MODELING OF ACOUSTIC WAVEGUIDES IN MATLAB

F. Rund

Dept. of Radioelectronics, FEE CTU in Prague

Abstract

Acoustic waveguides are widely used in many acoustic applications, for example in musical acoustics (music instruments), electro-acoustics (loudspeakers) etc. The underlying theory is also applicable to other special disciplines, e.g. for modeling of human ear canal or human vocal tract. The achieved results can be used not only in medicine but also in psychoacoustics, as a part of the overall model of human auditory system. In the paper a Matlab script with a GUI is presented, which can be used for modeling of transfer properties of acoustic waveguides.

1 Introduction

Models of properties of acoustic waveguides are useful in many applications where acoustic waveguides are used. The following article describes a simple and useful tool implemented in MATLAB®, which allows rapid analysis of transfer properties of an acoustic waveguide with certain shape and dimensions.

The theoretical background of the tool is based on the method of electro-acoustic analogy. The wave in the waveguide is described by the Webster's wave equation (one-dimensional – see e.g. [1]), thus the frequency range is limited to plane-wave condition only. From the wave equation the chain matrix of the waveguide is computed [2], from which the transfer function can be derived.

2 Description of the GUI

The MATLAB® script which computes the desired characteristics is provided with a GUI to allow simple entering of the parameters of the waveguide – see Fig. 1.

At first, the cross-section (elliptic or rectangular) and the shape of the waveguide have to be chosen. The shape of the waveguide can be exponential, sine, cosine, catenoid and Bessel's (specifications of the shapes are given in Tab. 2).

Subsequently, waveguide dimensions can be entered. The y- and z-plane dimensions of an exponential waveguide can differ, for the other shapes only the y dimension is accepted due to their rotational symmetry. The script sets the z dimensions to the value of y in case only the y dimension is entered. The dimensions are expected to be in mm.

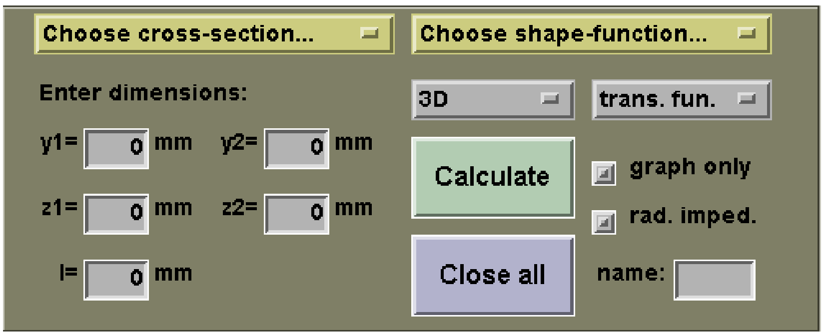

Figure 1: The MATLAB® GUI for waveguide modeling.

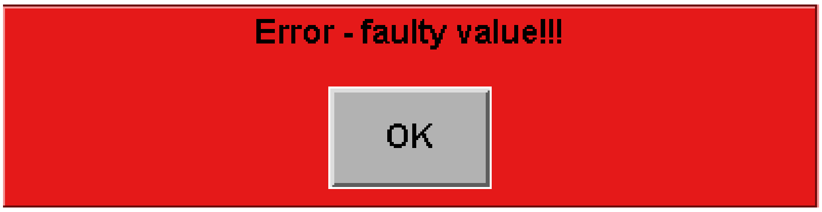

Figure 2: The error message.

The above mentioned parameters of the waveguide are obligatory. The script will generate an error message if any of them is omitted (see Fig 2). The remaining parameters are optional. The two popup menus below the shape-function menu as well as the graph only checkbox allow the user to control the appearance of the result window. If the rad. imped. checkbox is left unchecked, the script computes the transfer function of the specified waveguide terminated by a rigid closure. If the checkbox is checked, the waveguide is supposed to be terminated with radiation impedance of a plane acoustic source placed in an infinite plane [1]. When the optional filename is given, a couple of files with the extension `.out` which contain the A_{11} parameter and the critical frequencies are saved in the working directory. The meaning of the remaining two buttons, Calculate and Close all, needs no explanation.

