WAVELET TRANSFORM

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1. Introduction to Wavelet Transform
2. Fourier Transform
   - Continuous Fourier Transform
   - Short-Time Fourier Transform (STFT)
3. Wavelet Transform
   - Multi-Resolution Analysis (MRA)
   - Continuous Wavelet Transform (CWT)
   - Wavelet Functions Properties
4. Discrete Wavelet Transform
   - Discrete and Continuous Wavelet Transform
   - Subband Coding Algorithm
   - Matrix Interpretation
5. Wavelet Transform Applications
   - Discontinuity Detection in the ECG Signal
   - Image Compression
   - Image Segmentation
   - Noise Reduction by Wavelet Shrinkage

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Wavelet Transform (WT) History

- 19th cent. - Jean B. Fourier: frequency analysis
- 1909 - Alfred Haar: Haar function (not yet WT)
- since 1990s - WT related research and applications

Wavelet Functions

- Wavelet = a small wave, i.e. an oscillatory function - zero outside a bounded interval (having compact support)
- Some real, some complex
- Designed as much smooth and symmetric as possible
- Seldom analytic expression, mostly parametric (e.g. Morlet, Haar or Daubechies fcns)

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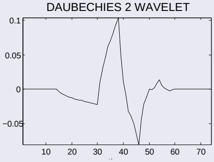

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WT Analysis

- For non-stationary signals (with time-varying frequency content)

WT Applications in Signal Processing

- Noise reduction
- Detection of trends and discontinuities in higher derivatives
- Compression (JPEG2000, FBI fingerprints database)
- Image edge detection
- Watermarking
- Features extraction for image segmentation

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Continuous Fourier Transform (FT)

- Decomposes signals (real or complex) into the orthogonal basis of complex exponentials $e^{-j2\pi ft}$ of frequencies f
- FT for the angular frequency $\omega = 2\pi f$ and time t:

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \tag{1}$$

  where $X(\omega)$ is the Fourier transform of the function $x(t)$
- Converges for a piece-wise smooth $x(t)$ or finite energy:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty \tag{2}$$

Inverse FT

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega \tag{3}$$

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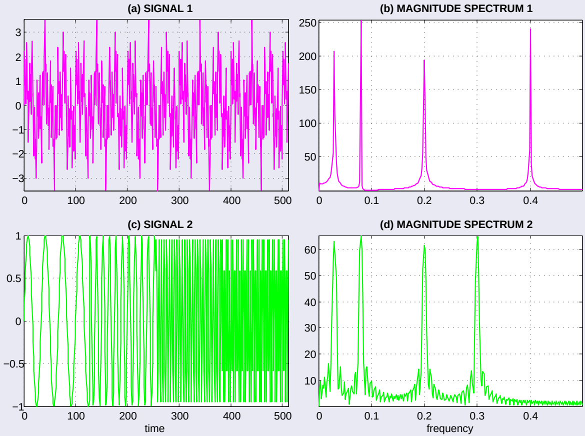
FT Analysis of Non-Stationary Signals

- Discrete FT for the discrete freq. index $k = 0, 1, \ldots, N-1$ and discrete time n (for the unit sampling frequency)

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \tag{4}$$



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Discrete Short-Time Fourier Transform (STFT)

- Non-stationary signal analysis (localizes frequency components within time intervals)
- STFT of a discrete signal $x(n)$ for $k=0,1,\ldots,N-1$:

$$X(u,k) = \sum_{n=0}^{N-1} x(n)\, w(n-u) e^{-j2\pi kn/N} \tag{5}$$

where u denotes the window position

Uncertainty (Heisenberg) Principle

$$\Delta T \, \Delta f = 1 \tag{6}$$

- $\Delta f = F_s/N$ ... frequency resolution (distance between adjacent spectral samples)
- $\Delta T = N\, T_s$ ... time resolution (window length)
- N ... number of samples per window
- $F_s = 1/T_s$ ... sampling frequency

