

# MATLAB ALGORITHMS FOR THE LAPLACE TRANSFORM INVERSION

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## Abstract

**There are currently no MATLAB functions to perform a numerical Laplace transform or a numerical inversion of the Laplace transform, officially supported by the MathWorks, Inc. The author introduces five MATLAB algorithms for the Laplace transform inversion found by means of Internet. The present contribution deals with using them to the numerical inversion of several complicated Laplace transforms described in literature. From the results, the conclusion follows that all the used MATLAB algorithms might be precise and reliable tools to calculate original from the Laplace transform.**

## 1 Introduction

The modern approach to the study of physical world phenomena relies upon the use of mathematical modelling. The description of physical processes leads to a number of equations, the most familiar of which are differential and integral ones. In many cases of importance, the complexity of the equations far exceeds the power of our mathematical capabilities. A useful tool for the solution of ordinary and partial differential equations and integral equations arising in many areas of engineering mathematics is the Laplace transform.

The Laplace transform of an original function  $f(t)$  of a real variable  $t$ , for  $t \geq 0$ , is defined by the integral (if it exists)

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt \quad (1)$$

The Laplace transform (image)  $F(s)$  is a function of a complex variable  $s$ . The integral (1) converges in a half plane

$$Re(s) > c \quad (2)$$

where the value  $c$  is referred to as the abscissa of convergence of Laplace transform. It is the rightmost real part of all singularities of the image  $F(s)$ .

The fundamental importance of Laplace transform consists in its ability to lower the level of equations. The ordinary differential or integral equations involving  $f(t)$  are transformed to the algebraic equations for  $F(s)$ , the partial differential equations in  $f(t)$  are transformed to the ordinal differential equations in  $F(s)$ . Such equations are more simply solvable than the original ones.

Then one has to go back to the original domain by the inversion of resulting Laplace transforms. The inversion of the image  $F(s)$  is the original  $f(t)$ , defined for  $t \geq 0$  by the Bromwich integral

$$f(t) = \frac{1}{2\pi j} \oint_B F(s)e^{st} ds \quad (3)$$

where  $B$  is the Bromwich contour in the complex plane. If all singularities of  $F(s)$  lay in the half plane  $Re(s) < c$ , it is possible to integrate along the line  $s = c + j\omega$  where  $\omega$  varies from  $-\infty$  to  $+\infty$  and the inversion is then defined by the integral

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)e^{st} ds \quad (4)$$

The inversion of Laplace transform is a topic of fundamental importance in many areas of applied mathematics and is found in many applications, such as circuit theory, process control, spectroscopy, medicine, pharmacy, geology etc.

The inversion of Laplace transform may be accomplished

- a) analytically by employing some basic properties of Laplace transform, if  $F(s)$  is a simple function,
- b) by the use of Laplace transform tables (dictionaries), if  $F(s)$  is a common frequently occurring function,
- c) by a numerical approach.

It is not always easy to find the inverse analytically or by the use of Laplace transform tables; therefore, the algorithms for the numerical inversion of Laplace transform are required.

## 2 Algorithms for the numerical inversion of Laplace transform

A survey of more than 1500 publications about the methods of numerical inversion of Laplace transforms together with a Mathematica program to generate and solve the suggested set of test problems and with a Java program to solve inversion problems for a restricted class of image functions have been presented on a www-page [1].

From the point of view of the user, a suitable algorithm should be chosen in accordance with the following requirements:

Algorithm has to

- be written and executable in an universally used standard programming environment,
- be accessible or simply programmable,
- have the minimum number of unknown free parameters,
- be user friendly.

One of the most frequently employed software environment for technical computing applications in standard use at many world's universities is MATLAB. There are currently no functions to perform a numerical Laplace transform or a numerical inversion of the Laplace transform, officially supported by the MathWorks, Inc. [2].

But several algorithms for Laplace transform inversion written in MATLAB do exist. Two algorithms are accessible on www pages of Weideman: `direct` [3] and `weeks` [4]. The third one, `invlap`, has been downloaded from www page [5]. This function has been written by [6]. The algorithms `nilt` and `niltd` have been obtained from [7, 8]. Since no single method gives optimum results for all purposes and all occasions, it is highly desirable to have more such algorithms based on different numerical methods to compare the results.

These algorithms are based on the following different numerical methods:

- `weeks` is based on the expansion of the given Laplace transform in terms of the Laplace transforms of the orthonormal Laguerre functions; the latter expansion is then reduced to a cosine series whose approximate expansion coefficients are obtained by means of trigonometric interpolation [9];
- `direct` and `invlap` are based on the trapezoidal approximation of the Bromwich integral; the resulting series converges slowly and, therefore, a sequence acceleration methods are used to improve the rate of convergence; the code of `direct` is based on equation (2) on page 335 of Ref. [10]; the code of `invlap` is based on Ref. [11];
- the task of `nilt` is to evaluate eq. 4 numerically not only accurately enough but also very fast in the whole interval  $\langle 0; tm \rangle$ , therefore the fast Fourier transformation is applied in order to ensure the high speed of computation, and the method is improved from point of view of reaching higher accuracy by using  $\epsilon$ -algorithm [7].
- a quotient-difference algorithm utilized in the `niltd` algorithm accelerates the convergence of a complex Fourier series replacing it by a continued fraction; as compared with the  $\epsilon$ -algorithm, the q-d algorithm is numerically more stable [8].

Several algorithms have been somewhat modified by the present author to the relatively unified form of MATLAB functions with the following headings:

function `f = weeks (F, t, N, sig, b)`

input formal parameters:

- `F*` Laplace transform to be inverted (a string containing the name of an existing MATLAB m-file, e.g.  $F = 'F7'$ , where function  $f=F7(s)$ ;  $f=1./(s.*(s+1))$  has to be written and saved into the work directory before the execution of the `weeks` function);
- `t` abscissa(s) where original  $f(t)$  is to be calculated (scalar or vector);
- `N` free parameter; number of terms in the Laguerre expansion;
- `sig, b` free parameters;  $sig$  has to satisfy  $sig > c$ , where  $c$  is the Laplace convergence abscissa,  $b$  is an arbitrary real value  $> 0$ ; the optimum values of  $sig$  and  $b$  may be estimated by some other algorithms [4, 12];

output:

- `f` approximate value(s) of the original  $f(t)$  (scalar or vector);

function `f = direct (F, N, t, at)`

input formal parameters:

- `F*` Laplace transform to be inverted; (a string containing the name of an existing MATLAB m-file, like in `weeks`);
- `N` free parameter; number of terms employed in the summation ( $50 < N < 100$  is recommended);
- `t` abscissa where original  $f(t)$  is to be calculated (scalar only);
- `at` free parameter ( $7 < at < 12$  is recommended);

output:

- `f` approximate value of the original  $f(t)$ ;

function `f = invlap (F, t, alpha, tol, P1, P2, P3, P4, P5, P6, P7, P8, P9)`

input formal parameters:

- `F` Laplace transform to be inverted (a string containing the name of an existing MATLAB m-file, like in `weeks`);
- `t` abscissa(s) where original  $f(t)$  is to be calculated (scalar or vector);
- `alpha` free parameter; largest pole of  $F(s)$  (default zero);
- `tol` free parameter; numerical tolerance of approaching pole (default  $1e-9$ );
- `P1, ..., P9` optional parameters to be passed on to  $F(s)$ ;

output:

- `f` approximate value(s) of the original  $f(t)$  (scalar or vector);

The functions `nilt` and `niltqd` have analogical headings:

function `[ft, t] = nilt (F, tm, M, Er)`;

function `[ft, t] = niltqd (F, tm, M, Er)`;

with following identical parameters:

input formal parameters:

- `F` Laplace transform to be inverted (a string containing the name of an existing MATLAB m-file, like in `weeks` function);
- `tm` the maximum value of abscissa where original is to be calculated;
- `M†` number of points to be calculated; it must be  $M = 2^m$ ,  $m$  integer;
- `Er‡` relative error.

output:

- `ft` vector of approximate values of the original  $f(t)$ ;

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\* The original parameter `F` was a symbolic function of  $s$ , e.g.  $F = '1./(s.*(s+1))'$ ; the author's modification consists in the replacement of the `eval(F)` statement by the `feval(F,s)` one in the body of the function.

†‡ Parameters `M` and `Er` were included in the input formal parameters by the author for more easy changeability of their values. The corresponding assignment statements in the function bodies were cancelled.

$t$  vector of the abscissas where original  $f(t)$  is calculated,  $t(1) = 0$ ,  $t(\text{end}) = tm$ .  
The next parameters can be changed in the bodies of both last functions if necessary:  
alfa exponential order of the real function  $f(t)$ ;  
P the parameter of the  $\varepsilon$ -algorithm ( $P = 2$  or  $3$  seems to be a good choice);

The advantage of the last two algorithms – the obtaining of original function at  $M$  discrete points on the whole abscissa interval  $\langle 0; tm \rangle$  – appears to be disadvantageous for testing and comparison with other algorithms, where the choice of  $t$ -points is quite unrestrained and by no means need be equidistant.

A MATLAB script has been written for the use of the above-mentioned functions to the inversion of chosen Laplace image. Functions `nilt` and `niltqd` were called first. Their actual parameters were set for example at  $M = 1024$ ,  $TM = M - 1$ ,  $(M - 1)/10$ ,  $(M - 1)/100$ ,  $(M - 1)/1000$ , so that  $\Delta t = 1, 0.1, 0.01, 0.001$ , respectively,  $Er = 1e-14$ . Afterwards, the values  $K$  and  $tm < TM$  were determined.  $K$  specified which of output values  $t, f(t)$  obtained by the execution of `nilt` and `niltqd` were selected for comparison with the results of other algorithms, e.g. for  $K = 5$  every 5<sup>th</sup> value. The other functions were executed in a loop for 100  $t$ -values from  $\Delta t.K$  with step  $\Delta t.K$  to  $tm$ . All the results were arranged in the table of  $t$ -values and approximate values of the original calculated by individual algorithms.

### 3 Examples

The standard method used to test numerical inversion algorithms consists in employing images whose inverses are known exactly for the estimation of errors [13]. This work was done and described in the previous contributions [14-16]. Finding "more complicated" practical problems that have well-defined solutions is difficult [13]. The present paper deals with the numerical inversion of the complicated Laplace transforms that are archetypical of those found in the scientific and engineering literature [10].