As a result, a 3D wireframe model of the waveguide and its transfer function are depicted. Several examples of the result window are shown in Fig. 3 – Fig. 6. The shape of the waveguide in the top panel is displayed as a 3D model by default, but the user can choose a 2D model instead (Fig. 4 top right). The results of the simulation are displayed in the bottom panel. By default, the transfer function is shown (red curve). The blue line shows the limit of validity of the Webster's equation. In the top left corner the parameters of the shape function are summarized. The chosen shape function is on the first line, the remaining lines give the values of the parameters, critical frequency and coordinate of the zero value of the first-order derivative. The A_{11} parameter of the chain matrix can be displayed (Fig. 4 right bottom) instead of the transfer function. Finally, the top panel can be switched off (Fig. 6 right).

3 Implemented functions

The core of the presented tool uses several special functions for the chain matrix computing:

- `vmate(rin,rin2,x1,x2,alfa1,alfa2,ro,c,j,k)` – **exponential waveguide**
  This waveguide can have different dimensions in y- and z- direction – i.e. different `rin1`

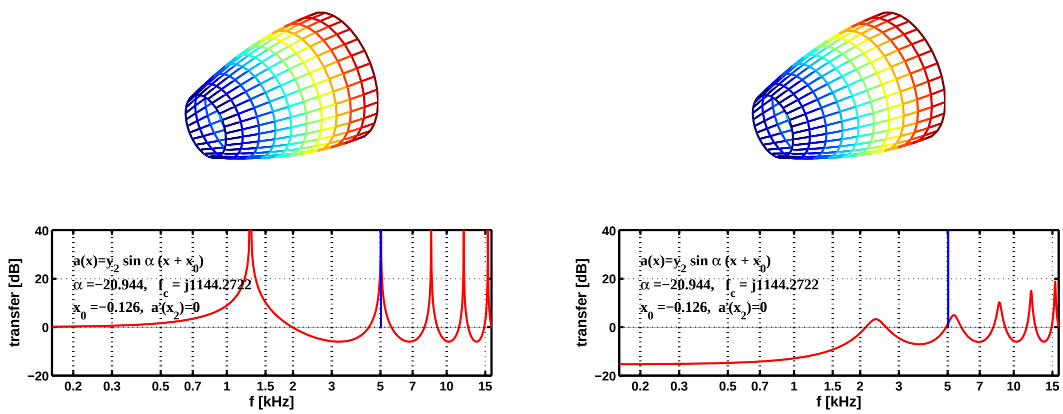

Figure 3: A sine waveguide – with rigid closure (left), with radiation impedance termination (right).

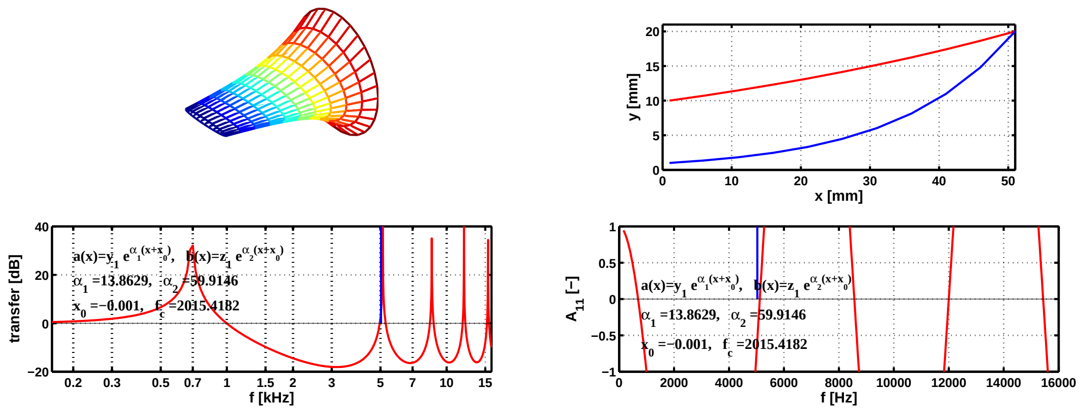

Figure 4: An asymmetric exponential waveguide – the 3D model and the transfer function (left), the 2D model and the A_{11}(right).

and `rin2`, `alfa1` and `alfa2`.

- `vmats(rin,x1,x2,alfa,ro,c,j,k)` – **sine waveguide**
- `vmatc(rin,x1,x2,alfa,ro,c,j,k)` – **cosine waveguide**
- `vmatch(rin,x1,x2,alfa,ro,c,j,k)` – **catenoid waveguide**
- `vmatb(rin,x1,x2,alfa,ro,c,j,k)` – **Bessel's waveguide**

The parameters of the chain-matrix-computing functions are listed in Tab. 1. These functions compute the chain matrix of the waveguide which is described by the *shape function* $a(x)$ (or $b(x)$). The method of deriving the chain-matrix from the shape function was published in [2]. The shape functions and their parameters are listed in Tab. 2.

