge Detector

Canny Algorithm

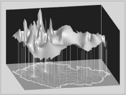
1. Convolution with 1D Gaussian masks in x and y-direction

$$G_{\sigma,0}(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{1}$$

2. Convolution with the derivatives of the 2D Gaussian in x-direction (and also in y-direction)

$$\frac{\partial G_{\sigma,0}(x,y)}{\partial x} = -\frac{x}{\sqrt{2\pi\sigma^3}} \cdot exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \tag{2}$$

3. Combining of these two matrices
4. Strong edges: pels value above the upper threshold
5. Weak edges:
   - Pels value above the lower threshold
   - The gradient $\equiv$ the direction of the strong edges in the neighborhood



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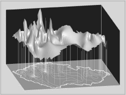

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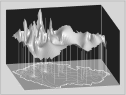

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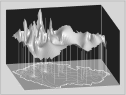

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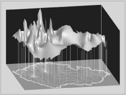
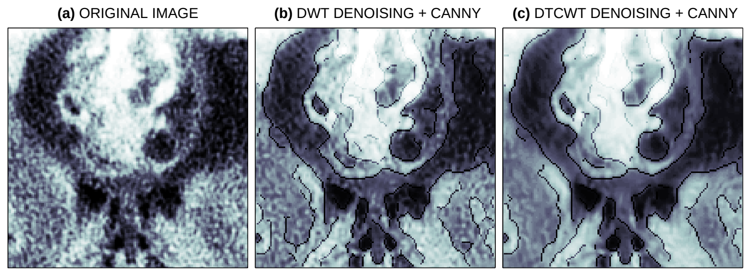


Canny Edge Detector

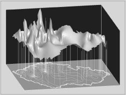
Canny Method for the De-Noised CT Image ($\sigma = 1.8$)



(a) ORIGINAL IMAGE **(b)** DWT DENOISING + CANNY **(c)** DTCWT DENOISING + CANNY

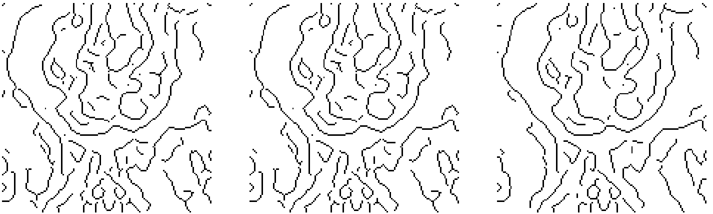
Denoising by wavelet shrinkage:

- DWT: 16-tap symlet filters, 4 levels
- DTCWT: 16-tap q-shift filters, 4 levels

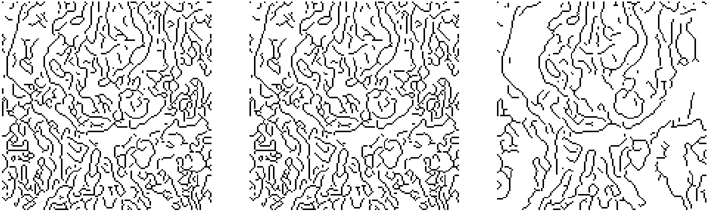


Canny Edge Detector

Edges of the De-Noised CT Image ($\sigma = 1.8, 1$)



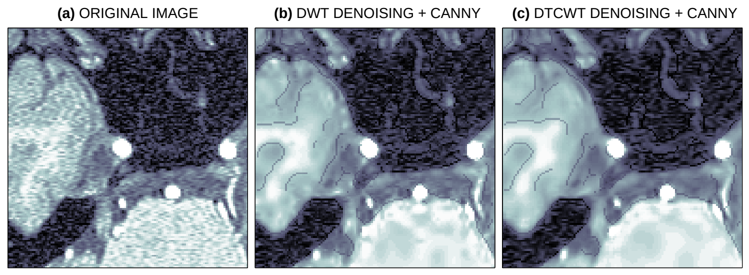
(a) EDGES: CANNY (σ=1.8) **(b)** DWT DEN. & CANNY (σ=1.8) **(c)** DTCWT DEN. & CANNY (σ=1.8)



(d) EDGES: CANNY (σ=1) **(e)** DWT DEN. & CANNY (σ=1) **(f)** DTCWT DEN. & CANNY (σ=1)

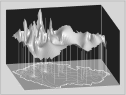
Canny Edge Detector

Canny Method for the De-Noised MR Image ($\sigma = 2.5$)



Denoising by wavelet shrinkage:

- DWT: 14-tap symlet filters, 3 levels
- DTCWT: 14-tap q-shift filters, 3 levels



Conclusions

Denoising by Wavelet Shrinkage

- DTCWT outperformes the DWT
  - Approximate shift invariance
  - Steady values of the magnitude across scale
  - Phase representation of edges orientation
  - Improved directional selectivity in higher dimensions

Edge Detection Methods Used

- Short gradient filters:
  - Insufficient for blurred or noisy images
- Canny detector:
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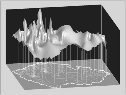

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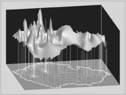

Further Reading

- I. W. Selesnick, R. G. Baraniuk, and N. G. Kingsbury. *The Dual-Tree Complex Wavelet Transform*. IEEE Signal Processing Magazine, 22(6): 123–151, IEEE, 2005.
- M. Petrou and P. Bosdogiann. *Image Processing*. John Wiley & Sons, 2000.
- R. M. Rangayyan. *Biomedical Image Analysis*. Biomedical Engineering Series, CRC Pres, USA, 2005.
- D. B. Percival and A. T. Walden. *Wavelet Methods for Time Series Analysis*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, USA, 2006.

Any questions?

http://dsp.vscht.cz