Our task is to find the inverses of the Laplace transforms shown in Table 1. Transform No. 1 arises in circuit theory [17]. In addition to the two poles at  $s = 0$  and  $s = -1$  it has an infinite number of poles on the imaginary axis and does not contain any branch point singularities. The authors [18] inverted a transform similar to No. 2. It has a simple pole at  $s = 0$  (which appears to be a branch point at first sight) together with poles lying along the negative real axis. Transform No. 3 arises in a viscous fluid mechanics problem [19] and incorporates a pole and three branch points. Transform No. 4 is similar to the previous one because it contains a pole at  $s = 0$  as well as four branch points. However,

Table 1: THE LAPLACE TRANSFORMS

No.	$F(s)$	parameter values
1	$\frac{1}{s(s+c)} \left[ \frac{1}{2sh} - \frac{e^{-2sh}}{1-e^{-2sh}} \right]$	$c = 1,$ $h = 1$
2	$\frac{(100s-1)\sinh\left(\frac{\sqrt{s}}{2}\right)}{s \left[ s \sinh(\sqrt{s}) + \sqrt{s} \cosh(\sqrt{s}) \right]}$	
3	$\frac{1}{s} \exp \left[ -r \sqrt{\frac{s(1+s)}{1+cs}} \right]$	$c = 0.4,$ $r = 0.5$
4	$\frac{\exp(-2\Psi)}{s}$ , where $\cosh(\Psi) = \sqrt{1+s^2+s^4/16}$	
5	$\frac{s - \sqrt{s^2 - c^2}}{\sqrt{s} \sqrt{s^2 - c^2} \sqrt{s - N \sqrt{s^2 - c^2}}}$	$N = 0.5,$ $c = 1$

unlike the previous example, the solution is not diffusive but is wavelike because it arises in the study of shock waves in diatomic chains [20]. Transform No. 5 arises in the theory of beams [21] and it has no poles but five branch points. Unlike the earlier transformations, it has a singularity in the right half-plane at  $s = c$ . All these transforms can be inverted analytically but the solutions are given by complicated equations containing infinite sums or integrals.

As an example, the original function corresponding to transform No. 2 can be evaluated by the series

$$f(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \left( 100 + \frac{1}{b_n^2} \right) \frac{2b_n \sin\left(\frac{b_n}{2}\right) \exp(-b_n^2 t)}{(2 + b_n^2) \cos(b_n)} \quad (5)$$

where the coefficient  $b_n$  can be obtained as the subsequent solutions to a nonlinear transcendent equation  $b_n \tan(b_n) = 1$  [10]. The combination of finding an (infinite) number of roots numerically and summing an (infinite) number of terms is not a trivial task, and one can hardly guarantee the accuracy of the independently calculated  $f(t)$  values. If the solution can be found only by another numerical method, which itself is complex enough, then it is practically impossible to estimate the error. It turns out that the most reliable way of calculation of the values of  $f(t)$  function is the numerical inversion of the Laplace transform [13].

Therefore the numerical inversions by means of the above-mentioned algorithms were carried out. The results of calculations are shown in Tables 2-6. Every table contains the values of the abscissa  $t$  and the values of the original  $f(t)$  calculated by means of individual algorithms (`weeks`, `direct`, `invlap`, `nilt` and `niltqd`). A survey of all MATLAB functions calls with their actual-parameter values is introduced under each table. The parameters were changed to find their optimum values in the cases of less accurate results only and otherwise they were kept constant. The results were printed in 6 decimal digits floating point format (7 digits altogether). All computations were executed in MATLAB environment version 6.5 on a standard PC (Pentium 4 1.5 GHz, 256 MB RAM, HD 40 GB, FDD 3.5") under Windows 98.

Table 2: THE ORIGINAL CORRESPONDING TO LAPLACE TRANSFORM No. 1

t	weeks	direct	invlap	nilt	niltqd
0.10	2.369339E-03	2.418709E-03	2.418709E-03	2.418718E-03	2.418717E-03
0.20	9.409020E-03	9.365377E-03	9.365377E-03	9.365382E-03	9.365382E-03
0.30	2.038798E-02	2.040911E-02	2.040911E-02	2.040912E-02	2.040911E-02
0.40	3.519140E-02	3.516002E-02	3.516002E-02	3.516003E-02	3.516003E-02
0.50	5.322490E-02	5.326533E-02	5.326533E-02	5.326534E-02	5.326533E-02
0.60	7.442305E-02	7.440582E-02	7.440582E-02	7.440582E-02	7.440582E-02
0.70	9.830095E-02	1.049546E-01	9.829265E-02	9.829266E-02	9.829266E-02
0.80	1.246454E-01	1.246645E-01	1.246645E-01	1.246645E-01	1.246645E-01
0.90	1.533091E-01	1.532848E-01	1.532848E-01	1.532848E-01	1.532848E-01
1.00	1.839394E-01	1.839397E-01	1.839397E-01	1.839397E-01	1.839397E-01
1.10	2.164279E-01	2.164355E-01	2.164355E-01	2.164356E-01	2.164356E-01
1.20	2.505891E-01	2.505971E-01	2.505971E-01	2.505971E-01	2.505971E-01
1.30	2.862640E-01	2.862659E-01	2.862659E-01	2.862659E-01	2.862659E-01
1.40	3.232842E-01	3.232985E-01	3.232985E-01	3.232985E-01	3.232985E-01
1.50	3.615370E-01	3.615651E-01	3.615651E-01	3.615652E-01	3.615651E-01
1.60	4.008965E-01	4.009483E-01	4.009483E-01	4.009484E-01	4.009482E-01
1.70	4.413557E-01	4.413418E-01	4.413417E-01	4.413420E-01	4.413413E-01
1.80	4.827653E-01	4.826494E-01	4.826491E-01	4.826500E-01	4.826472E-01
1.90	5.246307E-01	5.247846E-01	5.247678E-01	5.247856E-01	5.247842E-01
2.00	5.645221E-01	5.675350E-01	5.640827E-01	5.671672E-01	5.671614E-01
2.10	5.159326E-01	5.160658E-01	5.160235E-01	5.160774E-01	5.160661E-01
2.20	4.742741E-01	4.741323E-01	4.741272E-01	4.741357E-01	4.741337E-01
2.30	4.408289E-01	4.409476E-01	4.409463E-01	4.409491E-01	4.409507E-01

2.40	4.157724E-01	4.156790E-01	4.156778E-01	4.156799E-01	4.156847E-01
2.50	3.975141E-01	3.975732E-01	3.975741E-01	3.975737E-01	3.975779E-01
2.60	3.859536E-01	3.859484E-01	3.859483E-01	3.859489E-01	3.859511E-01
2.70	3.802501E-01	3.801881E-01	3.801878E-01	3.801884E-01	3.801896E-01
2.80	3.796282E-01	3.797340E-01	3.797339E-01	3.797343E-01	3.797351E-01
2.90	3.841508E-01	3.840813E-01	3.840814E-01	3.840815E-01	3.840821E-01
3.00	3.928219E-01	3.927730E-01	3.927758E-01	3.927732E-01	3.927736E-01
3.10	4.052779E-01	4.053957E-01	4.053954E-01	4.053959E-01	4.053962E-01
3.20	4.215744E-01	4.215753E-01	4.215750E-01	4.215755E-01	4.215758E-01
3.30	4.411130E-01	4.409734E-01	4.409754E-01	4.409736E-01	4.409738E-01
3.40	4.632963E-01	4.632836E-01	4.632762E-01	4.632838E-01	4.632840E-01
3.50	4.880589E-01	4.882289E-01	4.882246E-01	4.882291E-01	4.882292E-01
3.60	5.154353E-01	5.155584E-01	5.155848E-01	5.155586E-01	5.155587E-01
3.70	5.451764E-01	5.450451E-01	5.450713E-01	5.450455E-01	5.450456E-01
3.80	5.768540E-01	5.764963E-01	5.762954E-01	5.764845E-01	5.764845E-01
3.90	6.100349E-01	6.097054E-01	6.112868E-01	6.096896E-01	6.096896E-01
4.00	6.399598E-01	6.434129E-01	6.327244E-01	6.439868E-01	6.439868E-01
4.10	5.857975E-01	5.856098E-01	5.884037E-01	5.855802E-01	5.855803E-01
4.20	5.372596E-01	5.372829E-01	5.361573E-01	5.370320E-01	5.370320E-01
4.30	4.980155E-01	4.978597E-01	4.975929E-01	4.978617E-01	4.978617E-01
4.40	4.672762E-01	4.671763E-01	4.674094E-01	4.671770E-01	4.671770E-01
4.50	4.442414E-01	4.441698E-01	4.443054E-01	4.441705E-01	4.441705E-01
4.60	4.281730E-01	4.281111E-01	4.279838E-01	4.281116E-01	4.281115E-01
4.70	4.184018E-01	4.183385E-01	4.183175E-01	4.183389E-01	4.183389E-01
4.80	4.143244E-01	4.142539E-01	4.143115E-01	4.142544E-01	4.142543E-01
4.90	4.153971E-01	4.153177E-01	4.153945E-01	4.153167E-01	4.153166E-01
5.00	4.211289E-01	4.210355E-01	4.210266E-01	4.210361E-01	4.210360E-01
5.10	4.310758E-01	4.309687E-01	4.309302E-01	4.309693E-01	4.309692E-01
5.20	4.448350E-01	4.447147E-01	4.446567E-01	4.447154E-01	4.447153E-01
5.30	4.620394E-01	4.619105E-01	4.620145E-01	4.619115E-01	4.619114E-01
5.40	4.823530E-01	4.822285E-01	4.824978E-01	4.822293E-01	4.822292E-01
5.50	5.054666E-01	5.053709E-01	5.047243E-01	5.053717E-01	5.053716E-01
5.60	5.310945E-01	5.310729E-01	5.297608E-01	5.310699E-01	5.310699E-01
5.70	5.589743E-01	5.590749E-01	5.591980E-01	5.590806E-01	5.590809E-01
5.80	5.888633E-01	5.891662E-01	5.917828E-01	5.891838E-01	5.891843E-01
5.90	6.204779E-01	6.215458E-01	6.249755E-01	6.211801E-01	6.211810E-01
6.00	6.491033E-01	6.529017E-01	6.343713E-01	6.543833E-01	6.543848E-01
6.10	5.939099E-01	5.956539E-01	6.001269E-01	5.949867E-01	5.949889E-01
6.20	5.446294E-01	5.456030E-01	5.469007E-01	5.455423E-01	5.455455E-01
6.30	5.049569E-01	5.055548E-01	5.057896E-01	5.055610E-01	5.055652E-01
6.40	4.740614E-01	4.741465E-01	4.737429E-01	4.741424E-01	4.741478E-01
6.50	4.509624E-01	4.504741E-01	4.465854E-01	4.504718E-01	4.504782E-01
6.60	4.346211E-01	4.338180E-01	4.300110E-01	4.338119E-01	4.338192E-01
6.70	4.241063E-01	4.235014E-01	4.205320E-01	4.234957E-01	4.235038E-01
6.80	4.188398E-01	4.189257E-01	4.182813E-01	4.189194E-01	4.189281E-01
6.90	4.187264E-01	4.195434E-01	4.207753E-01	4.195368E-01	4.195461E-01
7.00	4.239284E-01	4.248604E-01	4.280977E-01	4.248536E-01	4.248634E-01
7.10	4.342763E-01	4.344296E-01	4.365569E-01	4.344226E-01	4.344330E-01
7.20	4.487762E-01	4.478465E-01	4.480673E-01	4.478391E-01	4.478500E-01
7.30	4.659109E-01	4.647441E-01	4.631921E-01	4.647370E-01	4.647485E-01
7.40	4.848377E-01	4.847928E-01	4.830062E-01	4.847850E-01	4.847971E-01
7.50	5.063479E-01	5.076888E-01	5.059901E-01	5.076833E-01	5.076961E-01
7.60	5.320107E-01	5.331740E-01	5.344100E-01	5.331606E-01	5.331742E-01
7.70	5.617340E-01	5.609673E-01	5.665941E-01	5.609716E-01	5.609860E-01

