CF TOOLBOX IN SOLAR CELL DIAGNOSTICS

P. Hruska[*]*, Z. Chobola*[**]*, L. Grmela*[*]

Department of Physics, Faculty of Electrical Engineering and Communication[*] and
Department of Physics, Faculty of Civil Engineering[**]
Brno Technology University

Abstract
Solar cell dc *i-u* curve is fitted with explicit theoretical function, expressed in terms of the Lambert W function. M-function fit_solar3 is compiled, using CF toolbox fitting curves. The function is applied to two sets of solar cells. Results are presented as graphs and tables.

Keywords:
Solar cell diagnostics, curve fitting, Lambert W function, Matlab, non-destructive diode diagnostics

1. Introduction

Solar cell non-destructive (ND) diagnostics is based on observations of selected cell parameters during the sample fabrication and their variations in the conditions of normal functioning and under accelerated aging. Based on the parameter values and their changes, diagnostics is, in many cases, able to describe the processes causing the changes, to characterize the cell quality and predict its reliability and effective lifetime.

It offers invaluable hints for technology modification and improvement. Selection of the parameters depends mainly on the technologist intentions. The parameters are frequently measured by miscellaneous instrumental set-ups of varying sensitivity. An interesting means of several parameters determination is described in the paper submitted. It is based on measurement of *i-u* curves and subsequent application of fitting procedures.

The procedures provide the parameters affecting the *i-u* curve as the outputs. Relatively high accuracy is reached in the series resistance determination (less than 0.05 Ω), and in the parallel resistance (units of Ω). The parameters determination is fast and non-destructive.

Experimental data were taken by measuring *i-u* dc curves of two sets of solar cells manufactured by an European manufacturer A. One set was created by samples labeled as c2 (c201, c202...), the other as c3 (c301, etc)

The parameters determination has been achieved by means of fitting the experimental curves with Matlab CFTOOL, or with the function FIT of the CF Toolbox, resp.

The great advantage of CFTOOL is the fact, that it supports relatively new concept of the Lambert W function.

Four to six relevant parameters appear in the forward *dc i-u* curve.

2. Series resistance. Lambert W function

The curve is described by a well known formula [1]

$$i = u/R + i_1 *(\exp(\beta_1 *(u - r_3 *i)).-1) \qquad (1)$$

The parameters are: series resistance r_3, parallel contact resistance R, and generation-recombination factors i_1 and β_1. Generation-recombination current, flowing via g-r centers in indirect semiconductors is present mainly in Si diodes. The current u/R is a parallel current, passing by the PN junction. We added term u/R, *which* describes this effect.

More accurate formula that takes into account the diffusion (Shockley) current is

$$i = u/R + i_1*(\exp(\beta_1*u) - 1) + i_2*(\exp(\beta_2*(u - r_3*i)) - 1) \tag{2}$$

Parameters i_2, β_2 are diffusion factors. Standard theory yields $\beta_2 = 2\beta_1 = e/kT$, k being the Boltzmann constant, e elementary charge, T absolute temperature. In Si devices the g-r current frequently overlaps the diffusion current.

The term r3*i represents a potential drop on the cell series resistance, that has to be subtracted from the cell voltage to obtain the PN junction voltage. In the case of two non-ohmic currents the above subtraction takes place only at higher bias as seen in Eq. (2). Factor -1 in the diffusion current plays role only for voltage $u < 120$ mV (error < 1%).

Brief inspection shows that both the functions (1, 2) are of implicit form, without possibility to express the current i in terms of elementary math functions.

