MODELLING OF LQR CONTROL WITH MATLAB

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Abstract

In the paper is used LQR control for regulation of the pressure in air-operated spring of the absorber of the vibrating mechanical system. The LQR is the control, which minimizes the accuracy of control opposite power exacting. The LQR control radiates from complete vector states, which in real life must be not in the feedback to position. In our case, we have to dispose the output parameters from the accelerometers.

1 Model of the mechanical system

Consider the vibrating system as a mechanical system, which is compiled of a driven motor, gearbox and mechanisms with elastic and damping parts and driven parts. Vibration transmitted the to the frame of this system is possible due to unbalanced rotors and mechanisms, the crank gear of the engine or oscillation of the moving driven parts.

The amplitude of the induced vibration is a function of the applied force and its frequency. An exciting force has the greatest effect when applied at the fundamental frequency of the system. The system is then excited at resonance, and in the case of a lightly damped system, the induced movement can be many times greater than the deflection caused by the equivalent static force. The ratio between the two effects is called the magnification factor. Vibrations in a structure have this effect. The very high peak accelerations can mean that the effective weight of the vibrating system increases several-fold, and this may cause its destruction. The vibration absorber is advantageous primarily in that it reduces the amplitude of the vibrations in the system by an oscillating force $F_{(t)}$ acting (Fig. 1), for two alternatives from an unbalanced rotor on the system:

I. $F_{(t)}=me\omega^2 \sin\omega t=F\sin\omega t$, where m is the mass of the unbalanced rotor, e is the eccentricity of the unbalanced rotor and ω is the angular velocity of the unbalanced rotor,

II. squere wave course of acting force $F_{(t)}$ with the amplitude force F and frequency ω.

The model of system with reduced mass m_1 and affiliate mass m_2 of the absorber is possible to illustrate as a two mass system located on two springs with coefficients of elasticity k_1 and k_2 and coefficients of stiffness $b1$ and $b2$ (Fig.1). The spring 2 is air-operated sprig and the absorber is the affiliate mass 2. The equations of motion for the model are

$$\begin{aligned} m_1\ddot{y}_1 &= -k_1 y_1 + k_2(y_2 - y_1) - b_1\dot{y}_1 + b_2(\dot{y}_2 - \dot{y}_1) + F_{(t)}, \\ m_2\ddot{y}_2 &= -k_2(y_2 - y_1) - b_2(\dot{y}_2 - \dot{y}_1). \end{aligned} \tag{1}$$

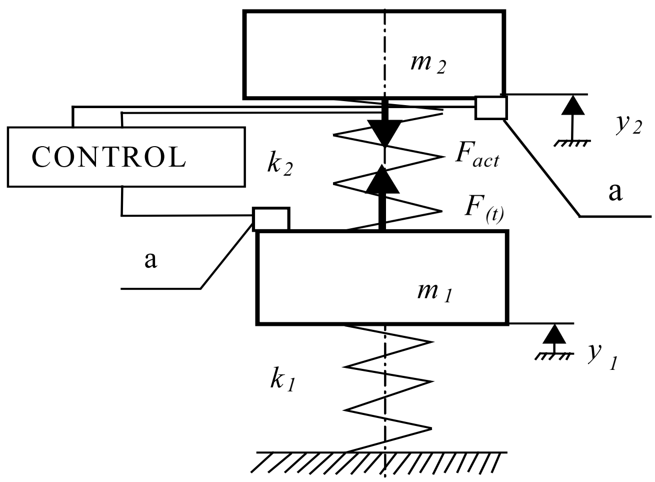

Fig. 1 The model of the mechanical system with an affiliate control absorber

The possibility to reduce the vibration is the tuneable absorber, where an air-operated spring with changed coefficient of elasticity k_2 through the changed pressure supply of the air is incorporated. Substituting $y_1=Y_1e^{i\omega t}$ and $y_2=Y_2e^{i\omega t}$ in equations (1) yields two simultaneous equations (reason of the influence of the damping *b1, b2* of the springs are very small):

$$Y_1 = \frac{(k_1 - m_2\omega^2)}{a}F, \quad Y_2 = \frac{k_2}{a}F, \tag{2}$$

with $a = (k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2.$ (3)