7.80	5.927381E-01	5.910557E-01	6.001362E-01	5.908941E-01	5.909094E-01
7.90	6.224092E-01	6.220818E-01	6.226735E-01	6.227270E-01	6.227434E-01
8.00	6.479180E-01	6.527544E-01	6.247478E-01	6.557826E-01	6.558001E-01
8.10	5.930868E-01	5.958857E-01	6.001738E-01	5.962526E-01	5.962713E-01
8.20	5.479589E-01	5.467818E-01	5.554065E-01	5.466878E-01	5.467079E-01
8.30	5.099467E-01	5.062689E-01	5.120714E-01	5.065976E-01	5.066194E-01
8.40	4.745498E-01	4.752192E-01	4.744126E-01	4.750807E-01	4.751042E-01
8.50	4.474154E-01	4.512953E-01	4.480478E-01	4.513212E-01	4.513466E-01
8.60	4.345715E-01	4.346080E-01	4.317582E-01	4.345808E-01	4.346083E-01
8.70	4.287849E-01	4.241793E-01	4.235284E-01	4.241917E-01	4.242216E-01
8.80	4.198285E-01	4.195517E-01	4.214044E-01	4.195494E-01	4.195819E-01
8.90	4.146934E-01	4.201161E-01	4.215273E-01	4.201072E-01	4.201426E-01
9.00	4.253870E-01	4.253787E-01	4.262795E-01	4.253701E-01	4.254087E-01
9.10	4.413331E-01	4.348972E-01	4.285402E-01	4.348905E-01	4.349326E-01
9.20	4.471284E-01	4.482779E-01	4.437886E-01	4.482632E-01	4.483093E-01
9.30	4.577386E-01	4.651529E-01	4.543300E-01	4.651218E-01	4.651722E-01
9.40	4.886346E-01	4.851595E-01	4.794174E-01	4.851345E-01	4.851896E-01
9.50	5.156046E-01	5.080123E-01	5.070860E-01	5.080012E-01	5.080617E-01
9.60	5.262274E-01	5.330230E-01	5.438271E-01	5.334506E-01	5.335169E-01
9.70	5.554619E-01	5.611620E-01	5.707905E-01	5.612368E-01	5.613095E-01
9.80	6.028668E-01	5.917495E-01	6.117139E-01	5.911376E-01	5.912175E-01
9.90	6.229370E-01	6.205661E-01	6.235114E-01	6.229519E-01	6.230395E-01
10.00	6.353139E-01	6.519169E-01	6.110757E-01	6.559916E-01	6.560879E-01

M = 1024; TM = 10.23; K = 10; tm = 10;  
[ft6, t6] = nilt (F, TM, M, 1e-14); [ft7, t6] = niltqd (F, TM, M, 1e-14); f2 = weeks(F, t, 1024, 1, 10);  
f4 = direct(F, 100, t, 12); f5 = invlap(F, t, 1, 1e-14);

Table 3: THE ORIGINAL CORRESPONDING TO LAPLACE TRANSFORM No. 2

t	weeks	direct	invlap	nilt	niltqd
0.10	2.133334E+01	2.133334E+01	2.133334E+01	2.133334E+01	2.133334E+01
0.20	2.966946E+01	2.966946E+01	2.966946E+01	2.966946E+01	2.966946E+01
0.30	3.056792E+01	3.056792E+01	3.056792E+01	3.056792E+01	3.056792E+01
0.40	2.929484E+01	2.929484E+01	2.929484E+01	2.929484E+01	2.929484E+01
0.50	2.746094E+01	2.746094E+01	2.746094E+01	2.746094E+01	2.746094E+01
0.60	2.555633E+01	2.555633E+01	2.555633E+01	2.555633E+01	2.555633E+01
0.70	2.372527E+01	2.372527E+01	2.372527E+01	2.372527E+01	2.372527E+01
0.80	2.200556E+01	2.200556E+01	2.200556E+01	2.200556E+01	2.200556E+01
0.90	2.040258E+01	2.040258E+01	2.040258E+01	2.040258E+01	2.040258E+01
1.00	1.891213E+01	1.891213E+01	1.891213E+01	1.891213E+01	1.891213E+01
1.10	1.752743E+01	1.752743E+01	1.752743E+01	1.752743E+01	1.752743E+01
1.20	1.624136E+01	1.624136E+01	1.624136E+01	1.624136E+01	1.624136E+01
1.30	1.504698E+01	1.504698E+01	1.504698E+01	1.504698E+01	1.504698E+01
1.40	1.393779E+01	1.393779E+01	1.393779E+01	1.393779E+01	1.393779E+01
1.50	1.290774E+01	1.290774E+01	1.290774E+01	1.290774E+01	1.290774E+01
1.60	1.195117E+01	1.195117E+01	1.195117E+01	1.195117E+01	1.195117E+01
1.70	1.106285E+01	1.106285E+01	1.106285E+01	1.106285E+01	1.106285E+01
1.80	1.023791E+01	1.023791E+01	1.023791E+01	1.023791E+01	1.023791E+01
1.90	9.471816E+00	9.471816E+00	9.471816E+00	9.471816E+00	9.471816E+00
2.00	8.760382E+00	8.760382E+00	8.760382E+00	8.760382E+00	8.760382E+00
2.10	8.099705E+00	8.099705E+00	8.099705E+00	8.099705E+00	8.099705E+00
2.20	7.486164E+00	7.486164E+00	7.486164E+00	7.486164E+00	7.486164E+00
2.30	6.916395E+00	6.916395E+00	6.916395E+00	6.916395E+00	6.916395E+00
2.40	6.387277E+00	6.387277E+00	6.387277E+00	6.387277E+00	6.387277E+00

2.50	5.895908E+00	5.895908E+00	5.895908E+00	5.895908E+00	5.895908E+00
2.60	5.439595E+00	5.439595E+00	5.439595E+00	5.439595E+00	5.439595E+00
2.70	5.015838E+00	5.015838E+00	5.015838E+00	5.015838E+00	5.015838E+00
2.80	4.622314E+00	4.622314E+00	4.622314E+00	4.622314E+00	4.622314E+00
2.90	4.256865E+00	4.256865E+00	4.256865E+00	4.256865E+00	4.256865E+00
3.00	3.917489E+00	3.917489E+00	3.917489E+00	3.917489E+00	3.917489E+00
3.10	3.602326E+00	3.602326E+00	3.602326E+00	3.602326E+00	3.602326E+00
3.20	3.309647E+00	3.309647E+00	3.309647E+00	3.309647E+00	3.309647E+00
3.30	3.037850E+00	3.037850E+00	3.037850E+00	3.037850E+00	3.037850E+00
3.40	2.785444E+00	2.785444E+00	2.785444E+00	2.785444E+00	2.785444E+00
3.50	2.551046E+00	2.551046E+00	2.551046E+00	2.551046E+00	2.551046E+00
3.60	2.333371E+00	2.333371E+00	2.333371E+00	2.333371E+00	2.333371E+00
3.70	2.131225E+00	2.131225E+00	2.131225E+00	2.131225E+00	2.131225E+00
3.80	1.943502E+00	1.943502E+00	1.943502E+00	1.943502E+00	1.943502E+00
3.90	1.769172E+00	1.769172E+00	1.769172E+00	1.769172E+00	1.769172E+00
4.00	1.607279E+00	1.607279E+00	1.607279E+00	1.607279E+00	1.607279E+00
4.10	1.456936E+00	1.456936E+00	1.456936E+00	1.456936E+00	1.456936E+00
4.20	1.317320E+00	1.317320E+00	1.317320E+00	1.317320E+00	1.317320E+00
4.30	1.187664E+00	1.187664E+00	1.187664E+00	1.187664E+00	1.187664E+00
4.40	1.067259E+00	1.067259E+00	1.067259E+00	1.067259E+00	1.067259E+00
4.50	9.554434E-01	9.554434E-01	9.554434E-01	9.554434E-01	9.554434E-01
4.60	8.516056E-01	8.516056E-01	8.516056E-01	8.516056E-01	8.516056E-01
4.70	7.551760E-01	7.551760E-01	7.551760E-01	7.551760E-01	7.551760E-01
4.80	6.656262E-01	6.656262E-01	6.656262E-01	6.656262E-01	6.656262E-01
4.90	5.824652E-01	5.824652E-01	5.824652E-01	5.824652E-01	5.824652E-01
5.00	5.052373E-01	5.052373E-01	5.052373E-01	5.052373E-01	5.052373E-01
5.10	4.335192E-01	4.335192E-01	4.335192E-01	4.335192E-01	4.335192E-01
5.20	3.669178E-01	3.669178E-01	3.669178E-01	3.669178E-01	3.669178E-01
5.30	3.050680E-01	3.050680E-01	3.050680E-01	3.050680E-01	3.050680E-01
5.40	2.476309E-01	2.476309E-01	2.476309E-01	2.476309E-01	2.476309E-01
5.50	1.942916E-01	1.942916E-01	1.942916E-01	1.942916E-01	1.942916E-01
5.60	1.447577E-01	1.447577E-01	1.447577E-01	1.447577E-01	1.447577E-01
5.70	9.875781E-02	9.875782E-02	9.875782E-02	9.875782E-02	9.875782E-02
5.80	5.603977E-02	5.603977E-02	5.603977E-02	5.603977E-02	5.603977E-02
5.90	1.636942E-02	1.636942E-02	1.636942E-02	1.636942E-02	1.636942E-02
6.00	-2.047067E-02	-2.047068E-02	-2.047068E-02	-2.047068E-02	-2.047068E-02
6.10	-5.468243E-02	-5.468244E-02	-5.468244E-02	-5.468244E-02	-5.468244E-02
6.20	-8.645336E-02	-8.645337E-02	-8.645337E-02	-8.645337E-02	-8.645337E-02
6.30	-1.159576E-01	-1.159576E-01	-1.159576E-01	-1.159576E-01	-1.159576E-01
6.40	-1.433569E-01	-1.433568E-01	-1.433569E-01	-1.433569E-01	-1.433569E-01
6.50	-1.688014E-01	-1.688014E-01	-1.688014E-01	-1.688014E-01	-1.688014E-01
6.60	-1.924306E-01	-1.924306E-01	-1.924306E-01	-1.924306E-01	-1.924306E-01
6.70	-2.143740E-01	-2.143740E-01	-2.143740E-01	-2.143740E-01	-2.143740E-01
6.80	-2.347518E-01	-2.347385E-01	-2.347518E-01	-2.347518E-01	-2.347518E-01
6.90	-2.536758E-01	-2.536758E-01	-2.536758E-01	-2.536758E-01	-2.536758E-01
7.00	-2.712497E-01	-2.712497E-01	-2.712497E-01	-2.712497E-01	-2.712497E-01
7.10	-2.875697E-01	-2.875697E-01	-2.875697E-01	-2.875697E-01	-2.875697E-01
7.20	-3.027255E-01	-3.027255E-01	-3.027255E-01	-3.027255E-01	-3.027255E-01
7.30	-3.167999E-01	-3.167999E-01	-3.167999E-01	-3.167999E-01	-3.167999E-01
7.40	-3.298702E-01	-3.298702E-01	-3.298702E-01	-3.298702E-01	-3.298702E-01
7.50	-3.420080E-01	-3.420080E-01	-3.420080E-01	-3.420080E-01	-3.420080E-01
7.60	-3.532799E-01	-3.532799E-01	-3.532799E-01	-3.532799E-01	-3.532799E-01
7.70	-3.637475E-01	-3.637475E-01	-3.637475E-01	-3.637475E-01	-3.637475E-01
7.80	-3.734684E-01	-3.734684E-01	-3.734684E-01	-3.734684E-01	-3.734684E-01