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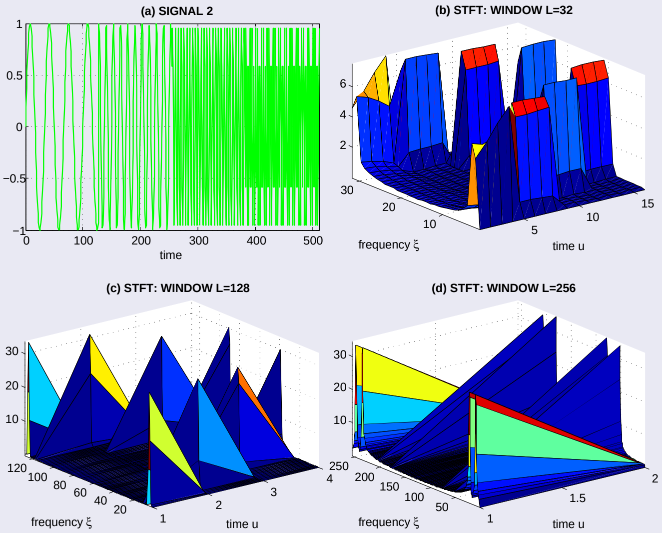
STFT - Uncertainty Tradeoff



(Using the Hamming window w - gently dropping to zero)

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Multi-Resolution Analysis (MRA)

- The wavelet transform deals with the uncertainty principle by MRA
- Analyzing signals at different frequency bands with different resolution:
  - Higher frequencies: good ΔT, poor Δf
  - Lower frequencies: good Δf, poor ΔT
- Convenient for most real-world signals composed of:
  - Long-lasting lower frequencies (approximations)
  - Short-lasting higher frequencies (details - main information)

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- FT: correlation of a signal of finite energy with complex exponentials $e^{j2\pi ft}$ of different frequencies f
- WT: correlation of a signal of finite energy with delated and shifted versions $\psi_{u,s}$ of the mother wavelet ψ:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{t-u}{s}\right) \quad (7)$$

  - where u is the shift of ψ along the signal $\rightarrow$ time
  - and s is scale (dilation of $\psi \rightarrow$ the inverse of frequency)
  - $1/\sqrt{s}$ ensures energy normalization

- CWT of the signal $x(t)$:

$$W_x^{\psi}(s,u) = \langle x, \psi_{u,s}\rangle = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} x(t)\,\psi^{*}\left(\frac{t-u}{s}\right)dt \quad (8)$$

  - where * denotes the complex conjugate pair
  - and $\langle\rangle$ stands for the inner product

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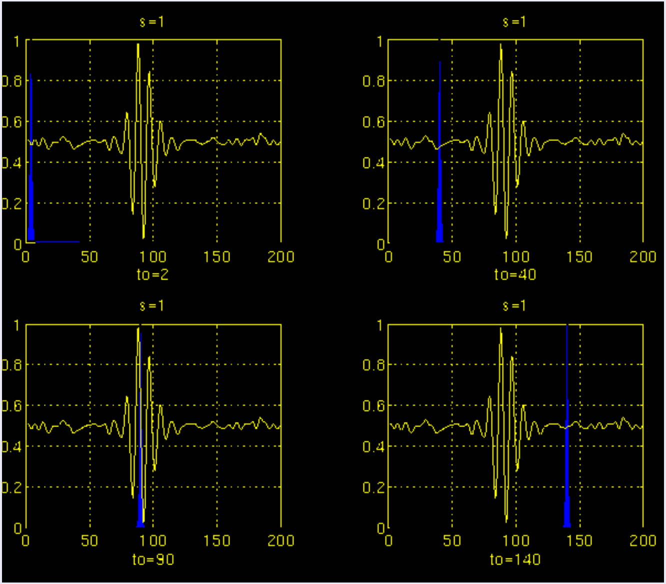
CWT Computation Procedure ($s = 1$)