Explicit $i(u)$ formula in terms of Lambert W function [Cor], replacing Eq. (1) reads

$$i = (b1*r3*u + \text{lambertw}(b1*r3*i1*\exp(b1*(u*R - r3*u + r3*i1*R)/R))*R -$$
$$-\beta_1*r_3*i_1*R)/\beta_1/r_3/R. \tag{3}$$

and

$$i = (\text{LambertW}(beta_2*r_3*i_2*\exp(-beta_2*(-u*R+r_3*u+r_3*i_1*R*\exp(beta_1*u)-r_3*i_1*R-$$
$$r_3*i_2*R)/R))*R+beta_2*r_3*u +beta_2*r_3*i_1*R*\exp(beta_1*u)-beta_2*r_3*i_1*R-beta_2*r_3*i_2*R) /(beta_2*r_3*R)$$
$$\tag{4}$$

The experimental i-u curves of the two sets are shown in Fig.1, using semilog scales.
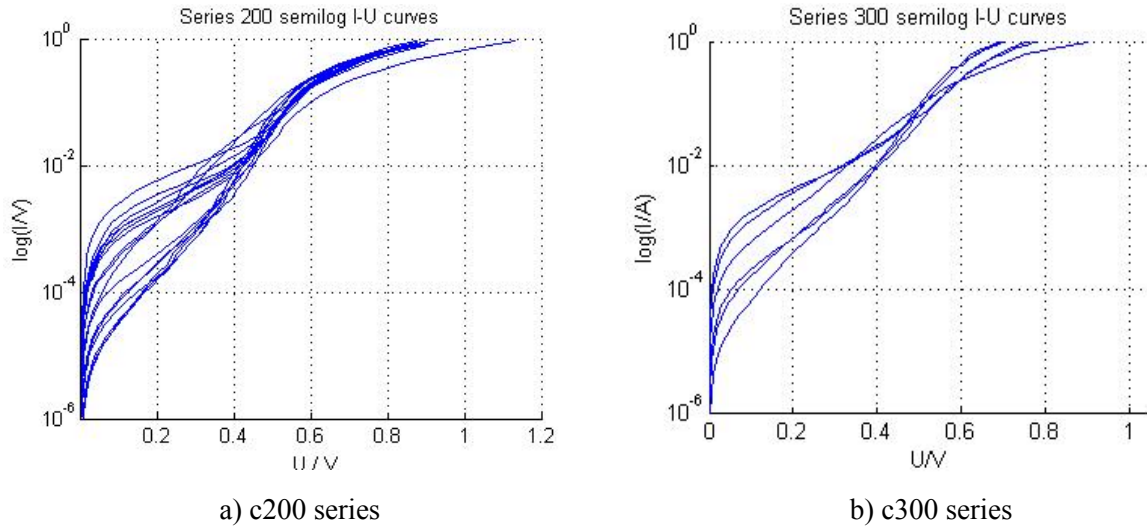

a) c200 series   b) c300 series

Fig. 1. Solar cell semilog i-u curves. Series c200 and c300

Distinct bending of the curves above 0.6 V is due to series resistance r_3 potential drop. Even though its value is of the order of several tenths of ohm, the drop is of the order of units or tens of milivolts, which causes the curve deviation from that of the ideal PN junction. The results obtained by fitting are given in tableT1.

Table T1. Parameters i_1 and r_3 of solar cells, sets c200 and c300

sample	i_1/A	r_3/Ω	sample	i1/A	r3/Ω
c201	4.6e-006	0.501	c301	6.2e-008	0.182
c201	4.6e-006	0.501	c302	2.1e-006	0.121
c202	3.9e-007	0.334	c308	3.9e-005	0.274
c203	9.5e-005	0.290	c310	3.7e-008	0.126

c204	5.2e-008	0.345		c312	1e-007	0.122
c205	1.3e-007	0.329		c315	3e-008	0.122
c207	1.6e-007	0.349		c318	5.4e-008	0.122
c208	1.4e-005	0.356				
c210	3e-008	0.341				
c211	.6e-008	0.339		Mean =	6.89e-006	0.148
c212	9.5e-008	0.374		Std =	1.58e-005	0.062
c213	3.5e-008	0.370				
c214	4.3e-008	0.358				
c215	3e-008	0.351				
c217	4.6e-008	0.346				
c218	2.4e-008	0.340				
c219	3.1e-008	0.337				
c220	8.3e-008	0.331				
c208	1.4e-005	0.356				
Mean=	7.01e-006	3.6e-001				
Std	2.17e-005	5.3e-002				

3. Parallel resistance

Experimental *i-u* curves display ohmic-like region at low cell currents. The region is visualized in loglog graphs, which exaggerate this region, Fig. 2.