7.90	-3.824957E-01	-3.824957E-01	-3.824957E-01	-3.824957E-01	-3.824957E-01
8.00	-3.908790E-01	-3.908790E-01	-3.908790E-01	-3.908790E-01	-3.908790E-01
8.10	-3.986642E-01	-3.986642E-01	-3.986642E-01	-3.986642E-01	-3.986642E-01
8.20	-4.058939E-01	-4.058939E-01	-4.058939E-01	-4.058939E-01	-4.058939E-01
8.30	-4.126079E-01	-4.126079E-01	-4.126079E-01	-4.126079E-01	-4.126079E-01
8.40	-4.188428E-01	-4.188428E-01	-4.188428E-01	-4.188428E-01	-4.188428E-01
8.50	-4.246329E-01	-4.246329E-01	-4.246329E-01	-4.246329E-01	-4.246329E-01
8.60	-4.300100E-01	-4.300100E-01	-4.300100E-01	-4.300100E-01	-4.300100E-01
8.70	-4.350034E-01	-4.350034E-01	-4.350034E-01	-4.350034E-01	-4.350034E-01
8.80	-4.396405E-01	-4.396405E-01	-4.396405E-01	-4.396405E-01	-4.396405E-01
8.90	-4.439468E-01	-4.439468E-01	-4.439468E-01	-4.439468E-01	-4.439468E-01
9.00	-4.479459E-01	-4.479459E-01	-4.479459E-01	-4.479459E-01	-4.479459E-01
9.10	-4.516597E-01	-4.516597E-01	-4.516597E-01	-4.516597E-01	-4.516597E-01
9.20	-4.551085E-01	-4.551085E-01	-4.551085E-01	-4.551085E-01	-4.551085E-01
9.30	-4.583112E-01	-4.583113E-01	-4.583112E-01	-4.583113E-01	-4.583113E-01
9.40	-4.612855E-01	-4.612855E-01	-4.612855E-01	-4.612855E-01	-4.612855E-01
9.50	-4.640476E-01	-4.640476E-01	-4.640476E-01	-4.640476E-01	-4.640476E-01
9.60	-4.666126E-01	-4.666126E-01	-4.666126E-01	-4.666126E-01	-4.666126E-01
9.70	-4.689946E-01	-4.689946E-01	-4.689946E-01	-4.689946E-01	-4.689946E-01
9.80	-4.712067E-01	-4.712067E-01	-4.712066E-01	-4.712067E-01	-4.712067E-01
9.90	-4.732609E-01	-4.732609E-01	-4.732609E-01	-4.732609E-01	-4.732609E-01
10.00	-4.751686E-01	-4.751686E-01	-4.751688E-01	-4.751686E-01	-4.751686E-01

M = 1024; TM = 10.23; K = 10; tm = 10;  
[ft6, t6] = nilt (F, TM, M, 1e-14); [ft7, t6] = niltqd (F, TM, M, 1e-14);  
f2 = weeks(F, t, 1024, 0.4, 10); f4 = direct(F, 50, t, 8); f5 = invlap(F, t, 0.4, 1e-14);

Table 4: THE ORIGINAL CORRESPONDING TO LAPLACE TRANSFORM No. 3

t	weeks	direct	invlap	nilt	niltqd
0.04	5.450944E-03	5.450944E-03	5.450944E-03	5.450944E-03	5.450944E-03
0.08	5.241004E-02	5.241004E-02	5.241004E-02	5.241004E-02	5.241004E-02
0.12	1.194896E-01	1.194896E-01	1.194896E-01	1.194896E-01	1.194896E-01
0.16	1.861622E-01	1.861622E-01	1.861622E-01	1.861622E-01	1.861622E-01
0.20	2.470948E-01	2.470948E-01	2.470948E-01	2.470948E-01	2.470948E-01
0.24	3.015363E-01	3.015363E-01	3.015363E-01	3.015363E-01	3.015363E-01
0.28	3.499390E-01	3.499390E-01	3.499390E-01	3.499390E-01	3.499390E-01
0.32	3.930096E-01	3.930096E-01	3.930096E-01	3.930096E-01	3.930096E-01
0.36	4.314408E-01	4.314408E-01	4.314408E-01	4.314408E-01	4.314408E-01
0.40	4.658434E-01	4.658434E-01	4.658434E-01	4.658434E-01	4.658434E-01
0.44	4.967383E-01	4.967383E-01	4.967383E-01	4.967383E-01	4.967383E-01
0.48	5.245659E-01	5.245659E-01	5.245659E-01	5.245659E-01	5.245659E-01
0.52	5.496988E-01	5.496988E-01	5.496988E-01	5.496988E-01	5.496988E-01
0.56	5.724536E-01	5.724536E-01	5.724536E-01	5.724536E-01	5.724536E-01
0.60	5.931014E-01	5.931014E-01	5.931014E-01	5.931014E-01	5.931014E-01
0.64	6.118753E-01	6.118753E-01	6.118753E-01	6.118753E-01	6.118753E-01
0.68	6.289772E-01	6.289772E-01	6.289772E-01	6.289772E-01	6.289772E-01
0.72	6.445829E-01	6.445829E-01	6.445829E-01	6.445829E-01	6.445829E-01
0.76	6.588465E-01	6.588465E-01	6.588465E-01	6.588465E-01	6.588465E-01
0.80	6.719034E-01	6.719034E-01	6.719034E-01	6.719034E-01	6.719034E-01
0.84	6.838733E-01	6.838733E-01	6.838733E-01	6.838733E-01	6.838733E-01
0.88	6.948621E-01	6.948621E-01	6.948621E-01	6.948621E-01	6.948621E-01
0.92	7.049642E-01	7.049642E-01	7.049642E-01	7.049642E-01	7.049642E-01
0.96	7.142637E-01	7.142637E-01	7.142637E-01	7.142637E-01	7.142637E-01
1.00	7.228359E-01	7.228359E-01	7.228359E-01	7.228359E-01	7.228359E-01