In addition to the above mentioned functions, two more functions have to be necessarily implemented. The `nm(mat1,mat2,k)` function is useful for computing the product of two chain matrices, while the `prm(matice,k,prvni,druhy)` function can pick up the parameter with coordinates `prvni`, `druhy` from the matrix `matice`.

4 How it works?

The tool consists of the GUI routines, the computing functions presented above and the main script. The main script reads the inputs from the GUI and calls the core functions to compute the desired characteristics. The function of the main script will be described below.

Table 1: Parameters of the chain-matrix-computing functions.

`rin`	y_1	input radius
`x1`	x_1	input coordinate
`x2`	x_2	output coordinate
`alfa`	α	parameter – see Tab. 2
`ro`	ρ	density of the medium
`c`	c	sound velocity
`j`	j	imaginary unit
`k`	k	wavenumber

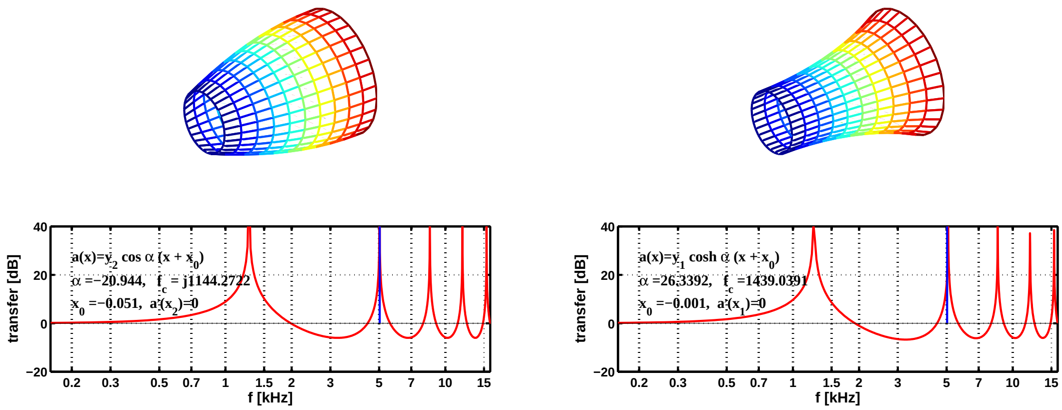

Figure 5: A cosine (left) and a catenoid (right) waveguide

The first step is reading the input data and checking it. An error message is generated (Fig. 2) in case of invalid inputs, otherwise the input data are converted to SI units (from mm to m) and the $x-$coordinate vector is generated.

Afterwards the optional control `switch` is used to choose of the shape of the waveguide. For the chosen shape the parameters of the shape function (Tab. 2) are computed. The vectors y and z are subsequently derived from the computed parameters. Also, the text comments are prepared. Finally, the proper function for matrix computing is called with the computed parameters. The result of this part of the script is the chain matrix `matice` of the waveguide.

In case the open-space-termination of the waveguide was chosen, the radiation impedance is computed. A simplified equation (see [1]) for the radiation impedance of a circle radiator in an infinite plane was used in the script. For a non-circle cross-section the equivalent area is used. The new matrix `matice` is then computed, as the product (function `nm`) of the old `matice` and the chain matrix of the radiation impedance. If the rigid termination of the waveguide is used, the matrix remains unchanged. At last the A_{11} parameter of the matrix `matice` is picked up (function `prm`).

