Algorithm for Time Domain Simulation of Analog Filters Using the Symbolic Math Toolbox

Karel Zaplatilek

University of Defence, Department of Electrical Engineering, Kounicova 65, 612 00 Brno, Czech Republic

karel.zaplatilek@unob.cz

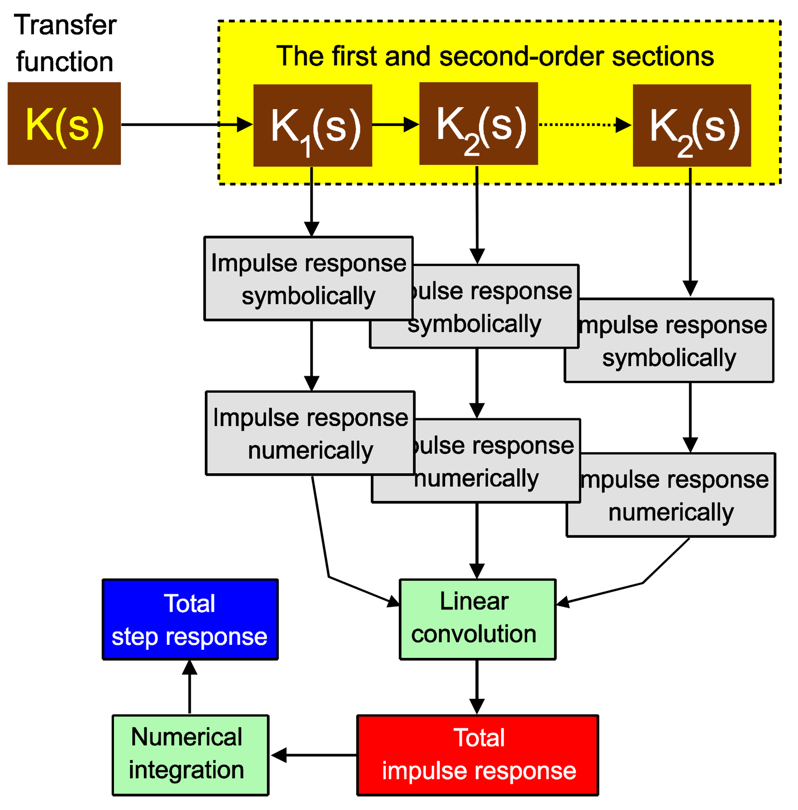
TCP2006

1. Abstract

- a set of new MATLAB functions was created to extend the current possibilities of time-domain analysis of analog filters
- the transfer function of the analog filter is an input expression of the algorithm
- the transfer function of the filter is divided into the first and second-order sections
- the impulse response of each section is derived symbolically using the MATLAB Symbolic Toolbox
- the impulse response and step response of the whole filter are obtained using a linear convolution and integration
- the descibed technique is suitable for accurate and robust calculation of the time domain responses of arbitrary-order analog filters

TCP2006

2. Basic idea of the algorithm



TCP2006

3. Impulse response in a symbolic form

- the impulse response of each section is derived using the MATLAB Symbolic Toolbox (L-Laplace transform)

$$g(t) = L^{-1}\{K(s)\}$$

Example 1

$$K(s) = \frac{s^2+\omega_r^2}{s^2+\frac{\omega_r}{Q}.s+\omega_r^2}$$

2^{nd}- order band stop filter

```
syms s wr Q
K=(s^2+wr^2)/(s^2+wr/Q*s+wr^2)
g=ilaplace(K)
```

MATLAB source code

$$g(t) = \delta(t) + 2xe^{xt}\cos(yt) + 2\frac{x^2}{y}e^{xt}\sin(yt)$$

$$g(t)_{Q=0,5} = \delta(t) - (2\omega_r - 2\omega_r^2 t).e^{-\omega_r t}$$

$$x = -\frac{1}{2}\frac{\omega_r}{Q}, \qquad y = \frac{1}{2}\frac{\omega_r}{Q}\sqrt{4Q^2-1}$$