1.04	7.307481E-01	7.307481E-01	7.307481E-01	7.307481E-01	7.307481E-01
1.08	7.380610E-01	7.380610E-01	7.380610E-01	7.380610E-01	7.380610E-01
1.12	7.448288E-01	7.448288E-01	7.448288E-01	7.448288E-01	7.448288E-01
1.16	7.511007E-01	7.511007E-01	7.511007E-01	7.511007E-01	7.511007E-01
1.20	7.569208E-01	7.569208E-01	7.569208E-01	7.569208E-01	7.569208E-01
1.24	7.623293E-01	7.623293E-01	7.623293E-01	7.623293E-01	7.623293E-01
1.28	7.673621E-01	7.673621E-01	7.673621E-01	7.673621E-01	7.673621E-01
1.32	7.720521E-01	7.720521E-01	7.720521E-01	7.720521E-01	7.720521E-01
1.36	7.764288E-01	7.764288E-01	7.764288E-01	7.764288E-01	7.764288E-01
1.40	7.805191E-01	7.805191E-01	7.805191E-01	7.805191E-01	7.805191E-01
1.44	7.843474E-01	7.843474E-01	7.843474E-01	7.843474E-01	7.843474E-01
1.48	7.879358E-01	7.879358E-01	7.879358E-01	7.879358E-01	7.879358E-01
1.52	7.913045E-01	7.913045E-01	7.913045E-01	7.913045E-01	7.913045E-01
1.56	7.944718E-01	7.944718E-01	7.944718E-01	7.944718E-01	7.944718E-01
1.60	7.974542E-01	7.974542E-01	7.974542E-01	7.974542E-01	7.974542E-01
1.64	8.002669E-01	8.002669E-01	8.002669E-01	8.002669E-01	8.002669E-01
1.68	8.029238E-01	8.029238E-01	8.029238E-01	8.029238E-01	8.029238E-01
1.72	8.054373E-01	8.054373E-01	8.054373E-01	8.054373E-01	8.054373E-01
1.76	8.078190E-01	8.078190E-01	8.078190E-01	8.078190E-01	8.078190E-01
1.80	8.100792E-01	8.100792E-01	8.100792E-01	8.100792E-01	8.100792E-01
1.84	8.122275E-01	8.122275E-01	8.122275E-01	8.122275E-01	8.122275E-01
1.88	8.142725E-01	8.142725E-01	8.142725E-01	8.142725E-01	8.142725E-01
1.92	8.162223E-01	8.162223E-01	8.162223E-01	8.162223E-01	8.162223E-01
1.96	8.180840E-01	8.180840E-01	8.180840E-01	8.180840E-01	8.180840E-01
2.00	8.198642E-01	8.198642E-01	8.198642E-01	8.198642E-01	8.198642E-01
2.04	8.215691E-01	8.215691E-01	8.215691E-01	8.215691E-01	8.215691E-01
2.08	8.232041E-01	8.232041E-01	8.232041E-01	8.232041E-01	8.232041E-01
2.12	8.247742E-01	8.247742E-01	8.247742E-01	8.247742E-01	8.247742E-01
2.16	8.262842E-01	8.262633E-01	8.262842E-01	8.262842E-01	8.262842E-01
2.20	8.277383E-01	8.277114E-01	8.277383E-01	8.277383E-01	8.277383E-01
2.24	8.291403E-01	8.291403E-01	8.291403E-01	8.291403E-01	8.291403E-01
2.28	8.304938E-01	8.304938E-01	8.304938E-01	8.304938E-01	8.304938E-01
2.32	8.318021E-01	8.318021E-01	8.318021E-01	8.318021E-01	8.318021E-01
2.36	8.330681E-01	8.330681E-01	8.330681E-01	8.330681E-01	8.330681E-01
2.40	8.342946E-01	8.342946E-01	8.342946E-01	8.342946E-01	8.342946E-01
2.44	8.354841E-01	8.354841E-01	8.354841E-01	8.354841E-01	8.354841E-01
2.48	8.366389E-01	8.366389E-01	8.366389E-01	8.366389E-01	8.366389E-01
2.52	8.377612E-01	8.377612E-01	8.377612E-01	8.377612E-01	8.377612E-01
2.56	8.388529E-01	8.388529E-01	8.388529E-01	8.388529E-01	8.388529E-01
2.60	8.399158E-01	8.399158E-01	8.399158E-01	8.399158E-01	8.399158E-01
2.64	8.409515E-01	8.409515E-01	8.409515E-01	8.409515E-01	8.409515E-01
2.68	8.419616E-01	8.419616E-01	8.419616E-01	8.419616E-01	8.419616E-01
2.72	8.429476E-01	8.429476E-01	8.429476E-01	8.429476E-01	8.429476E-01
2.76	8.439106E-01	8.439106E-01	8.439106E-01	8.439106E-01	8.439106E-01
2.80	8.448519E-01	8.448519E-01	8.448519E-01	8.448519E-01	8.448519E-01
2.84	8.457726E-01	8.457726E-01	8.457726E-01	8.457726E-01	8.457726E-01
2.88	8.466737E-01	8.466737E-01	8.466737E-01	8.466737E-01	8.466737E-01
2.92	8.475562E-01	8.475562E-01	8.475562E-01	8.475562E-01	8.475562E-01
2.96	8.484209E-01	8.484209E-01	8.484209E-01	8.484209E-01	8.484209E-01
3.00	8.492687E-01	8.492687E-01	8.492687E-01	8.492687E-01	8.492687E-01
3.04	8.501004E-01	8.501004E-01	8.501004E-01	8.501004E-01	8.501004E-01
3.08	8.509165E-01	8.509165E-01	8.509165E-01	8.509165E-01	8.509165E-01
3.12	8.517179E-01	8.517179E-01	8.517179E-01	8.517179E-01	8.517179E-01
3.16	8.525050E-01	8.525050E-01	8.525050E-01	8.525050E-01	8.525050E-01

3.20	8.532784E-01	8.532784E-01	8.532784E-01	8.532784E-01	8.532784E-01
3.24	8.540388E-01	8.540388E-01	8.540388E-01	8.540388E-01	8.540388E-01
3.28	8.547865E-01	8.547865E-01	8.547865E-01	8.547865E-01	8.547865E-01
3.32	8.555221E-01	8.555221E-01	8.555221E-01	8.555221E-01	8.555221E-01
3.36	8.562460E-01	8.562460E-01	8.562460E-01	8.562460E-01	8.562460E-01
3.40	8.569585E-01	8.569585E-01	8.569585E-01	8.569585E-01	8.569585E-01
3.44	8.576601E-01	8.576601E-01	8.576601E-01	8.576601E-01	8.576601E-01
3.48	8.583511E-01	8.583511E-01	8.583511E-01	8.583511E-01	8.583511E-01
3.52	8.590318E-01	8.590318E-01	8.590318E-01	8.590318E-01	8.590318E-01
3.56	8.597026E-01	8.597026E-01	8.597026E-01	8.597026E-01	8.597026E-01
3.60	8.603638E-01	8.603638E-01	8.603638E-01	8.603638E-01	8.603638E-01
3.64	8.610157E-01	8.610157E-01	8.610157E-01	8.610157E-01	8.610157E-01
3.68	8.616585E-01	8.616585E-01	8.616585E-01	8.616585E-01	8.616585E-01
3.72	8.622924E-01	8.622924E-01	8.622924E-01	8.622924E-01	8.622924E-01
3.76	8.629178E-01	8.629178E-01	8.629178E-01	8.629178E-01	8.629178E-01
3.80	8.635348E-01	8.635348E-01	8.635348E-01	8.635348E-01	8.635348E-01
3.84	8.641436E-01	8.641436E-01	8.641436E-01	8.641436E-01	8.641436E-01
3.88	8.647445E-01	8.647445E-01	8.647445E-01	8.647445E-01	8.647445E-01
3.92	8.653377E-01	8.653377E-01	8.653377E-01	8.653377E-01	8.653377E-01
3.96	8.659234E-01	8.659234E-01	8.659234E-01	8.659234E-01	8.659234E-01
4.00	8.665016E-01	8.665016E-01	8.665016E-01	8.665016E-01	8.665016E-01

M = 1024; TM = 10.23; K = 4; tm = 4;  
[ft6, t6] = nilt (F, TM, M, 1e-14); [ft7, t6] = niltqd (F, TM, M, 1e-14); f2 = weeks(F, t, 1024, 1, 10);  
f4 = direct(F, 50, t, 7); f5 = invlap(F, t, 1, 1e-14);

Table 5: THE ORIGINAL CORRESPONDING TO LAPLACE TRANSFORM No. 4

t	weeks	direct	invlap	nilt	niltqd
0.12	3.429563E-05	3.429563E-05	3.429563E-05	3.429563E-05	3.429563E-05
0.24	5.362412E-04	5.362412E-04	5.362412E-04	5.362412E-04	5.362412E-04
0.36	2.612657E-03	2.612657E-03	2.612657E-03	2.612657E-03	2.612657E-03
0.48	7.826851E-03	7.826851E-03	7.826851E-03	7.826851E-03	7.826851E-03
0.60	1.784149E-02	1.784149E-02	1.784149E-02	1.784149E-02	1.784149E-02
0.72	3.403389E-02	3.403389E-02	3.403389E-02	3.403389E-02	3.403389E-02
0.84	5.716805E-02	5.716805E-02	5.716805E-02	5.716805E-02	5.716805E-02
0.96	8.719543E-02	8.719543E-02	8.719543E-02	8.719543E-02	8.719543E-02
1.08	1.232284E-01	1.232284E-01	1.232284E-01	1.232284E-01	1.232284E-01
1.20	1.636922E-01	1.636922E-01	1.636922E-01	1.636922E-01	1.636922E-01
1.32	2.066231E-01	2.066231E-01	2.066231E-01	2.066231E-01	2.066231E-01
1.44	2.500472E-01	2.500472E-01	2.500472E-01	2.500472E-01	2.500472E-01
1.56	2.923581E-01	2.923581E-01	2.923581E-01	2.923581E-01	2.923581E-01
1.68	3.326110E-01	3.326110E-01	3.326110E-01	3.326110E-01	3.326110E-01
1.80	3.706697E-01	3.706697E-01	3.706697E-01	3.706697E-01	3.706697E-01
1.92	4.071745E-01	4.071745E-01	4.071745E-01	4.071745E-01	4.071745E-01
2.04	4.433396E-01	4.433396E-01	4.433396E-01	4.433396E-01	4.433396E-01
2.16	4.806234E-01	4.806234E-01	4.806234E-01	4.806234E-01	4.806234E-01
2.28	5.203467E-01	5.203467E-01	5.203467E-01	5.203467E-01	5.203467E-01
2.40	5.633402E-01	5.633402E-01	5.633402E-01	5.633402E-01	5.633402E-01
2.52	6.097010E-01	6.097010E-01	6.097010E-01	6.097010E-01	6.097010E-01
2.64	6.587116E-01	6.587116E-01	6.587116E-01	6.587116E-01	6.587116E-01
2.76	7.089376E-01	7.089376E-01	7.089376E-01	7.089376E-01	7.089376E-01
2.88	7.584861E-01	7.584861E-01	7.584861E-01	7.584861E-01	7.584861E-01
3.00	8.053660E-01	8.053660E-01	8.053660E-01	8.053660E-01	8.053660E-01
3.12	8.478737E-01	8.478737E-01	8.478737E-01	8.478737E-01	8.478737E-01

