SIMULATION OF NON-ORTHOGONAL SPACE-TIME BLOCK CODED SYSTEM WITH CHANNEL ESTIMATION ERRORS

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Abstract

The purpose of this paper is present some numerical results of the concrete space-time block coded system with estimated CSI and compare the performance of a space-time coded system with perfect CSI (Channel State Information) and the same system with CSI estimated using Least Squares Method. Transmission over Multiple Input Multiple Output (MIMO) radio channels with rayleigh fading is considered. The numerical results are obtained using Matlab.

High transmission data rate, spectral efficiency, and reliability are necessary for future communications systems. In a multipath-rich wireless channel, deploying multiple antennas at both the transmitter and receiver achieves high data rate, without increasing the total transmission power or bandwidth. When the perfect knowledge of the wireless channel conditions is available at the receiver, the capacity has been shown to grow linearly with the number of antennas. The channel conditions so-called CSI (Channel State Information) must be estimated at the receiver side. To examine the system performance accurately, the assumption of the perfect CSI knowledge must be cancelled. System performance depends on the quality of channel estimate. Channel estimations are commonly obtained by transmitting pilot symbols known to the receiver, but added pilot symbols reduce the spectral efficiency.

Further, the performance is investigated on non-orthogonal space-time code with double data rate using MIMO system with two transmitting and two receiving antennas. Least Squares Method for MIMO channels estimation is described and its utilization for concrete non-orthogonal space-time block coded system is verified. Numerical results show the Bit Error Rate vs. signal-to-noise ratio for two estimation cases. First case assume lower and second case higher number of pilot symbols. The estimations are optimized for applied space-time code. Another numerical results show BER performance vs. number of the pilot symbols. It is useful for estimation, how many pilot symbols are required for near perfect CSI state.

One of the first space-time block codes is due to Alamouti [1], who suggested to simultaneously transmit two complex symbols c_1 and c_2 from two transmitting antennas during two symbol periods according the following matrix:

$$\mathbf{G} = \begin{pmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{pmatrix}. \qquad (1)$$

Symbols c_1 and c_2 are transmitted by two antennas at time t and then symbols $-c_2^*$ and c_1^* are transmitted at time $(t + T_s)$, where T_s denotes symbol interval. Columns of the matrix corresponds to the antennas and rows corresponds to the time slots. The rate of the Alamouti code is equal to $R = 2/2 = 1$. This code is orthogonal, it guarantees that the (coherent) ML detection of different symbols c_n is decoupled and diversity order is equal to $n_r n_t$, where n_r and n_t is number of receiving and transmitting antennas respectively. Orthogonal linear space-time block code has the following property:

$$\mathbf{GG}^H = \left[\sum_{n=1}^{n_s} |c_n|^2 \right] \mathbf{I}, \qquad (2)$$

where **I** is unit matrix 2 x 2. The space-time decoder combines the received signals as follows:

$$\begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = \begin{pmatrix} h_1^* & h_2 \\ h_2 & -h_1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2^* \end{pmatrix}, \quad (3)$$

where $\tilde{c}_1$, $\tilde{c}_2$ are estimates of symbols c_1, c_2 and h_1, h_2 are complex path gains from transmit antennas to the receive antenna and r_1, r_2 are received symbols at time t and $(t + T_s)$.

Non-orthogonal space-time block code

The two essential features of space-time block codes on orthogonal design are linearity and orthogonality [5]. These two properties do not fit well together with achievable symbol rate of the STBC. The orthogonality has to be sacrificed to increase the rate of the STBC [4]. The non-orthogonal space-time block code from [4] was adopted. This code has symbol-rate 2, employing 2 transmitting and 2 receiving antennas. Coding matrix 2 x 2 contains 4 complex symbols encoded by the following formula:

$$\mathbf{X}(c_1, c_2, c_3, c_4) = \begin{pmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{pmatrix} + \mathbf{U} \begin{pmatrix} c_3 & c_4 \\ -c_4^* & c_3^* \end{pmatrix} \quad (4)$$

where **U** is 2 x 2 unitary matrix, which satisfy $\det(\mathbf{U}) = -1$.

In [4], the non-orthogonal space time block code with diagonal matrix

$$\mathbf{U} = \begin{pmatrix} e^{j\frac{\pi}{4}} & 0 \\ 0 & e^{-j\frac{\pi}{4}} \end{pmatrix} \quad (5)$$

and with non-diagonal matrix

$$\mathbf{U} = \begin{pmatrix} e^{j\frac{7\pi}{20}} \cos\left(\frac{9\pi}{50}\right) & e^{j\frac{\pi}{4}} \sin\left(\frac{9\pi}{50}\right) \\ -e^{-j\frac{\pi}{4}} \sin\left(\frac{9\pi}{50}\right) & e^{-j\frac{7\pi}{20}} \cos\left(\frac{9\pi}{50}\right) \end{pmatrix} \quad (6)$$

was proposed.