These equations define the dynamics of the system with one-degree of freedom after it has been modified by attaching the secondary mass/spring system. The extra mass and spring are the absorber. Ideally, we completely want to stop the vibration of primary mass m_1. We can do this by setting $Y_1=0$ in the first equation (2). This yields:

$$\omega = \sqrt{\frac{k_2}{m_2}}, \text{ or } \omega = \omega_2. \tag{4}$$

That is, if the natural frequency of the added mass-spring system itself is the same as the excitation frequency, the primary mass will stop moving. What this means is that we can tune an absorber to single excitation frequency. The second equation (2) can be rearranged to find out how much the secondary mass m_2 will be vibrate:

$$Y_2 = \frac{-F}{k_2}. \tag{5}$$

This indicates that providing the original system is being driven by a force input, the motion of the added mass is bounded. It does not go infinite, even though it is being excited at a frequency that matches its original natural frequency.

2 Design of the parameters of the absorber

A wheel of m_1=350 kg rotates at n=48 revolutions per minute. A misbalance e= 1.7 mm in the wheel means that when it is running it produces a peak force of

$$F = m_1 e\omega^2 = 15\,N. \tag{7}$$

Assume there is a maximum permissible absorber deflection of Y_2=50 mm. The motion of the secondary mass m_2 is given by the equation (5), Assuming we use the entire clearance for the motion of the absorber. We can calculate the absorber stiffness as:

$$k_2 = \frac{F}{y_2} = \left|\frac{15}{0.05}\right| = 300\,Nm^{-1}. \tag{8}$$

Recall that for the absorber to work, its natural frequency (before it is fastened to the vibrating system) is the same as the excitation frequency (4):

$$m_2 = \frac{k_2}{\omega^2} = \frac{300}{5^2} = 12\,kg. \tag{9}$$

The result of this solution is shown in the graph of the function in dimensionless variables of the amplitude characteristic dependence displacements y_1/y_2 and angular velocities ω/ω_2 of the absorber m_2 (Fig.2). The affiliate mass m_2 of the absorber is not moveable in the case when the ratio $\omega/\omega_2=1$.The next result is the phase characteristic. The change of the motion of the affiliate mass m_2 of absorber is 180° in area when the ratio $\omega/\omega_2=1$ (Fig. 3).

3 LQR control of the tuneable absorber

The possibility to reduce the vibration by the transient state is the tuneable absorber, where an air-operated spring with changed coefficient of elasticity k_2 through the changed pressure supply of the air is incorporated.

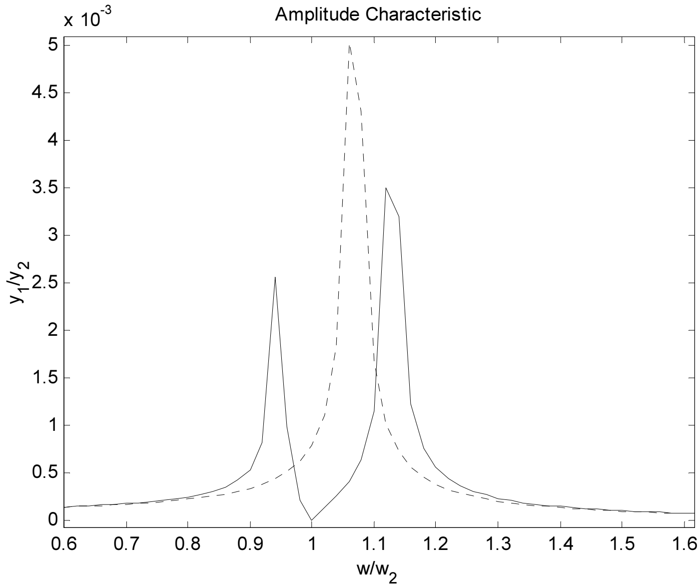

Fig. 2 The amplitude characteristic on dimensionless variables (without the absorber is stroke course)