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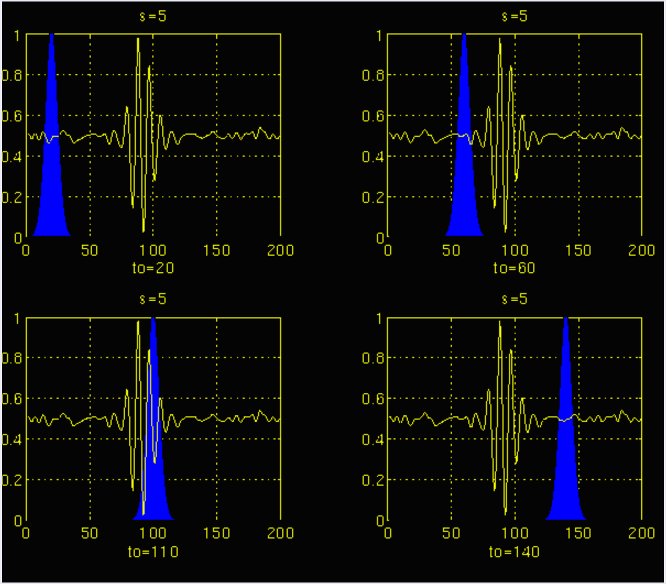
CWT Computation Procedure ($s=5$)



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CWT Properties

- Linearity: implied by the properties of the inner product
- Shift invariance:
  - A shift in the signal along the time axis $\mapsto$ the equivalent shift of the wavelet coefficients without any changes
  - Does not apply to the discrete WT (great drawback)

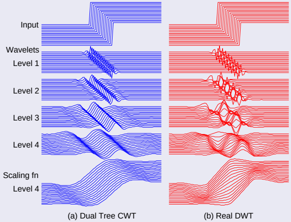

Figure: Comparison of DWT and nearly invariant complex WT (CWT is even better - completely shift invariant)

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Orthogonality and orthonormality

- Not necessary for wavelet analysis (biorthogonality for more freedom in wavelet filters construction)
- A set of shifted and dilated versions of the mother wavelet $\psi_{u,s}$, is orthonormal on the interval $[a, b]$ when:

$$\langle \psi_{u,s}, \psi_{v,r} \rangle = \int_a^b \psi_{u,s}(t)\, \psi^*_{v,r}(t)\, dt = \begin{cases} 1 & \text{for } u=v, s=r \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

- A set of $\psi_{u,s}$, is orthogonal on the interval $[a, b]$ when:

$$\langle \psi_{u,s}, \psi_{v,r} \rangle = \int_a^b \psi_{u,s}(t)\, \psi^*_{v,r}(t)\, dt = \begin{cases} c & \text{for } u=v, s=r \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where c is a constant, * denotes a complex conjugate

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Wavelet Functions Properties I

- Admissibility condition

$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < +\infty \tag{11}$$

  - where $\Psi(\omega)$ is the FT of $\psi(t)$
  - Required for no loss of information during the analysis nor the synthesis
  - The basis do not have to be orthogonal

Consequences of Admissibility Condition

- Oscillatory function (zero mean):

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0 \tag{12}$$

- Band-pass spectrum ($\Psi(\omega)$ vanishes at $\omega = 0$):

$$|\Psi(\omega)|^2|_{\omega=0} = 0 \tag{13}$$

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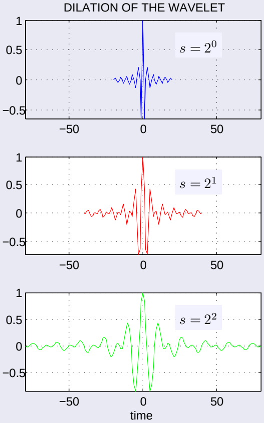
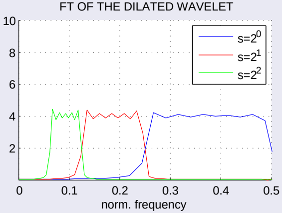
Wavelet Functions Properties II