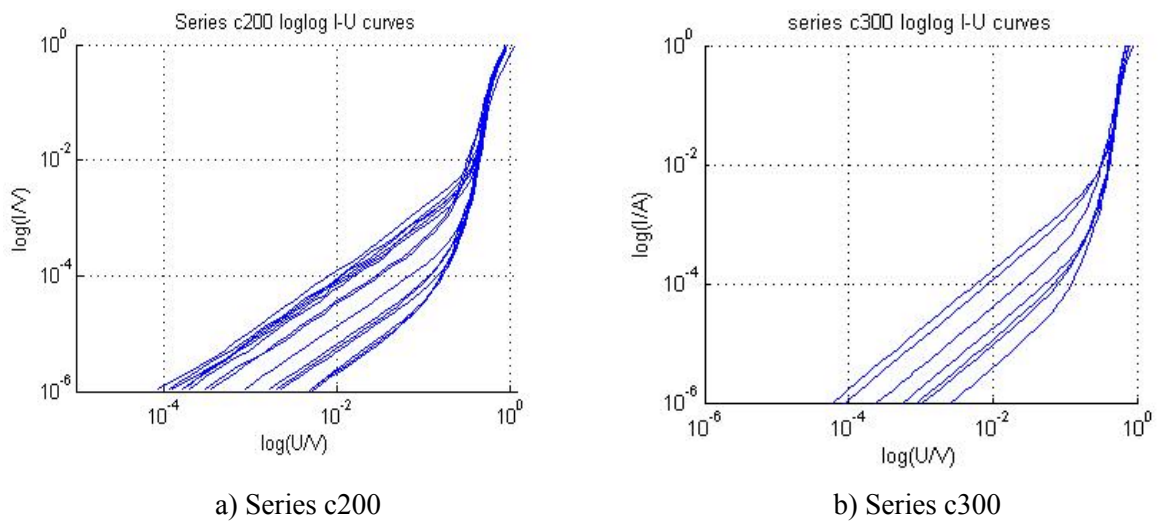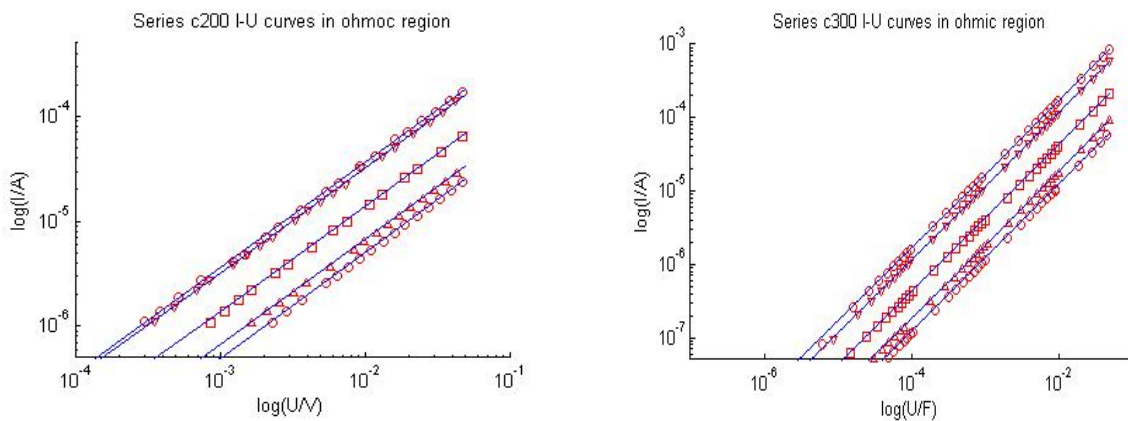a) Series c200   b) Series c300

Fig. 2. Solar cells *i-u* curves with exaggerated low current (ohmic-like) region. Series c200 and c300

Detailed graphs of the linear regions with several fits are shown in Fig. 3. The linearity of the curves is clearly seen.