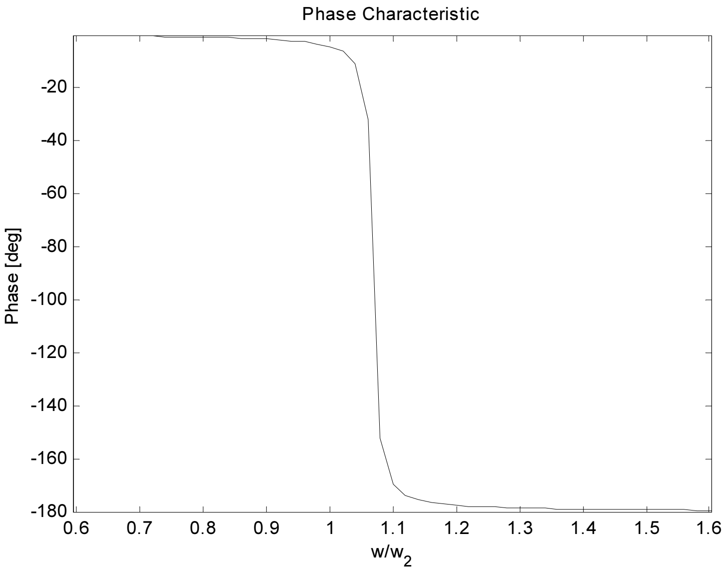

Fig. 3 The phase characteristic on the dimensionless variables

The model with affiliate mass m_2 of the LQR control absorber changed coefficient of elasticity k_2, an accelerometer a_1 located on the mass m_1 and an accelerometer a_2 located on the mass m_2 and control unit is shown on the Fig.1. It is possible to write the state description system and model (Fig.1) in the form:

$$\begin{aligned} &\dot{x} = Ax + B_{act}F_{act} + B_{tech}F_{tech}\,, \dot{y}_1 = Cx + Du\,, \\ &\dot{x}_m = Ax_m + B_{act}F_{act} + B_{tech}F_{tech} + LC\,, \\ &\dot{y}_{1m} = Cx_m + Du_m\,, \end{aligned} \tag{10}$$

where A is the state matrix, C the state matrix of output, D is the matrix of the coupling between input and output, F_{act} is the control force in the air-operated spring, F_{tech} is the spurious force from the technological process, B_{act} is the matrix of the input a control, B_{tech} is the matrix of input of the spurious force, x_m is the state vector of the model, y_{1m} is the displacement of the model and vector of input u is

$$F_{act} = -G.x_m\,, \tag{11}$$

where G is the matrix of control. It is possible to obtain this state bond with the help of the minimization of the integral criterion on the LQR (Linear Quadratic Regulator) control [1,2]

$$J = \int_0^{\infty} (x^T \cdot Q \cdot x + F_{act}^T \cdot R \cdot F_{act}) dt, \tag{12}$$