3.24	8.849183E-01	8.849183E-01	8.849183E-01	8.849183E-01	8.849183E-01
3.36	9.162180E-01	9.162180E-01	9.162180E-01	9.162180E-01	9.162180E-01
3.48	9.423228E-01	9.423228E-01	9.423228E-01	9.423228E-01	9.423228E-01
3.60	9.644604E-01	9.644604E-01	9.644604E-01	9.644604E-01	9.644604E-01
3.72	9.842390E-01	9.842390E-01	9.842390E-01	9.842390E-01	9.842390E-01
3.84	1.003272E+00	1.003272E+00	1.003272E+00	1.003272E+00	1.003272E+00
3.96	1.022808E+00	1.022808E+00	1.022808E+00	1.022808E+00	1.022808E+00
4.08	1.043443E+00	1.043443E+00	1.043443E+00	1.043443E+00	1.043443E+00
4.20	1.064986E+00	1.064986E+00	1.064986E+00	1.064986E+00	1.064986E+00
4.32	1.086496E+00	1.086496E+00	1.086496E+00	1.086496E+00	1.086496E+00
4.44	1.106492E+00	1.106492E+00	1.106492E+00	1.106492E+00	1.106492E+00
4.56	1.123285E+00	1.123285E+00	1.123285E+00	1.123285E+00	1.123285E+00
4.68	1.135356E+00	1.135356E+00	1.135356E+00	1.135356E+00	1.135356E+00
4.80	1.141707E+00	1.141707E+00	1.141707E+00	1.141707E+00	1.141707E+00
4.92	1.142095E+00	1.142095E+00	1.142095E+00	1.142095E+00	1.142095E+00
5.04	1.137112E+00	1.137112E+00	1.137112E+00	1.137112E+00	1.137112E+00
5.16	1.128076E+00	1.128076E+00	1.128076E+00	1.128076E+00	1.128076E+00
5.28	1.116778E+00	1.116778E+00	1.116778E+00	1.116778E+00	1.116778E+00
5.40	1.105116E+00	1.105116E+00	1.105116E+00	1.105116E+00	1.105116E+00
5.52	1.094721E+00	1.094721E+00	1.094721E+00	1.094721E+00	1.094721E+00
5.64	1.086634E+00	1.086634E+00	1.086634E+00	1.086634E+00	1.086634E+00
5.76	1.081119E+00	1.081119E+00	1.081119E+00	1.081119E+00	1.081119E+00
5.88	1.077643E+00	1.077643E+00	1.077643E+00	1.077643E+00	1.077643E+00
6.00	1.075036E+00	1.075036E+00	1.075036E+00	1.075036E+00	1.075036E+00
6.12	1.071784E+00	1.071784E+00	1.071784E+00	NaN	1.071784E+00
6.24	1.066402E+00	1.066402E+00	1.066402E+00	1.066402E+00	1.066402E+00
6.36	1.057794E+00	1.057794E+00	1.057794E+00	1.057794E+00	1.057794E+00
6.48	1.045529E+00	1.045529E+00	1.045529E+00	1.045529E+00	1.045529E+00
6.60	1.029971E+00	1.029971E+00	1.029971E+00	1.029971E+00	1.029971E+00
6.72	1.012230E+00	1.012230E+00	1.012230E+00	1.012230E+00	1.012230E+00
6.84	9.939493E-01	9.939493E-01	9.939489E-01	9.939493E-01	9.939493E-01
6.96	9.769735E-01	9.769735E-01	9.769719E-01	9.769735E-01	9.769735E-01
7.08	9.629736E-01	9.629736E-01	9.629753E-01	9.629736E-01	9.629736E-01
7.20	9.531035E-01	9.531035E-01	9.531054E-01	NaN	9.531035E-01
7.32	9.477699E-01	9.477699E-01	9.477697E-01	9.477699E-01	9.477699E-01
7.44	9.465620E-01	9.465620E-01	9.465471E-01	NaN	9.465620E-01
7.56	9.483567E-01	9.483567E-01	9.483555E-01	9.483567E-01	9.483567E-01
7.68	9.515764E-01	9.515764E-01	9.516186E-01	9.515764E-01	9.515764E-01
7.80	9.545419E-01	9.545419E-01	9.545114E-01	9.545419E-01	9.545419E-01
7.92	9.558438E-01	9.558438E-01	9.559754E-01	9.558438E-01	9.558438E-01
8.04	9.546512E-01	9.546512E-01	9.547515E-01	9.546512E-01	9.546512E-01
8.16	9.508900E-01	9.508900E-01	9.508887E-01	9.508900E-01	9.508900E-01
8.28	9.452529E-01	9.452529E-01	9.451369E-01	9.452529E-01	9.452529E-01
8.40	9.390390E-01	9.390390E-01	9.389613E-01	9.390390E-01	9.390390E-01
8.52	9.338594E-01	9.338594E-01	9.338125E-01	9.338594E-01	9.338594E-01
8.64	9.312722E-01	9.312722E-01	9.313750E-01	9.312722E-01	9.312722E-01
8.76	9.324287E-01	9.324287E-01	9.325611E-01	9.324287E-01	9.324287E-01
8.88	9.378073E-01	9.378073E-01	9.378236E-01	NaN	9.378073E-01
9.00	9.470938E-01	9.470938E-01	9.476306E-01	9.470938E-01	9.470938E-01
9.12	9.592336E-01	9.592336E-01	9.591619E-01	9.592336E-01	9.592336E-01
9.24	9.726451E-01	9.726451E-01	9.725155E-01	NaN	9.726451E-01
9.36	9.855469E-01	9.855469E-01	9.849172E-01	9.855469E-01	9.855469E-01
9.48	9.963273E-01	9.963273E-01	9.957879E-01	9.963273E-01	9.963273E-01
9.60	1.003876E+00	1.003876E+00	1.003826E+00	NaN	1.003876E+00

9.72	1.007807E+00	1.007807E+00	1.008053E+00	1.007807E+00	1.007807E+00
9.84	1.008519E+00	1.008519E+00	1.008810E+00	1.008519E+00	1.008519E+00
9.96	1.007090E+00	1.007090E+00	1.007470E+00	1.007090E+00	1.007090E+00
10.08	1.005023E+00	1.005023E+00	1.004976E+00	1.005023E+00	1.005023E+00
10.20	1.003898E+00	1.003898E+00	1.003606E+00	NaN	1.003898E+00
10.32	1.005017E+00	1.005017E+00	1.003123E+00	1.005017E+00	1.005017E+00
10.44	1.009100E+00	1.009100E+00	1.011769E+00	1.009100E+00	1.009100E+00
10.56	1.016125E+00	1.016125E+00	1.021522E+00	1.016125E+00	1.016125E+00
10.68	1.025321E+00	1.025321E+00	1.029831E+00	NaN	1.025321E+00
10.80	1.035332E+00	1.035332E+00	1.035003E+00	NaN	1.035332E+00
10.92	1.044509E+00	1.044509E+00	1.046285E+00	1.044509E+00	1.044509E+00
11.04	1.051267E+00	1.051267E+00	1.042964E+00	NaN	1.051267E+00
11.16	1.054428E+00	1.054428E+00	1.046129E+00	NaN	1.054428E+00
11.28	1.053478E+00	1.053478E+00	1.045437E+00	1.053478E+00	1.053478E+00
11.40	1.048678E+00	1.048678E+00	1.044895E+00	NaN	1.048678E+00
11.52	1.041008E+00	1.041008E+00	1.042411E+00	1.041008E+00	1.041008E+00
11.64	1.031954E+00	1.031954E+00	1.038511E+00	1.031954E+00	1.031954E+00
11.76	1.023193E+00	1.023193E+00	1.033052E+00	NaN	1.023193E+00
11.88	1.016229E+00	1.016229E+00	1.032007E+00	1.016229E+00	1.016229E+00
12.00	1.012078E+00	1.012078E+00	1.024150E+00	1.012078E+00	1.012078E+00

Comment: NaN means "not a number"

M = 2048; TM = 20.47; K = 12; tm = 12;

[ft6, t6] = nilt (F, TM, M, 1e-14); [ft7, t6] = niltqd (F, TM, M, 1e-14); f2 = weeks(F, t, 1024, 1, 10);

f4 = direct(F, 50, t, 8); f5 = invlap(F, t, 1, 1e-14);

Table 6: THE ORIGINAL CORRESPONDING TO LAPLACE TRANSFORM No. 5

t	weeks	direct	invlap	nilt	niltqd
0.10	3.537008E-03	3.537007E-03	3.537008E-03	3.537015E-03	3.537015E-03
0.20	1.416574E-02	1.416574E-02	1.416574E-02	1.416574E-02	1.416574E-02
0.30	3.193947E-02	3.193947E-02	3.193947E-02	3.193947E-02	3.193947E-02
0.40	5.694756E-02	5.694756E-02	5.694756E-02	5.694757E-02	5.694757E-02
0.50	8.931628E-02	8.931628E-02	8.931628E-02	8.931629E-02	8.931629E-02
0.60	1.292100E-01	1.292100E-01	1.292100E-01	1.292100E-01	1.292100E-01
0.70	1.768327E-01	1.768327E-01	1.768327E-01	1.768327E-01	1.768327E-01
0.80	2.324300E-01	2.324300E-01	2.324300E-01	2.324300E-01	2.324300E-01
0.90	2.962911E-01	2.962911E-01	2.962911E-01	2.962912E-01	2.962912E-01
1.00	3.687521E-01	3.687521E-01	3.687521E-01	3.687521E-01	3.687521E-01
1.10	4.501982E-01	4.501982E-01	4.501982E-01	4.501983E-01	4.501983E-01
1.20	5.410684E-01	5.410684E-01	5.410684E-01	5.410684E-01	5.410684E-01
1.30	6.418585E-01	6.418585E-01	6.418585E-01	6.418585E-01	6.418585E-01
1.40	7.531262E-01	7.531262E-01	7.531262E-01	7.531262E-01	7.531262E-01
1.50	8.754962E-01	8.754962E-01	8.754962E-01	8.754962E-01	8.754962E-01
1.60	1.009666E+00	1.009666E+00	1.009666E+00	1.009666E+00	1.009666E+00
1.70	1.156411E+00	1.156411E+00	1.156411E+00	1.156411E+00	1.156411E+00
1.80	1.316595E+00	1.316595E+00	1.316595E+00	1.316595E+00	1.316595E+00
1.90	1.491174E+00	1.491174E+00	1.491174E+00	1.491174E+00	1.491174E+00
2.00	1.681205E+00	1.681205E+00	1.681205E+00	1.681205E+00	1.681205E+00
2.10	1.887861E+00	1.887861E+00	1.887861E+00	1.887861E+00	1.887861E+00
2.20	2.112434E+00	2.112434E+00	2.112434E+00	2.112435E+00	2.112435E+00
2.30	2.356353E+00	2.356353E+00	2.356353E+00	2.356353E+00	2.356353E+00
2.40	2.621192E+00	2.621192E+00	2.621192E+00	2.621192E+00	2.621192E+00
2.50	2.908687E+00	2.908687E+00	2.908687E+00	2.908687E+00	2.908687E+00
2.60	3.220751E+00	3.220751E+00	3.220751E+00	3.220751E+00	3.220751E+00