- Time dilation of ψ causes shift and compression of the magnitude frequency spectrum $|\Psi|$





NEWLAND WAVELET:

$$\psi(t) = \frac{1}{j\,\frac{\pi}{2}\,t}\,(e^{j\,\pi\,t} - e^{j\,\frac{\pi}{2}\,t})$$

WAVELET SCALING & DILATION:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{t-u}{s}\right)$$

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Desired Wavelet Functions Properties

- Compact support (non-zero only on a restricted interval) - not necessary
- Regularity:
  - Smoothness of ψ and vanishing of $|\Psi(\omega)|$ for large ω (small scales)
  - *Vanishing moments* concept (see literature)
- Symmetry (liner phase)

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Discrete Wavelet Transform (DWT)

- CWT provides us with redundant signal representation
- DWT - derive by critically sampling the CWT (great reduction in the number of dilations and shifts)

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{t-u}{s}\right) \Rightarrow \psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}}\,\psi\left(\frac{t-k\,\tau_0\,s_0^j}{s_0^j}\right) \tag{14}$$

Sampling on the Dyadic Grid

- $s_0 = 2 \Rightarrow$ the scale $s = 2^j$
- $\tau_0 = 1 \Rightarrow$ the time translation $u = k\,2^j$
- t denotes time, j, k are integers

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\,\psi\left(\frac{t-k2^j}{2^j}\right) \tag{15}$$

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Discrete Wavelet Transform (DWT)

- CWT provides us with redundant signal representation
- DWT - derive by critically sampling the CWT (great reduction in the number of dilations and shifts)

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\,\psi\left(\frac{t-u}{s}\right) \Rightarrow \psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}}\,\psi\left(\frac{t-k\,\tau_0\,s_0^j}{s_0^j}\right) \tag{14}$$

Sampling on the Dyadic Grid

- $s_0 = 2 \Rightarrow$ the scale $s = 2^j$
- $\tau_0 = 1 \Rightarrow$ the time translation $u = k\,2^j$
- t denotes time, j, k are integers

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\,\psi\left(\frac{t-k2^j}{2^j}\right) \tag{15}$$

Wavelet Transform

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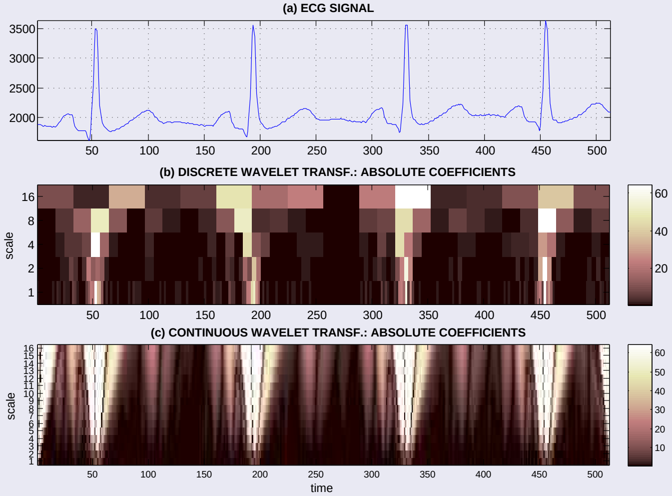
Discrete & Continuous Wavelet Transform



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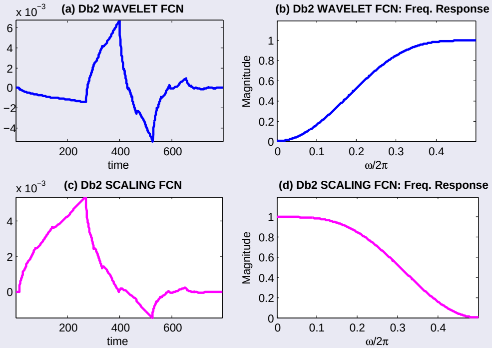
Wavelet and Scaling Filters