where *Q, R* are balance matrices. The LQR is the control, which minimizes the accuracy of control (first part of equation (12)) opposite power exacting (second part of equation (12)). The LQR control radiates from complete vector states, which in real life must be not in the feedback to position. In our case, we have to dispose the output parameters from the accelerometer *a* (Fig. 1). One-way, which this problem can be solved is use of the so-called state observer formulate with matrix *L,* where the parameters from the accelerometer are in to reconstructed a state of the system. The Matlab solution equations (1) for the parameters of system are in m-file on Fig. 4.The block diagrams of LQR control are on Fig. 5 and Fig. 6.

```
The constants:%---//
w = 5; %exciting frequency [rad/sec]
m1 = 350; % mass m1 [kg]
k1 = 10000;    %coefficient of elasticity k1 [N/m]
m2 = 12;    % affiliate masse m2 [kg] of absorber
k2 = w^2*m2;    %coefficient of elasticity k2 [N/m]
b1 = 10;    %damping coefficient b1 [Ns/m]
b2 = 0.1;    %damping coefficient b1 [Ns/m]
%x1 = dy1    velocity [m/s] of the the system
%x2 = dy2    velocity [m/s] of the absorber
%x3 = y1    position y1 [m] of the system
%x4 = y2    position y2 [m] of the absorber
%The system with damping %-------------------//
A = [   -(b1+b2)/m1   b2/m1   -(k1+k2)/m1   k2/m1;
        b2/m2   -b2/m2   k2/m2   -k2/m2;
        1   0   0   0;
        0   1   0   0  ];
B = [  1/m1;
       0;
       0;
       0  ];
C = [  0  0  1  0  ];
D = [  0  ];
        sys = ss(A,B,C,D);
%The system without damping%------//
A = [  -b1/m1  -k1/m1;
        1     0     ];
B = [  1/m1;
        0  ];
C = [  0  1  ];
D = [  0  ];
sys1 = ss(A,B,C,D);
%Bode characteristic%------//
W = 0.1*w:0.1:10*w;
[mag phase] = bode(sys, W);