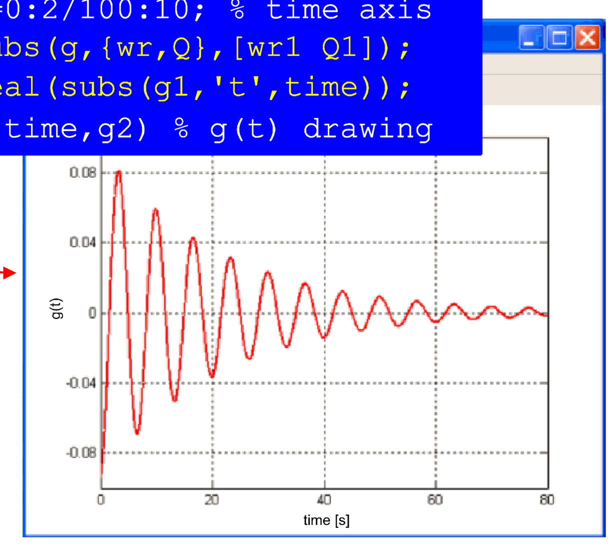
TCP2006

4. Impulse response in a numeric form

- the numeric form of the impulse response is obtained using a substitution

Example 2

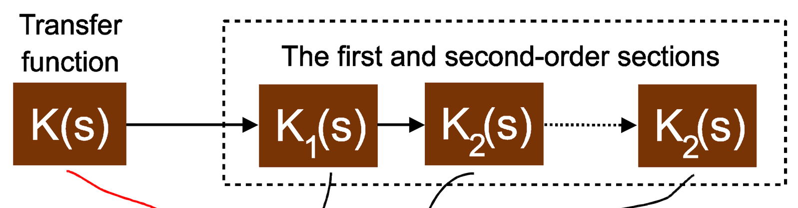
```
fr1=0.15; % resonant frequency
wr1=2*pi*fr1;
Q1=10; % Q-factor
time=0:2/100:10; % time axis
g1=subs(g,{wr,Q},[wr1 Q1]);
g2=real(subs(g1,'t',time));
plot(time,g2) % g(t) drawing
```



TCP2006

5. Total impulse response

- the total impulse response of the whole filter is computed using a linear convolution



$$g(t) = g_1(t) * g_{21}(t) * g_{22}(t)$$

```
gg=Ts*conv(g1,g21);
ggg=Ts*conv(g22,gg);
g=ggg(1,[length(time)]);
```

MATLAB source code

Ts - sampling period

TCP2006

Example 3

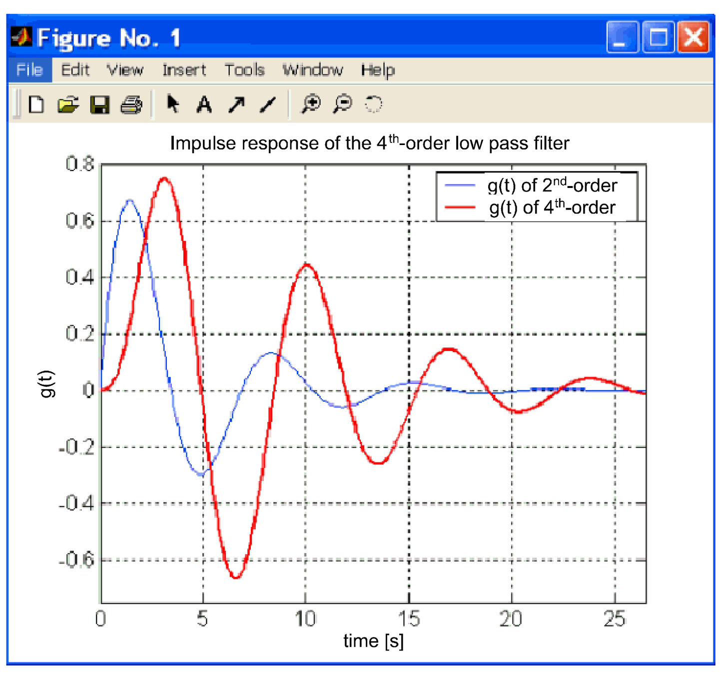
- computation of the impulse response of the 4th-order low-pass filter (two identical sections)

```
syms s wr Q
K=wr^2/(s^2+wr/Q*s+wr^2); % one section
g=ilaplace(K); % impulse response
  fr1=0.15;
  wr1=2*pi*fr1;
  Q1=2;
  maxtime=25/wr1;
  numstep=500; % number of steps
  time=0:maxtime/(numstep-1):maxtime;
  Ts=maxtime/numstep; % sampling period
g1=subs(g,{wr,Q},[wr1 Q1]);
g2=real(subs(g1,'t',time));
gg4=Ts*conv(g2,g2); % second section
g4=gg4(1,[1:length(time)]);
plot(time,g2) % g(t) of 2nd-order section
hold on
plot(time,g4) % g(t) of 4th-order filter
```