- The spectrum of the wavelet function corresponds to a high-pass filter (or a band-pass filter for higher levels)
- The spectrum of the scaling function corresponds to a low-pass filter

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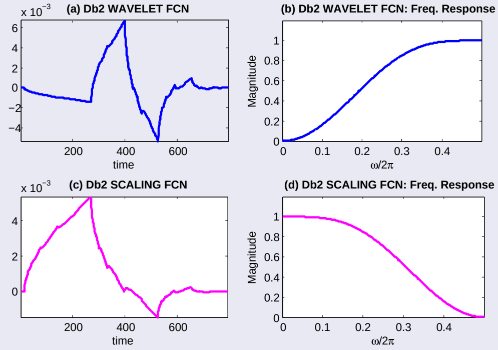
Wavelet and Scaling Filters



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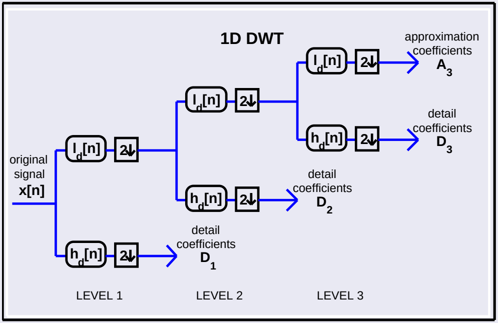
Subband Coding Algorithm



CWT versus DWT

- Correlation
- Dilating the wavelets

- Convolution
- Keeping filters the same and down-sampling the previous level output by 2

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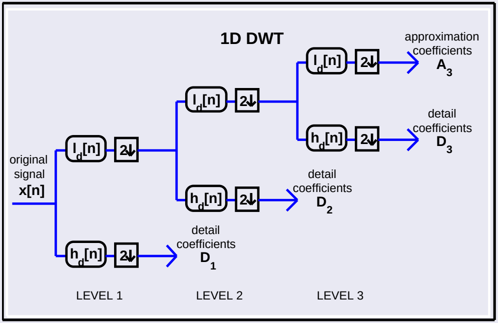

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Convolution and Downsampling by 2

- The taps of the high-pass filter h_d and the low-pass l_d filter are derived from the wavelet and the scaling function, resp., of a chosen family (e.g. Daubechies, symlets, etc.)

Convolution and Downsampling by 2

- Approximation coefficients of the first level

$$A_1[n] = \sum_{k=-\infty}^{\infty} l_d[k]\, x[2n-k] \tag{16}$$

- Detail coefficients of the first level

$$D_1[n] = \sum_{k=-\infty}^{\infty} h_d[k]\, x[2n-k] \tag{17}$$

- The j-th level $$A_j[n] = \sum_{k=-\infty}^{\infty} l_d[k]\, A_{j-1}[2n-k] \tag{18}$$

$$D_j[n] = \sum_{k=-\infty}^{\infty} h_d[k]\, A_{j-1}[2n-k] \tag{19}$$

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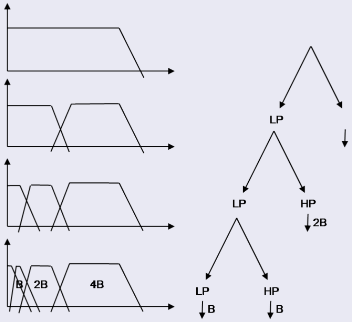
Subband Coding & the Frequency Spectrum



- Dilated by 2 $\Rightarrow$ the spectrum is compressed and shifted
- Finite number of dilations $\Rightarrow$ advantageous to use a low-pass filter - derived from the scaling function $\phi(t)$ (the counter part of the wavelet filter at each level - creating a filter bank)