[mag1 phase] = bode(sys1, W);
mag = squeeze(mag(1,1,:));
mag1 = squeeze(mag1(1,1,:));
phase = squeeze(phase(1,1,:));
figure(1);
plot(W./w,mag,'-k',W./w,mag1,':k');
axis([0.4 2 0 0.001]);
xlabel('w/w_h');
ylabel('y/y_h');
title('Amplitude Characteristic');
figure(2);
plot(W./w, phase,'-k');
axis([0 3 -180 0]);
xlabel('w/w_h');
ylabel('Phase [deg]');
   title('Phase Characteristic');
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Fig.4 The Matlab m-file for the solution of the absorber

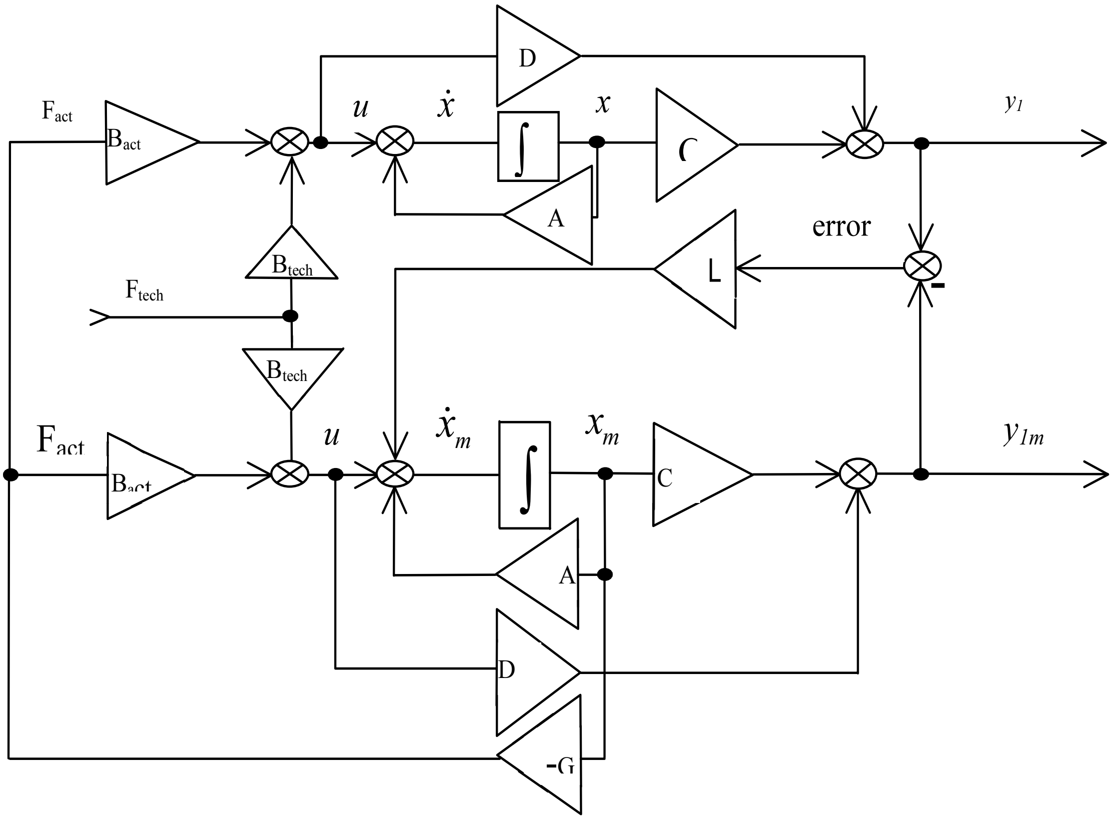

Fig. 5 LQR control block diagram for the control absorber

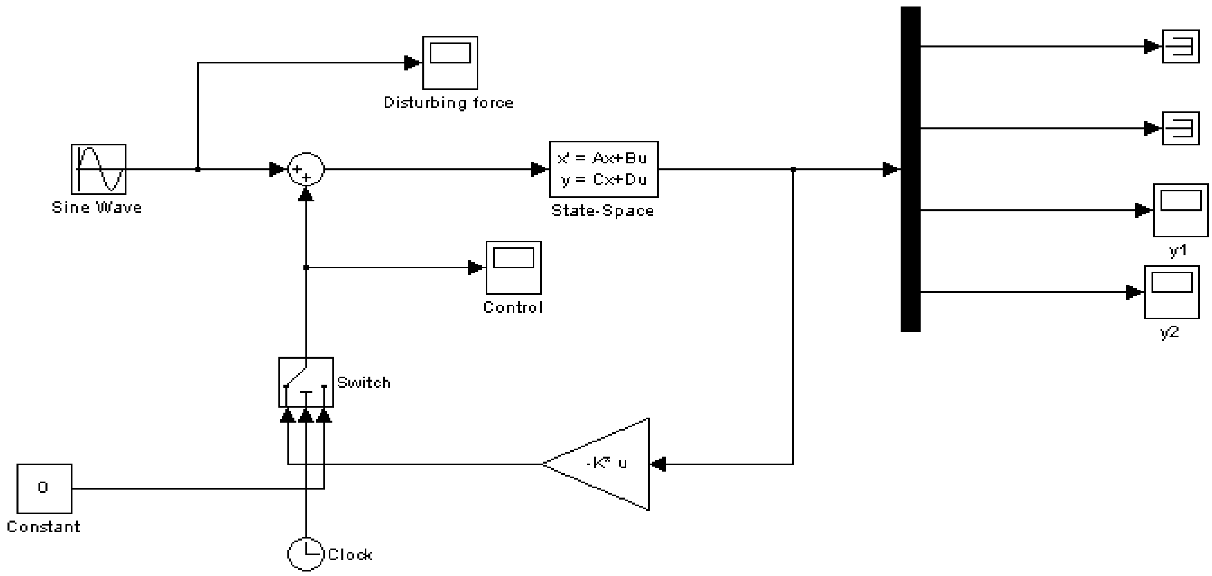

Fig. 6 The block diagram of LQR control

If we use the air-operated spring with the changed coefficient of elasticity k_2 with the possibility of regulation of the pressure p_{act} air in dependence to the displacement y_1, it is possible to reduce this displacement y_1 on the minimum. It is possible change some dependence for a pressure in the rubber bellows in the air-operated spring as the calculated force F_{act} . The coefficient of elasticity k_2 of the air-operated spring is

$$k_2 = \frac{2\,\gamma\, F_{act}\, S}{V}, \tag{13}$$

where γ is the ratio of specific heats for air, S is the effective cross-sectional are of the bellows for the air-operated spring. and V is the volume of the bellows.

4 Results

The present article discusses also a method of vibration control LQR for a structure by using the vibration absorber without damping. In the method, a variable stiffness vibration absorber is used for controlling the principle mode. The stiffness is controlled by the accelerometers *a* under the auto-tuning algorithm for creating an anti-resonance state. The optimal vibration absorber with damping with the air-operated spring is