Table 2: PARAMETERS OF THE SHAPE FUNCTIONS

waveguide	*shape function*	*parameters*	
exponential non-symmetric	$a(x) = y_1 e^{\alpha_1(x+x_0)}$	$\alpha_1 = \frac{\ln \frac{y_2}{y_1}}{x_2-x_1}$	$x_0 = -x_1$
	$b(x) = z_1 e^{\alpha_2(x+x_0)}$	$\alpha_2 = \frac{\ln \frac{z_2}{z_1}}{x_2-x_1}$	
sine divergent	$a(x) = y_2 \sin \alpha(x + x_0)$	$\alpha = \frac{\arccos \frac{y_1}{y_2}}{x_1-x_2}$	$x_0 = \frac{\pi}{2\alpha} - x_2$
sine convergent	$a(x) = y_1 \sin \alpha(x + x_0)$	$\alpha = \frac{\arccos \frac{y_2}{y_1}}{x_2-x_1}$	$x_0 = \frac{\pi}{2\alpha} - x_1$
cosine divergent	$a(x) = y_2 \cos \alpha(x + x_0)$	$\alpha = \frac{\arccos \frac{y_1}{y_2}}{x_1-x_2}$	$x_0 = -x_2$
cosine convergent	$a(x) = y_1 \cos \alpha(x + x_0)$	$\alpha = \frac{\arccos \frac{y_2}{y_1}}{x_2-x_1}$	$x_0 = -x_1$
catenoid divergent	$a(x) = y_1 \cosh \alpha(x + x_0)$	$\alpha = \frac{\operatorname{arccosh} \frac{y_2}{y_1}}{x_2-x_1}$	$x_0 = -x_1$
catenoid convergent	$a(x) = y_2 \cosh \alpha(x + x_0)$	$\alpha = \frac{\operatorname{arccosh} \frac{y_1}{y_2}}{x_1-x_2}$	$x_0 = -x_2$
Bessel's	$a(x) = \mathfrak{c} x^{\alpha}$	$\alpha = \frac{\ln \frac{y_2}{y_1}}{\ln \frac{x_2}{x_1}}$	$\mathfrak{c} = \frac{y_1}{x_1^q}$

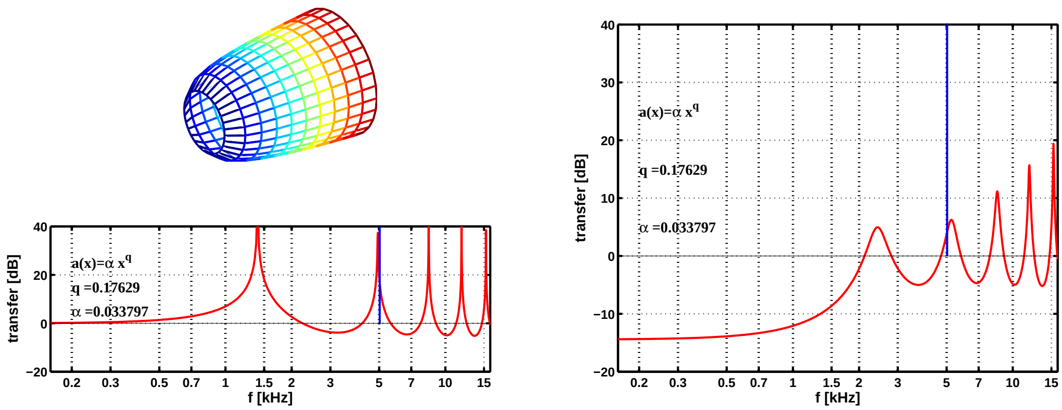

Figure 6: A Bessel's waveguide (left) and its transfer function with freefield termination (right)

If a filename was specified in the GUI, the basic parameters (critical frequency, A_{11}) are saved in the working directory.

Creating the images is the last step. At first, the frequency limits of the theory is computed for each cross-section. If the top panel is switched on, the 3D or 2D model of the waveguide is shown. In the bottom panel, either the A_{11} parameter alone or the transfer function is displayed. In the latter case the modulus of $1/A_{11}$ is computed and depicted. The text comments are displayed as well.

5 Conclusions

In the paper a tool with a GUI for acoustic waveguide analysis & design written in MATLAB® was presented. The theory for the computations originates from [2]. Several special functions for computing of the parameters were newly implemented for the tool. The way the tool works was described in detail and many examples of analyzed waveguides were given.

The presented tool is useful for rapid analysis of acoustic waveguides. The area of possible applications is not explicitly determined – it can be used for any acoustic problem which deals with acoustic waveguides. The frequency range is limited to the frequencies when only plane wave is present. The cross-section of the waveguide can be elliptical or rectangular, and the exponential, catenoidal, sine, cosine and Bessel's waveguides are supported.

6 Acknowledgement

This work has been supported by the research project VZ MSM6840770016.

References

[1] ŠKVOR, Zd., Akustika a elektroakustika. Praha : Academia, 2001, pp. 527. ISBN 80-200-0464-0

[2] RUND, F. Modeling of Acoustic Waveguides Using Electro-Acoustic Analogy: Application to the External Human Ear Canal. In *Proceedings of Inter-Noise 2004* [CD-ROM]. Prague: Czech Acoustic Society, 2004, s. 793. ISBN 80-01-03055-5.

František Rund
Katedra radioelektroniky, FEL ČVUT Praha, Technická 2, 166 27, Praha 6
tel. 22435 2108, e-mail: xrund@feld.cvut.cz