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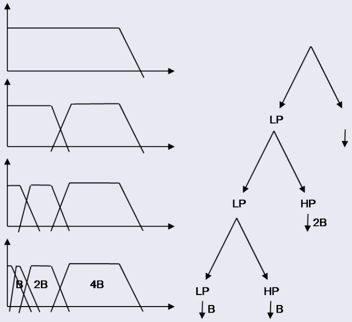

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DWT Matrix

- The Haar filters

$$l_d = 1/\sqrt{2} \cdot [1,\ 1] \tag{20}$$

$$h_d = 1/\sqrt{2} \cdot [1,\ -1] \tag{21}$$

- DWT matrix for the Haar filters

$$\mathbf{W} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix} \tag{22}$$

- $\mathbf{W}$ includes both convolution and down-sampling by 2 (the filters are shifted by 2 samples)
- $\mathbf{W}$ is orthonormal $\Rightarrow \mathbf{W} \cdot \mathbf{W}^T = \mathbf{I}$
  where $\mathbf{I}$ stands for the identity matrix

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DWT of Signal **x** Using the Haar Filters

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 1 & \dots & 0 & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ \vdots \\ x(N-1) \\ x(N) \end{pmatrix} = \begin{pmatrix} A_1(0) \\ D_1(0) \\ A_1(1) \\ D_1(1) \\ \vdots \\ A_1(\frac{N}{2}-1) \\ D_1(\frac{N}{2}-1) \end{pmatrix}$$

DWT Decomposition and Reconstruction

- Signal decomposition

$$\mathbf{W} \cdot \mathbf{x} = \mathbf{w} \tag{23}$$

- Signal reconstruction

$$\mathbf{x} = \mathbf{W}^{-1} \cdot \mathbf{w} \tag{24}$$

- Signal reconstruction for the orthonormal DWT

$$\mathbf{x} = \mathbf{W}^{T} \cdot \mathbf{w} \tag{25}$$

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Discontinuity Detection in the ECG Signal

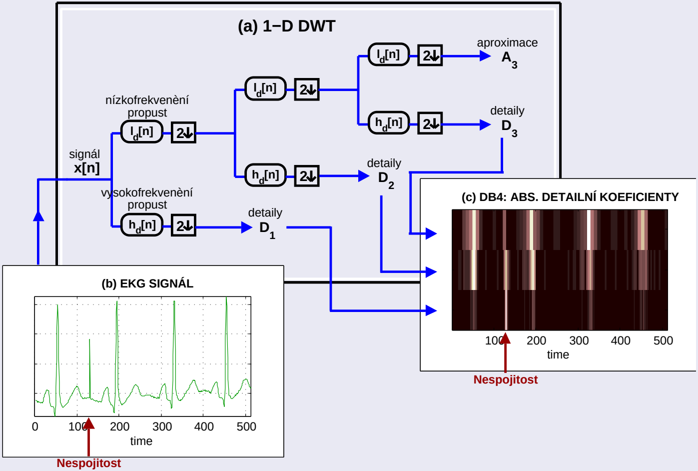

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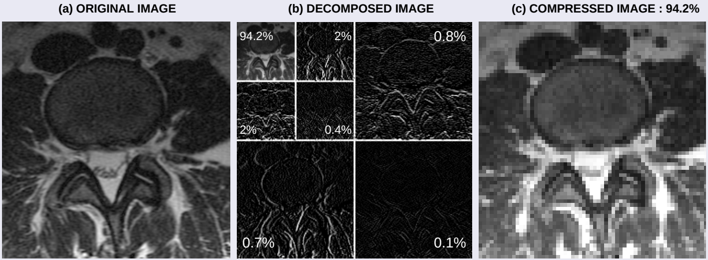
Image Compression

- The Parseval theorem: the energy ε_x conveyed in the signal **x** equals the energy of the coefficients **w** obtained through an orthonormal transform

$$\varepsilon_x = \sum_{n=0}^{N-1} |x_n|^2 = \sum_{k=0}^{N-1} |w_k|^2 \qquad (26)$$