2.70	3.559489E+00	3.559489E+00	3.559489E+00	3.559489E+00	3.559489E+00
2.80	3.927219E+00	3.927219E+00	3.927219E+00	3.927219E+00	3.927219E+00
2.90	4.326494E+00	4.326494E+00	4.326494E+00	4.326494E+00	4.326494E+00
3.00	4.760119E+00	4.760119E+00	4.760119E+00	4.760119E+00	4.760119E+00
3.10	5.231182E+00	5.231182E+00	5.231182E+00	5.231183E+00	5.231183E+00
3.20	5.743079E+00	5.743079E+00	5.743079E+00	5.743079E+00	5.743079E+00
3.30	6.299540E+00	6.299540E+00	6.299540E+00	6.299540E+00	6.299540E+00
3.40	6.904670E+00	6.904670E+00	6.904670E+00	6.904670E+00	6.904670E+00
3.50	7.562979E+00	7.562979E+00	7.562979E+00	7.562979E+00	7.562979E+00
3.60	8.279423E+00	8.279423E+00	8.279423E+00	8.279423E+00	8.279423E+00
3.70	9.059449E+00	9.059449E+00	9.059449E+00	9.059449E+00	9.059449E+00
3.80	9.909043E+00	9.909043E+00	9.909043E+00	9.909043E+00	9.909043E+00
3.90	1.083478E+01	1.083478E+01	1.083478E+01	1.083478E+01	1.083478E+01
4.00	1.184389E+01	1.184389E+01	1.184389E+01	1.184389E+01	1.184389E+01
4.10	1.294431E+01	1.294431E+01	1.294431E+01	1.294431E+01	1.294431E+01
4.20	1.414476E+01	1.414476E+01	1.414476E+01	1.414476E+01	1.414476E+01
4.30	1.545483E+01	1.545483E+01	1.545483E+01	1.545483E+01	1.545483E+01
4.40	1.688504E+01	1.688504E+01	1.688504E+01	1.688504E+01	1.688504E+01
4.50	1.844695E+01	1.844695E+01	1.844695E+01	1.844695E+01	1.844695E+01
4.60	2.015328E+01	2.015328E+01	2.015328E+01	2.015328E+01	2.015328E+01
4.70	2.201797E+01	2.201797E+01	2.201797E+01	2.201798E+01	2.201798E+01
4.80	2.405637E+01	2.405637E+01	2.405637E+01	2.405637E+01	2.405637E+01
4.90	2.628532E+01	2.628532E+01	2.628532E+01	2.628532E+01	2.628532E+01
5.00	2.872332E+01	2.872332E+01	2.872332E+01	2.872333E+01	2.872333E+01
5.10	3.139072E+01	3.139072E+01	3.139072E+01	3.139072E+01	3.139072E+01
5.20	3.430984E+01	3.430984E+01	3.430984E+01	3.430984E+01	3.430984E+01
5.30	3.750524E+01	3.750524E+01	3.750524E+01	3.750524E+01	3.750524E+01
5.40	4.100388E+01	4.100388E+01	4.100388E+01	4.100388E+01	4.100388E+01
5.50	4.483540E+01	4.483540E+01	4.483540E+01	4.483540E+01	4.483540E+01
5.60	4.903236E+01	4.903236E+01	4.903236E+01	4.903236E+01	4.903236E+01
5.70	5.363054E+01	5.363054E+01	5.363054E+01	5.363054E+01	5.363054E+01
5.80	5.866926E+01	5.866926E+01	5.866926E+01	5.866929E+01	5.866929E+01
5.90	6.419173E+01	6.419173E+01	6.419173E+01	6.419173E+01	6.419173E+01
6.00	7.024544E+01	7.024544E+01	7.024544E+01	7.024544E+01	7.024544E+01
6.10	7.688258E+01	7.688258E+01	7.688258E+01	7.688259E+01	7.688259E+01
6.20	8.416053E+01	8.416053E+01	8.416053E+01	8.416053E+01	8.416053E+01
6.30	9.214232E+01	9.214232E+01	9.214232E+01	9.214233E+01	9.214233E+01
6.40	1.008973E+02	1.008973E+02	1.008973E+02	1.008973E+02	1.008973E+02
6.50	1.105016E+02	1.105016E+02	1.105016E+02	1.105016E+02	1.105016E+02
6.60	1.210390E+02	1.210390E+02	1.210390E+02	1.210390E+02	1.210390E+02
6.70	1.326015E+02	1.326015E+02	1.326015E+02	1.326015E+02	1.326015E+02
6.80	1.452905E+02	1.452905E+02	1.452905E+02	1.452905E+02	1.452905E+02
6.90	1.592171E+02	1.592171E+02	1.592171E+02	1.592171E+02	1.592171E+02
7.00	1.745038E+02	1.745038E+02	1.745038E+02	1.745038E+02	1.745038E+02
7.10	1.912850E+02	1.912850E+02	1.912850E+02	1.912850E+02	1.912850E+02
7.20	2.097088E+02	2.097088E+02	2.097088E+02	2.097088E+02	2.097088E+02
7.30	2.299377E+02	2.299377E+02	2.299377E+02	2.299377E+02	2.299377E+02
7.40	2.521508E+02	2.521508E+02	2.521508E+02	2.521508E+02	2.521508E+02
7.50	2.765448E+02	2.765448E+02	2.765448E+02	2.765448E+02	2.765448E+02
7.60	3.033359E+02	3.033359E+02	3.033359E+02	3.033360E+02	3.033360E+02
7.70	3.327624E+02	3.327624E+02	3.327624E+02	3.327624E+02	3.327624E+02
7.80	3.650858E+02	3.650858E+02	3.650858E+02	3.650858E+02	3.650858E+02
7.90	4.005941E+02	4.005941E+02	4.005941E+02	4.005941E+02	4.005941E+02
8.00	4.396040E+02	4.396040E+02	4.396040E+02	4.396040E+02	4.396040E+02

8.10	4.824639E+02	4.824639E+02	4.824639E+02	4.824639E+02	4.824639E+02
8.20	5.295570E+02	5.295570E+02	5.295570E+02	5.295570E+02	5.295570E+02
8.30	5.813050E+02	5.813050E+02	5.813050E+02	5.813050E+02	5.813050E+02
8.40	6.381717E+02	6.381717E+02	6.381717E+02	6.381717E+02	6.381717E+02
8.50	7.006674E+02	7.006674E+02	7.006674E+02	7.006675E+02	7.006675E+02
8.60	7.693537E+02	7.693537E+02	7.693537E+02	7.693537E+02	7.693537E+02
8.70	8.448482E+02	8.448482E+02	8.448482E+02	8.448482E+02	8.448482E+02
8.80	9.278308E+02	9.278308E+02	9.278308E+02	9.278308E+02	9.278308E+02
8.90	1.019049E+03	1.019049E+03	1.019049E+03	1.019050E+03	1.019050E+03
9.00	1.119327E+03	1.119327E+03	1.119327E+03	1.119327E+03	1.119327E+03
9.10	1.229570E+03	1.229570E+03	1.229570E+03	1.229570E+03	1.229570E+03
9.20	1.350775E+03	1.350775E+03	1.350775E+03	1.350775E+03	1.350775E+03
9.30	1.484038E+03	1.484038E+03	1.484038E+03	1.484038E+03	1.484038E+03
9.40	1.630568E+03	1.630568E+03	1.630568E+03	1.630568E+03	1.630568E+03
9.50	1.791692E+03	1.791692E+03	1.791692E+03	1.791693E+03	1.791693E+03
9.60	1.968874E+03	1.968874E+03	1.968874E+03	1.968875E+03	1.968875E+03
9.70	2.163724E+03	2.163724E+03	2.163724E+03	2.163724E+03	2.163724E+03
9.80	2.378012E+03	2.378012E+03	2.378012E+03	2.378012E+03	2.378012E+03
9.90	2.613690E+03	2.613690E+03	2.613690E+03	2.613690E+03	2.613690E+03
10.00	2.872905E+03	2.872905E+03	2.872905E+03	2.872905E+03	2.872905E+03

$M = 1024$ ;  $TM = 10.23$ ;  $K = 10$ ;  $tm = 10$ ;  
 $[ft6, t6] = \text{nilt}(F, TM, M, 1e-16)$ ;  $[ft7, t6] = \text{niltqd}(F, TM, M, 1e-16)$ ;  
 $f2 = \text{weeks}(F, t, 1024, 1.4, 14)$ ;  $f4 = \text{direct}(F, 50, t, 15)$ ;  
 $f5 = \text{invlap}(F, t, 1.4, 1e-14)$ ;

The plots of calculated originals  $f(t)$  are shown in Figures 1-5.