MATLAB source code

TCP2006

- impulse responses of the 2nd-order section and the whole 4th-order filter



Impulse responses diagram

TCP2006

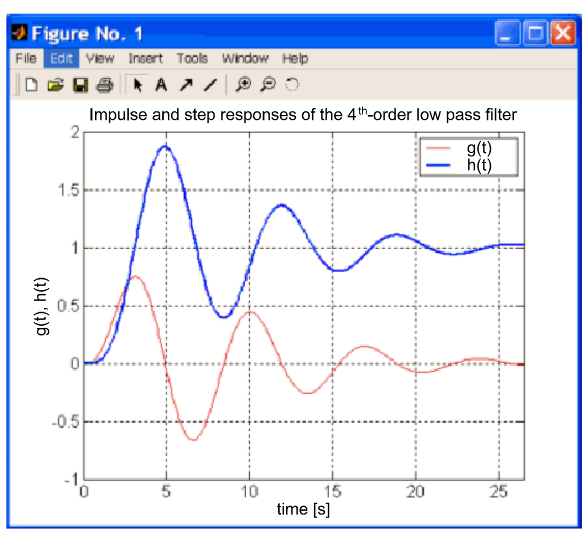
6. Total step response

- the step response of the whole filter is computed using a digital integrator (a simple IIR digital filter)

Digital integrator $K(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$

```
h=Ts*filter(1,[1 -1],g4);
h=h(1,[length(time)]);
```

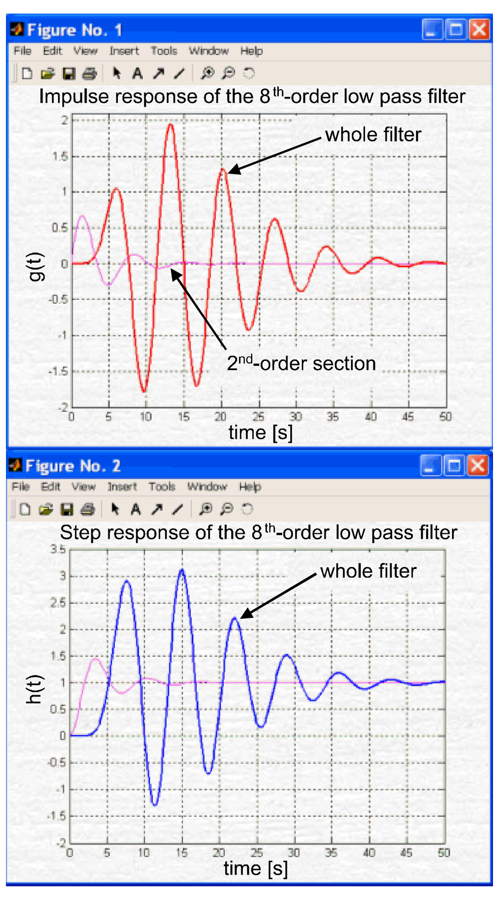
MATLAB source code



TCP2006

7. Illustrational example

- time domain analysis of the 8th-order low-pass filter



TCP2006

8. Generalized mathematical formalism

- the 1st and 2nd-order sections have different models according to the Q-factor (Q=0,5[-] and an arbitrary value)
- nevertheless we can derive common models using a limit calculus

2nd-order high-pass filter

$$g(t)_Q = \lim_{q\to Q}\left\{\delta(t) + 2xe^{x.t}.\cos(yt) + 2\frac{x^2}{y}(1-2q^2)e^{x.t}.\sin(yt)\right\} ✓$$

$$g(t)_{0,5} = \lim_{q\to 0,5}\left\{\delta(t) + 2xe^{x.t}.\cos(yt) + 2\frac{x^2}{y}(1-2q^2)e^{x.t}.\sin(yt)\right\} =$$

$$= \lim_{y\to 0}\left\{\delta(t) + 2xe^{x.t}.\cos(yt) + 2\frac{x^2}{y}(1-2q^2)e^{x.t}.\sin(yt)\right\} =$$

platí $x = -\frac{1}{2}\frac{\omega_r}{Q}$ pro $Q = 0,5 \Rightarrow x = -\omega_r$

$$\lim_{y\to 0}\frac{\sin(yt)}{y} = \lim_{y\to 0}\frac{\sin(yt)'}{y'} = \lim_{y\to 0}\frac{t.\cos(yt)}{1} = t$$

$$= \;\ldots\ldots\; \delta(t) + (\omega_r^2 t - 2\omega_r)e^{-\omega_r.t} ✓$$

TCP2006

9. Conclusion

- the MATLAB system has insufficient support for time domain analysis of analog filters
- a new function was developed to extend MATLAB possibilities
- our approach is based on rigorous symbolic preprocessing; impulse responses of the 1st and 2nd-order sections are derived in a symbolic form
- the described algorithm is also suitable for higher-order filters and it is accurate and robust
- the algorithm can detect the Dirac function of some impulse responses

TCP2006

Acknowledgement

The Czech Science Foundation supports this project under number 102/03/1181

www.gacr.cz