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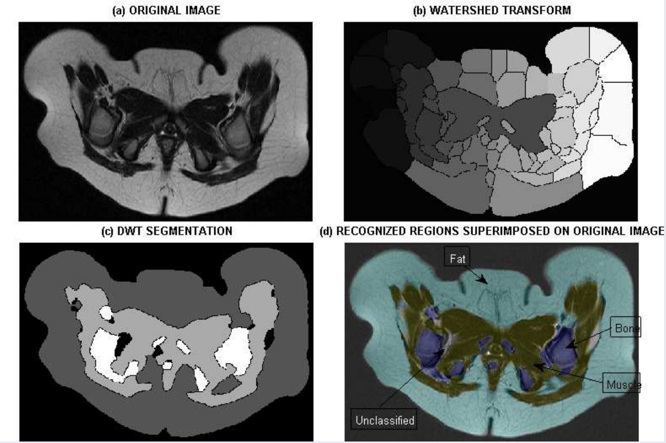
Magnetic Resonance Image Segmentation



1. Image preprocessing
2. Watershed transform
3. DWT features extraction
4. Features classification using a neural network

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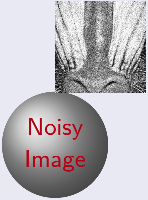

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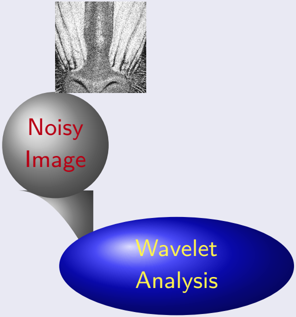

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  - $LoLo_1$
    - $LoLo_2$
    - $LoHi_2$
    - $HiLo_2$
    - $HiHi_2$
  - $LoHi_1$
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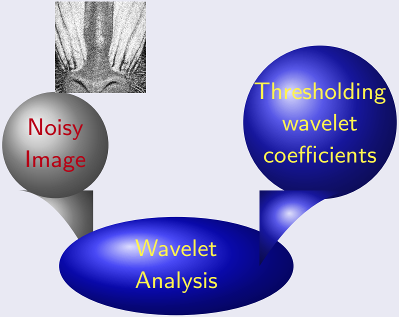

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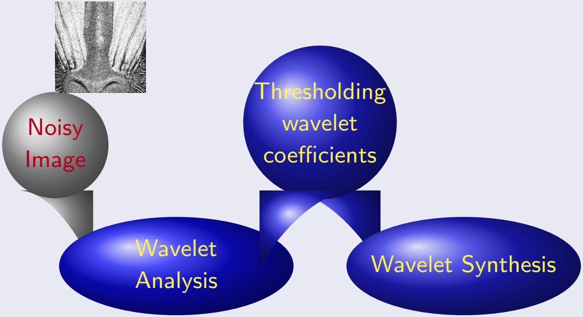

Image → $LoLo_1$ → $LoLo_2$, $LoHi_2$, $HiLo_2$, $HiHi_2$

Image → $LoHi_1$, $HiLo_1$, $HiHi_1$

$LoLo_1$, $LoHi_1$, $HiLo_1$, $HiHi_1$ → $LoLo_2$

$LoLo_2$, $LoHi_2$, $HiLo_2$, $HiHi_2$ → Image

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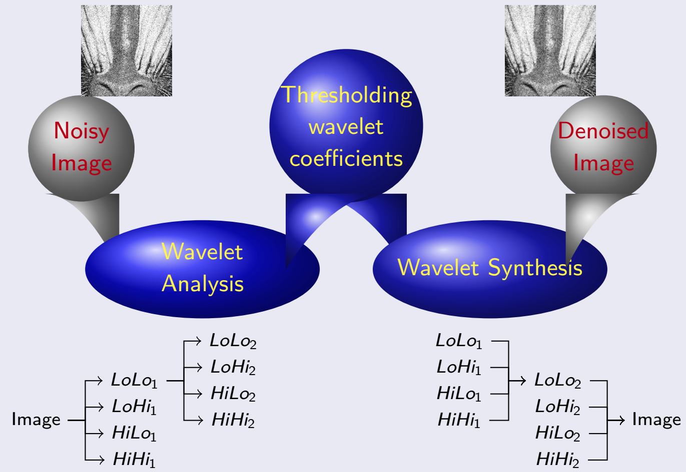