Channels estimation and system model

We investigate a communication system over Multiple Input, Multiple Output (MIMO) Rayleigh flat fading channels with two transmitting and two receiving antennas. The channel realizations between the antennas are expressed by the matrix:

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}, \quad (6)$$

where the $h_{i,j}$ are assumed uncorrelated fading path gains.

It is necessary to know the matrix **H** at receiver side for decoding of MIMO signals. It was mentioned, that this information is called CSI (*Channel State Information*) and it can be estimated from received signals by several methods. One of these methods is the method of Least Squares (LS) which is described below.

Least squares (LS) estimation requires pilot symbols and matrix inversion. A training sequence (known to the receiver) is transmitted by each transmit antenna at the beginning of each data burst. The received signal matrix during the two symbol periods is given by

$$\mathbf{R} = \mathbf{HX} + \mathbf{V}, \quad (7)$$

where **H** is the channel matrix, **X** is the transmitted matrix and **V** is the noise matrix.

The ML estimate for the channel matrix is given by [6]:

$$\begin{aligned}\widetilde{\mathbf{H}} &= \mathbf{R}\mathbf{X}^{+}\\ \widetilde{\mathbf{H}} &= (\mathbf{H}\mathbf{X}+\mathbf{V})\mathbf{X}^{+}\\ \widetilde{\mathbf{H}} &= \mathbf{H}+\mathbf{V}\mathbf{X}^{+}\end{aligned} \tag{8}$$

where $\mathbf{X}^{+} = (\mathbf{X}^{H}\mathbf{X})^{-1}\mathbf{X}^{H}$ is the Moore-Penrose Pseudoinverse.

The estimation error is given by

$$\Delta\mathbf{H} = \mathbf{V}\mathbf{X}^{+}\,. \tag{9}$$

The computational complexity of LS estimation depends on the number of transmit antennas and the number of training symbols per antenna. To achieve the same BER performance, more pilot symbols are required as the number of transmit antennas increases. A longer training sequence matrix results in a more complex matrix inversion.

At first, we assume quasi-static flat fading, it means that h_{ij} is constant for the duration of *M* symbol periods. Further, a MIMO channel matrix is estimated also after each *M* symbols periods. It is obvious [4], that according two symbol intervals, block of four symbols z_i, $i = 1...4$ is transmitted. For 2-PSK signal modulation, there are only $2^4 = 16$ possible matrices **X**. Due to low number of possible received matrices, maximum likelihood detection was used. It can be shown, that code [4] with diagonal matrix (5) using 2-PSK symbols produces also singular matrices. Obviously, only non-singular matrices are used for estimation by (8). The space-time code with non-diagonal matrix (6) produces only non-singular matrices. If the noise matrix **V** contains Gaussian random variable, the estimation error matrix $\Delta\mathbf{H}$ also contains gaussian random variables. It can be shown, that SNR of the signal **R** and SNR of the CSI (matrix **H**) is for orthogonal code the same, when Least Squares Method is used. Numerical results show, that for used non-orthogonal code with diagonal matrix, SNR of the received signals is about 3 dB better than SNR of the CSI, obtained by Least Squares method. If using the non-diagonal matrix (6), the SNR penalty is 2,1 dB.

Numerical results

The performace of system is evaluated by BER (*Bit Error Rate*), without any concatenated coding for three simulated cases. At first, we assume the perfect CSI knowledge at receiver side in figure 1. Then, in “case 1”, the CSI is estimated by LS (Least Squares) method from only one received matrix **X** every *M* symbol periods.

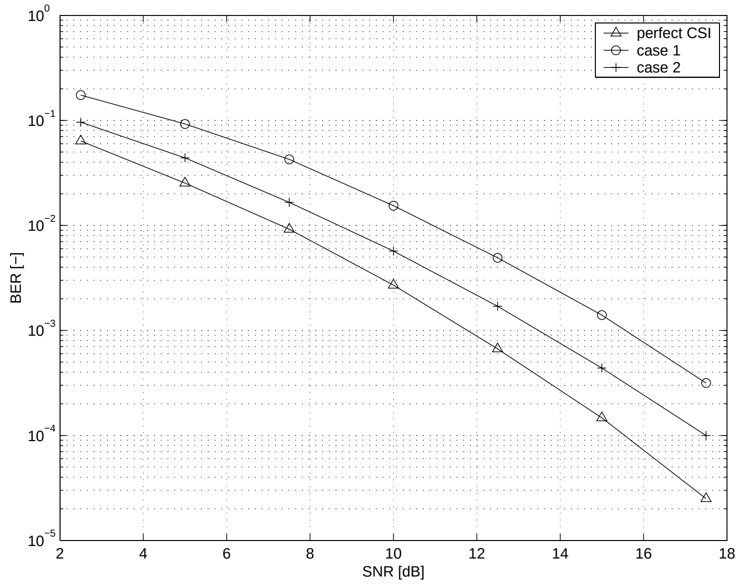

Fig. 1: Uncoded BER performance for diagonal matrix **U**

Further, if two received matrices are used for the estimation in “case 2”, the error of the CSI estimation and BER is lower. The numerical results for M = 100 and several signal-to-noise conditions are shown in figure 1.