also utilized for controlling higher modes. The analyses and Matlab m-file for the auto-tuning control are developed. A method to obtain the optimal parameters has been presented for the vibration absorber, which controls higher modes. In order to validate the control method and the analysis, experimental tests will be carried out in the next phase of research.

Vibrations of machines and structures vanish perfectly at a certain frequency when they have a vibration absorber without damping. But if forced frequencies vary from the anti-resonance frequency, their vibration amplitudes increase significantly. Then, the absorber without damping cannot be applied to the structure subjected to variable frequency loads or to the loads having high frequency components. A vibration absorber can be used to eliminate unwanted steady-state vibrations from a one-degree of freedom system. We do this by tuning the natural frequency of the absorber (by itself) to the excitation frequency. The following must be kept in mind when designing an absorber: The vibration of the primary mass is reduced to zero (for no damping), or very small (with light damping). However, this is only at a single operating speed. If the system operates over a wide speed range, the vibrations away from the absorber frequency may still be large. The original system had one-degree of freedom. The modified system has an additional natural frequency. One of the new natural frequencies will be lower, and one higher, than the original frequency.

The difference between the two new natural frequencies, what we can solve from the equations (1) depends on the ratio of the secondary to primary masses $m_{2/}\ m_1$. The bigger the mass ratio, the further apart are the new natural frequencies.