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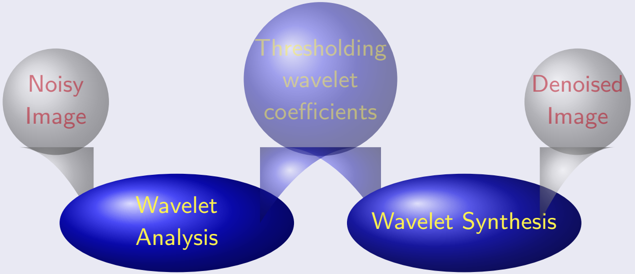

Wavelet function? Number of levels?

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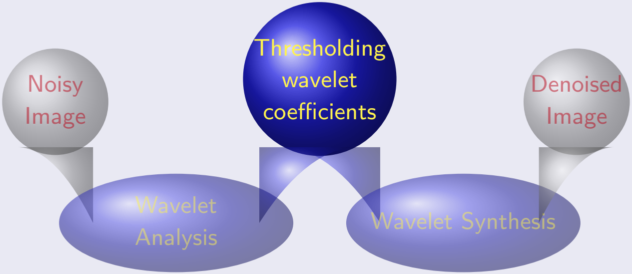
Thresholding procedure type? Threshold level?



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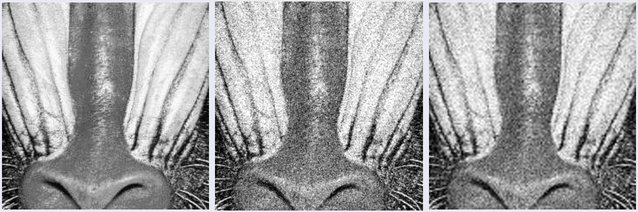
(a) ORIGINAL | (b) WITH ADD. GAUSSIAN NOISE | (c) DENOISED



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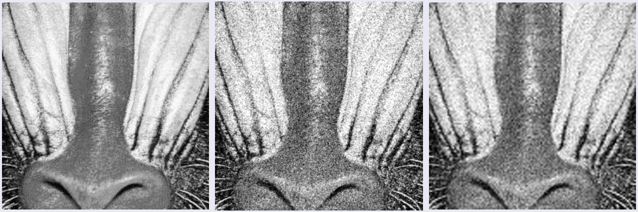
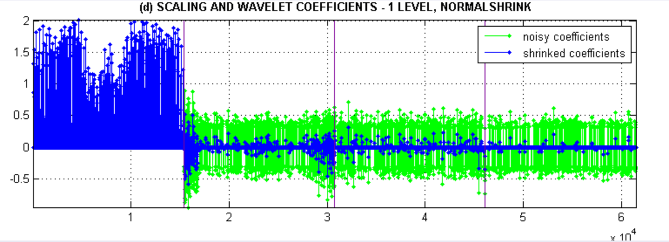
Noise Reduction by Wavelet Shrinkage

(a) ORIGINAL (b) WITH ADD. GAUSSIAN NOISE (c) DENOISED





Wavelet Transform

Eva Hostalkova

Introduction

Fourier Transform
- Continuous Fourier Transform
- STFT

Wavelet Transform
- MRA
- CWT
- Wavelets Properties

DWT
- DWT and CWT
- Subband Coding
- Matrix Interpretation

WT Applications
- Discontinuity Detection
- Image Compression
- Image Segmentation
- Noise Reduction

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