Fig. 3. Solar cells *i-u* low current curves with power one-fit. Series c200 and c300

The fits of data in Fig.3 are evaluated by M-function fit_R(x,y). Its short form is given below.

Table T3. fit_R.m

1. function fit_R(x2,y2) 2. %fits as power one 3. fo_ = fitoptions('method','NonlinearLeastSquares', ... 4. 'Lower',0); 5. set(fo_,'Startpoint',[0.2]); 6. p_w = 'a*x'; 7. ft_ = fittype(p_w,... 8. 'dependent',{'y'},'independent',{'x'},... 9. 'coefficients',{'a'}); 10.[cf_ ,gof,output] = fit(x2,y2,ft_,fo_); 11.disp(cf_) %cfit	Comments on fit_R(x2,y2): It fits data as power-one function, i.e. x ^ 1, without absolute term (see p_w). It corresponds to the functional dependence of Ohm's law. The result of the fit is displayed on call disp(cf_) It is shown here: • fit_R(x,y) • General model: • cf_(x) = a*x • Coefficients (with 95% confidence bounds): • a = 0.01309 (0.008956, 0.01722)

Parameters *R* of the two sets of solar cells obtained by function fit_R(x,y) are given below, in

Table T4   Parameters *R* of solar cells, sets c200 and c300

Sample	c201	c202	c203	c204	c205	c207	c208	c210
R/Ω	729.5	80.9	279.0	133.7	141.5	39.1	311.6	1712
Sample	c301	c302	c308	c310	c312	c315	c318	
R/Ω	543.5	58.8	234.6	811.7	2313	984.1	86.0	

Sample	c211	c212	c213	c214	c215	c217	c218
R/Ω	2031	3688	4464	4145	1520	113.1	101.8

4. Fitting procedures

The Curve Fitting Toolbox, as mentioned above, supports the concept of Lambert W function. The functions fitoptions, fittype, fit, syms, subs of this and other toolboxes have been used in the cells *i-u* curves fitting.

Direct fit suffered by great numerical errors. The fitting formula (line 9 below) has to evaluate the difference between two unknowns: R-r_3. The two quantities are of distinct difference in magnitude – 10^{-1} and 10^3 which results in relatively uncertain value of R. Therefore, we determined R within he linear range for voltage below 0.05 V , function fit_R(x,y) above, and then passed the value as parameter rho into another fitting function fit_solar3(x1,y1,rho).
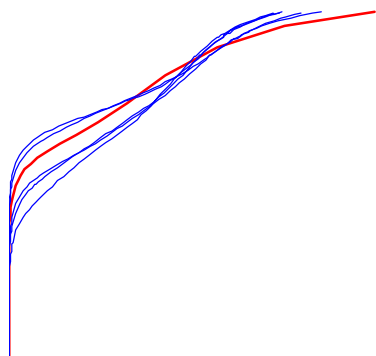
The short form of the function is given below. Its full version is relatively complex, but many details are not relevant in this connection.

 Table T4. Function   [s1,s2,s3,s4] = fit_solar3(x1,y1,rho);

1. function [s1,s2,s3,s4] = fit_solar3(x1,y1,rho);
 2. % Fitting i-u curve of a solar cell
 3. % Fit setting:
 4. fo_ = fitoptions('method','NonlinearLeastSquares',...
     .
 5.   'Lower',[0 0 0]);
 6. set(fo_,'Startpoint',[10 1e-010 0.05]);
 7. syms R1 R
 8. R1=rho;
 9. itty='(beta1*r3*x+lambertw(beta1*…
     r3*i1*exp(beta1*(x*R- r3*x+r3*i1*R)/R))*R-
     beta1*r3*i1*R)/beta1/r3/R';
 10.ittyR=subs(itty,R,R1);
 11.ittyRc=char(ittyR);
 12.ft_ = fittype(ittyRc,...
 13.   'dependent',{'y'},'independent',{'x'},...
 14.   'coefficients',{'beta1', 'i1', 'r3'});
 15. [cf_ gof] = fit(x1,y1,ft_ ,fo_);
 16.res_ = y1 - cf_(x1);
 17.figure(1);
 18.subplot(2,1,1),plot(cf_,'r',x1,y1,'b.')
 19.subplot(2,1,2);plot(x1,res_,'r.',x1,res_,'r-')
 20.% Results:
 21.s1=R1;s2=cf_.i1;s3=cf_.beta1;s4=cf_.r3;
 22.disp(s1, s2, s3, s4);
	Comments on fit_solar3(x1,y1,rho):
	