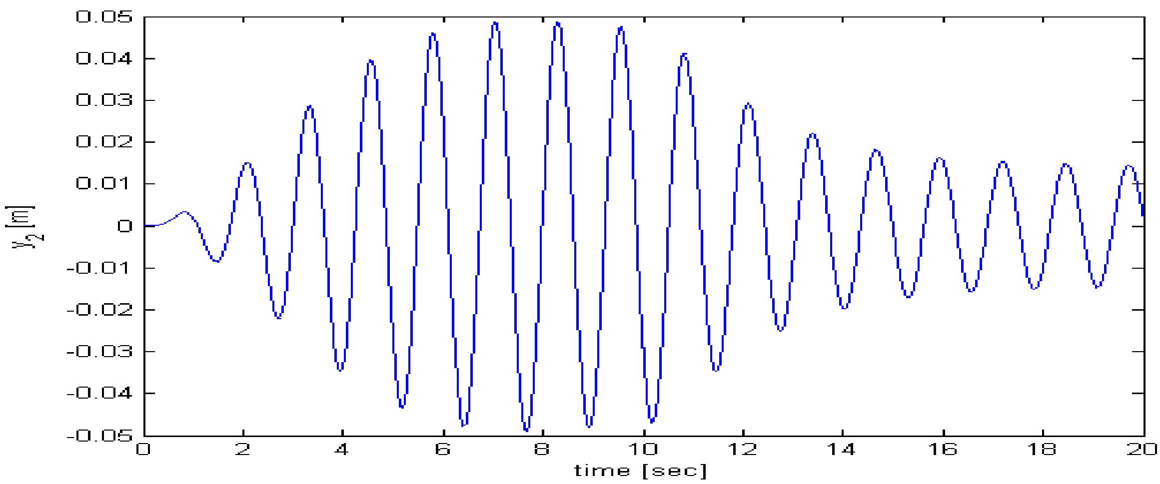

Fig. 7 The displacement y_2 after starting (10 sec) of LQR control (alternative I)

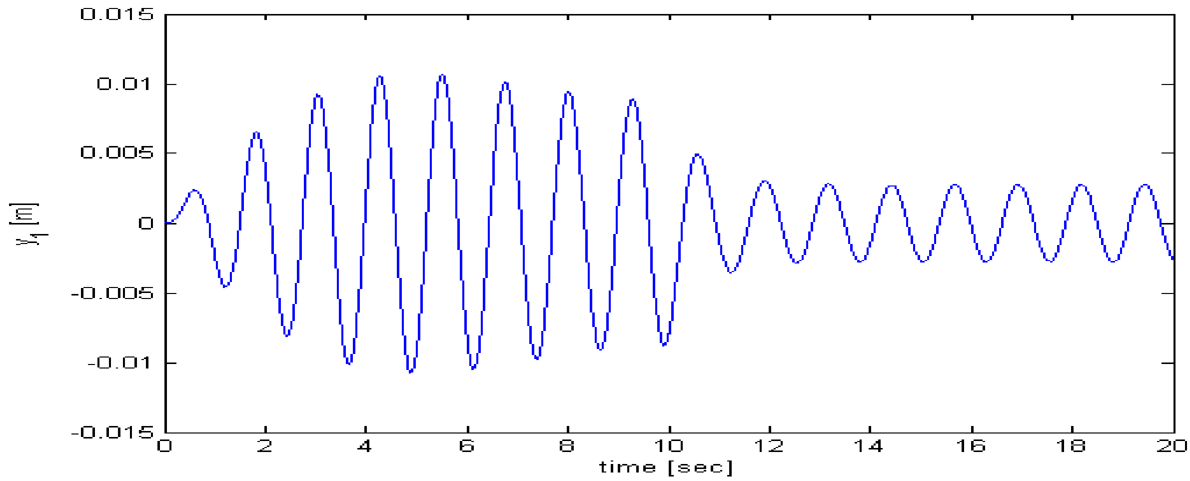

Fig. 9 The displacement y_1 after starting (10 sec) of LQR control (alternative I)

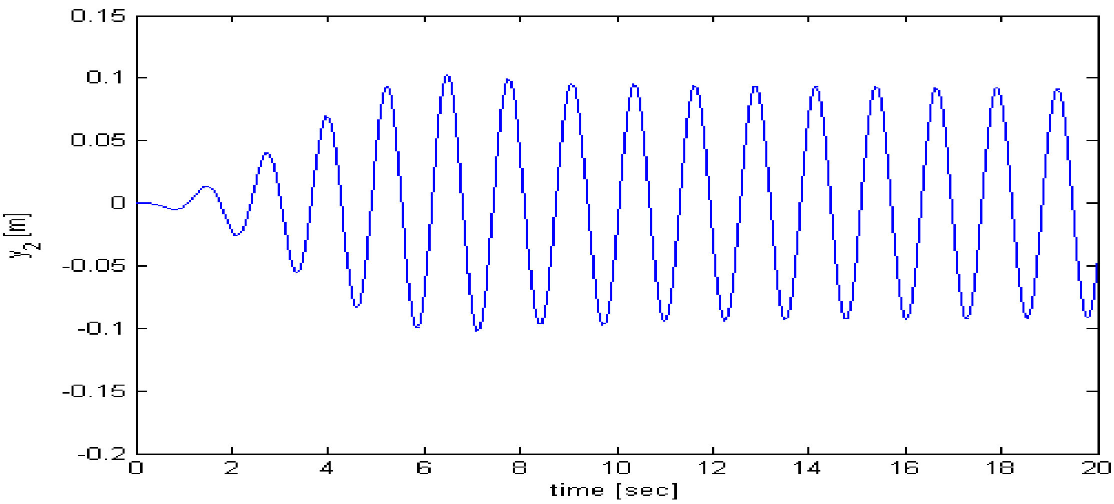

Fig. 9 The displacement y_2 after starting (10 sec) of LQR control (alternative II)

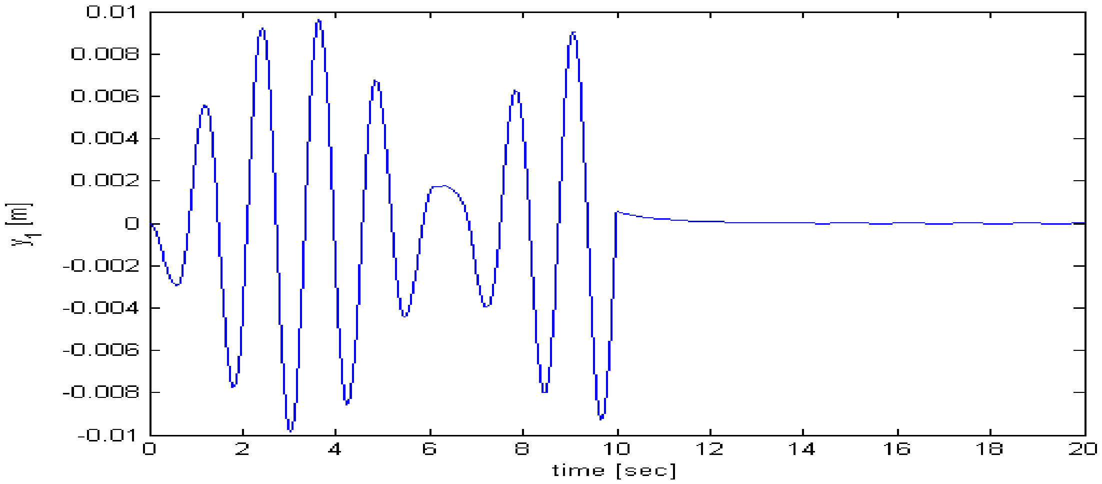

Fig. 10 The displacement y_1 after starting (10 sec) of LQR control (alternative II)

The aim of the paper is to acquaint the order with design of the incorporated absorber to the vibration of system, which makes possible the reduction of the vibration of the system to a minimum. In the case when the frequency ω of the acting force $F_{(t)}$ (alternative I and II) driving the system is the same as frequency ω_2 of the vibration of the affiliate mass m_2 of the absorber is the displacement of the mass m_1 after starting (t=10 sec) of LQR control y_1= 1,95 mm (Fig. 8) for the alternative I and for the alternative II is the displacement of the mass m_1 after starting (t=10 sec) of LQR control y_1= 0 mm (Fig. 10). The displacements of the mass m_2 the absorber is show on the Fig. 7 and Fig. 9. If we use the air-operated spring with the changed coefficient of elasticity k_2 with the possibility of regulation of the pressure $p_{act}=F_{act}/S$ for the air in the operated spring in dependence to the displacement y_1 of is possible to reduce this displacement y_1 on the minimum (Fig. 8).

References

[1] G. F. Franklin, J. D. Powell, M. L. Lewis. *Control of Dynamic Systems* Adison Wesley, New Jersey, second edition, 1990.

[2] Vl. Havlena, J. Štecha, *Moderní teorie řízen* Vydavatelství ČVUT, Praha, 2000.

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