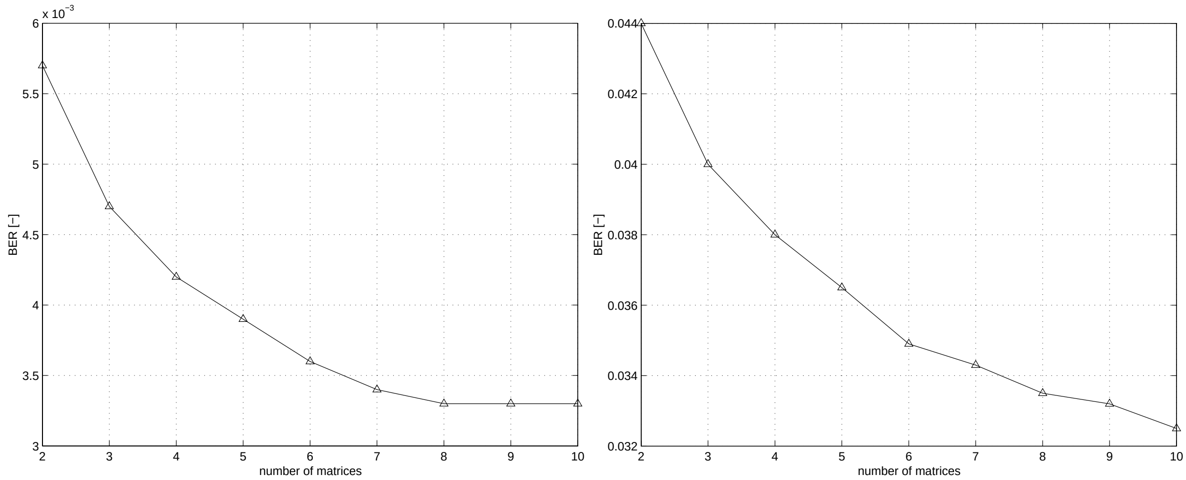

Fig. 2: BER performance vs. number of matrice used for estimation for SNR = 5 dB (left), SNR = 10 dB (right)

The numerical results on the next figures (fig. 2) show the BER as a function of the number of matrices $\mathbf{X}$ used for estimation. We can estimate, how many matrices (training symbols) are required for “near” perfect CSI.

The next simulation verify the sensitivity of the code (4) to the channel state estimation errors. Error of the CSI is expessed as signal-to-noise ratio of the components of matric $\widetilde{\mathbf{H}}$ compared to perfect CSI.

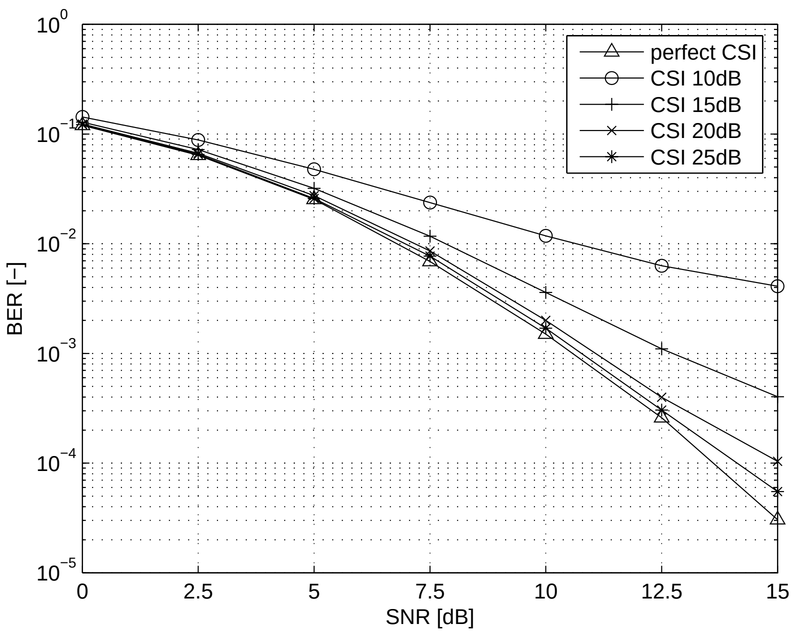

Fig. 3: Sensitivity of the non-orthogonal code to CSI error

Conclusion

We have considered a MIMO 2 x 2 scheme with non-orthogonal space-block coding. Numerical results show performance of concrete code in some simulated cases. We assumed perfect CSI knowledge and the remaining cases are for Least Squares estimation from one or two received matrices (case 1, case 2). It is shown, that a very simple estimation scheme ("case 2") may produce only approximately 1.2 dB signal-to-noise penalty at BER level 10^{-2}, compared to perfect CSI. Next results show the sensitivity of the code to channel estimation errors.

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