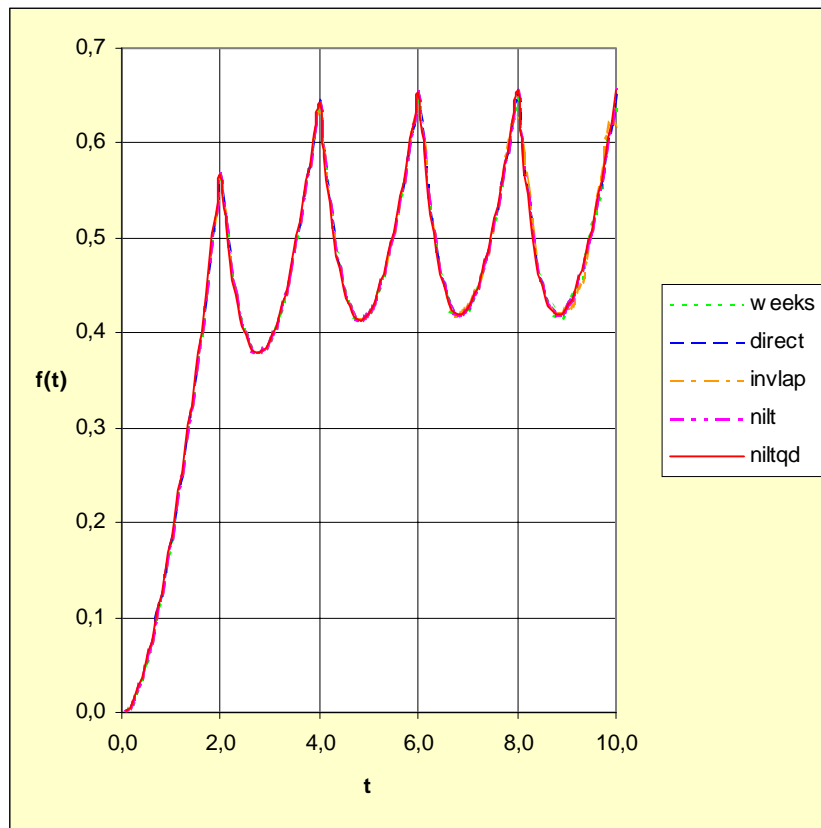


Figure 1: The original corresponding to Laplace transform No. 1

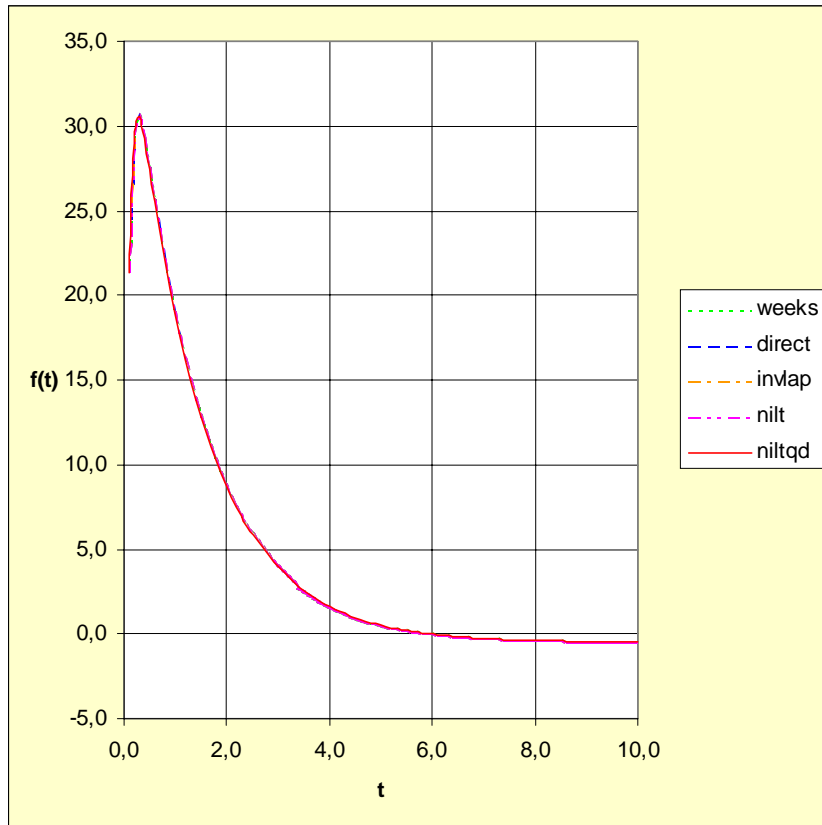


Figure 2: The original corresponding to Laplace transform No. 2

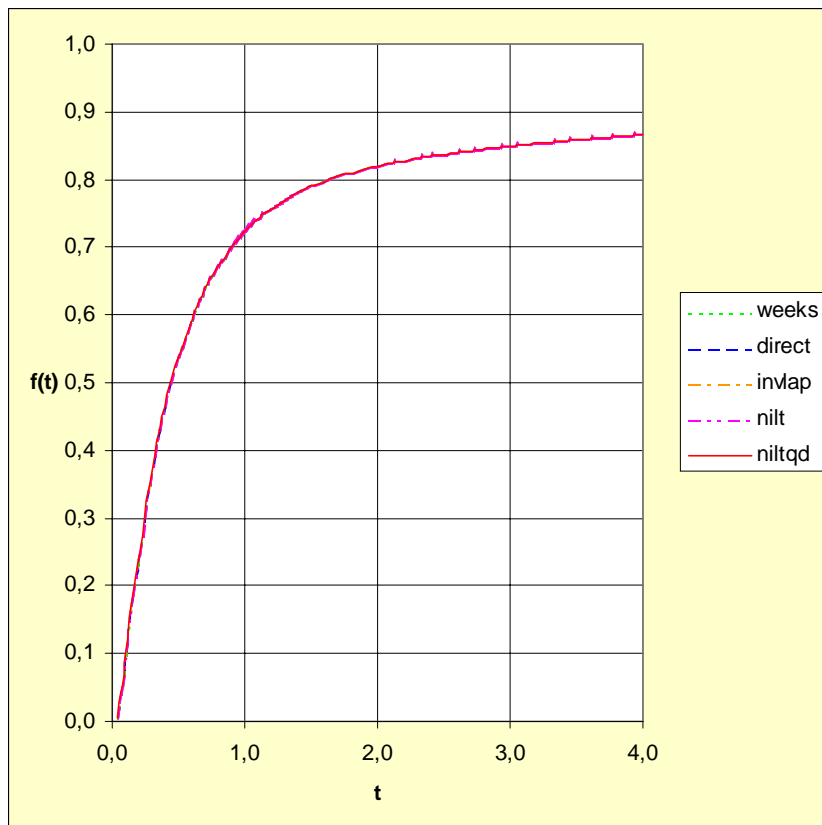


Figure 3: The original corresponding to Laplace transform No. 3



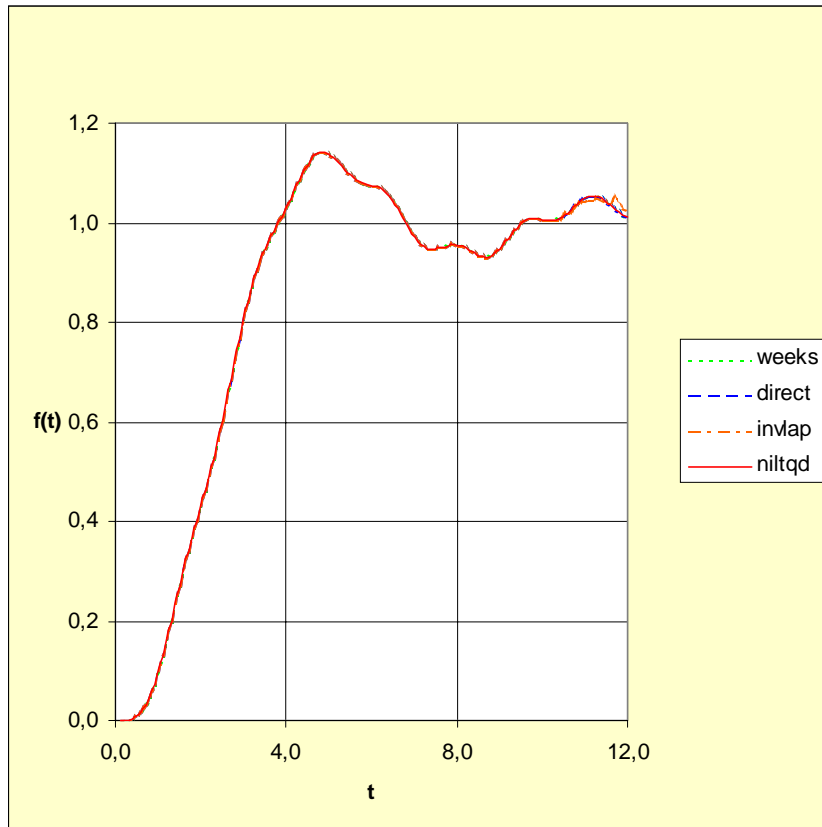


Figure 4: The original corresponding to Laplace transform No. 4

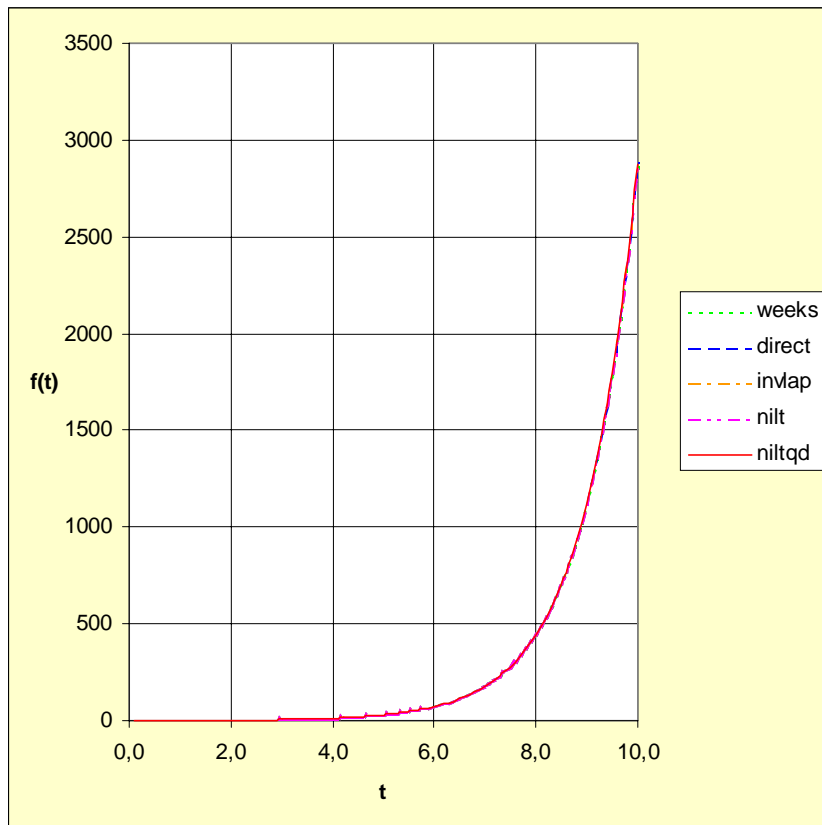


Figure 5: The original corresponding to Laplace transform No. 5

## 4 Discussion

As the exact values of the originals  $f(t)$  were not calculated, it was impossible to estimate the errors. The coincident digits of  $f(t)$  obtained by several different algorithms were taken for correct while the results differing from those obtained by other algorithms were considered erroneous.

The numerical inversion of Laplace transform No. 1 was the key test for all algorithms used. The accuracy of the solution was not particularly good in this case. The coincidence of the  $f(t)$  values calculated by the individual algorithms was the best at  $t < 2$  and became worse at the higher  $t$  values and near the points  $t = 2, 4, 6, 8, 10$  at which  $f(t)$  is non-differentiable – see Figure 1. From the point of view of technical computing, the results obtained by the algorithms `direct`, `nilt` and `niltqd` were acceptable, while those obtained by `weeks` and `invlap` were less accurate.

All the results of the numerical inversion of the Laplace transform No. 2 are practically identical within the first 7 digits, with exception of the five isolated  $f(t)$  values obtained by the `weeks` algorithm which differ from others in the last 1 digit, one isolated  $f(t)$  value obtained by the `direct` algorithm differs from others in the last 1 digit only, one isolated `direct` algorithm value at  $t = 6.80$  differs in the last 3 digits.

All the calculated  $f(t)$  values were identical within the first 7 digits at all  $t$ -values in the case of Laplace transform No. 3, except for the two  $f(t)$  values at  $t = 2.16$  and  $t = 2.20$  calculated by the `direct` algorithm differing from those obtained by other algorithms in the last 3 digits.

The numerical inversion of Laplace transform No. 4 calculated by individual algorithms was exact in all 7 decimal digits at  $t \leq 6$ . The instability of `nilt` algorithm and a fall in the accuracy of `invlap` algorithm occurred at greater  $t$  values.

The results of the numerical inversion of Laplace transform No. 5 obtained by the algorithms `weeks`, `direct` and `invlap` are identical in all 7 decimal digits over all the examined  $t$ -interval except for  $t = 0.10$ , where the unit difference in the last decimal digit occurred. The results of `nilt` and `niltqd` algorithms differ from those obtained by other algorithms in the last 2 or 3 digits by the standard value  $Er = 1e-14$  but, after the reduction of it to  $Er = 1e-16$ , some isolated  $f(t)$  values obtained by the `nilt` and `niltqd` algorithms differ from others in the last 1 digit only.

The described differences are practically not observable in the plots shown in Figures 1-5.

## 5 Conclusion

All the algorithms used have given acceptable results. The choice of right algorithm depends upon the problem solved. Since every method has its weak points, a simultaneous execution of several different algorithms in a program and comparison of results is recommended. This approach is especially useful when the original cannot be represented by an analytical formula or can be found by another complex numerical method only, which makes it practically impossible to estimate the error. And last but not least: One often has not time enough to find whether an analytical solution exists and – in the case of positive answer – to execute it. Then the numerical inversion of the Laplace transform in MATLAB might be a fast and reliable method to calculate the original with the accuracy satisfying (and mostly exceeding) the technical computing requirements.

## Acknowledgements

The work was supported by the grant of the Ministry of Education, Youth and Sports of the Czech Republic No. 0021527505. This support is very gratefully acknowledged.

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