	   In the fitting procedure we use a two step procedure. Input parameters are x1, y1 coordinates of i-u curve and the parallel resistance R. They are determined elsewhere.
	
	   Selected fit options and starting parameters are defined in line 4 – 6. The fitting formula itty (line 9) with the Lambert W function, see (3), contains parameter R, which has to be included with its numerical value. Substitution is done on lines 11 – 12.
	
	   The actual fit is performed on line 15 by the function fit. The function output parameters are of class struct. Requested parameters i1, beta1 and r3 are obtained on line 21.
	
	   The results of the fitting procedure are visualized in two subplots:
	 1 – line 18
	 h1=plot(cf_,'r',x1,y1);
	 2 – line 19
	 h2=plot(x1,res_,'r.',x1,res_,'r-');
	   Plot h1 visualizes the measured *i-u* curve (x1,y1) and the fit cf_, i.e. curve on line 9 with numerical parameters.
	   Plot h2 visualizes residua of fitting, line 16

The full version of the function incorporates both the functions fit_R and fit_solar3.

    The fitting procedure described was applied to a set of solar cells manufactured by an European manufacturer A. The relevant results were saved in form of graphs and tables. An example of the fit output, obtained for a solar cell c308, is shown in Fig. 4. For comparison, *i-u* curves of several   solar cells are added, part a).

Custom Equation:

cf_(x) = 1/234.6*(beta1*r3*x+ 8255983405410741*234.6* lambertw(beta1*r3*i1*exp(1/234.6*beta1* (8255983405410741*234.6*x-r 3*x+8255983405410741*234.6*r3*i1)))- 8255983405410741*234.6*beta1*r3*i1) beta1/r3

b) General model with Lambert W function for c308

Coefficients (with 95% confidence bounds):

beta1 =   16.44 (15.75, 17.14)

i1 = 3.897e-005 (2.527e-005, 5.268e-005)

r3 =   0.2745 (0.2695, 0.2795)

a) Solar cell c301, c302, c308, c310, c312 and c318 with the FIT of c308

c) Coefficients of c308 fitting

Fig. 4. a) *i-u* curves of a group of solar cells b) Custom Equation containing the Lambert W function for sample c308 c) Results of 2nd step fitting. Parallel resistance R = 234.6 Ω obtained in first step.

5. Conclusions

Fitting procedure based on measured *i-u* curves of solar cells are described and demonstrated on series c200 and c300 samples. The procedure utilizes explicit *i-u* curve, taking in account the cell series resistance. Functions of Matlab CF toolbox are widely used. The results of the procedure are parameters: parallel resistance R , r-g factors β_1, i_1 and series resistance r_3. The accuracy of the parameters is better than several percent points. The parameters play central role in solar cell diagnostics.

Acknowledgements

References

[1] S. M. Sze: Physics of Semiconductor Devices, 2nd edition, J.Wiley, New York 1981
[2] R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffrey, and D.E. Knuth. "On The LambertW Function". *Advances in Computational Mathematics 5* (1996): 329-359.
[3] P. Hruska, Z. Chobola and J. Sikula: "Transport reliability indicators for solar cells", *2nd ELEN Workshop*, October 25-27, 1995, Grenoble, France.

Contact information:

Doc. RNDr. Pavel Hruška, CSc.   Technická 8, 616 00 Brno
Ústav fyziky, FEKT VUT v Brně,   Tel. +420 541 143 258, email: hruskap@feec.vutbr.cz.