CONSTITUTIVE MATERIAL MODEL OF FIBER-REINFORCED COMPOSITES AT FINITE STRAINS IN COMSOL MULTIPHYSICS

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Constitutive equations of anisotropic hyperelasticity

The free energy for the material with one family of fiber is:

$$\Psi = \Psi\left(\mathbf{C}, \mathbf{A}_0\right) \qquad \mathbf{A_0} = \mathbf{a_0} \otimes \mathbf{a_0},\ |\mathbf{a_0}| = 1.$$

$$\Psi = \Psi\left[I_1\left(\mathbf{C}\right), I_2\left(\mathbf{C}\right), I_3\left(\mathbf{C}\right), I_4\left(\mathbf{C}, \mathbf{a_0}\right), I_5\left(\mathbf{C}, \mathbf{a_0}\right)\right]$$

For the two families of fibers, the free energy is

$$\Psi = \Psi\left(\mathbf{C}, \mathbf{A_0}, \mathbf{B_0}\right) \qquad \mathbf{A_0} = \mathbf{a_0} \otimes \mathbf{a_0},\ \mathbf{B_0} = \mathbf{b_0} \otimes \mathbf{b_0}.$$

$$\Psi = \Psi\left[I_1\left(\mathbf{C}\right), I_2\left(\mathbf{C}\right), I_3\left(\mathbf{C}\right), I_4\left(\mathbf{C}, \mathbf{a_0}\right), I_5\left(\mathbf{C}, \mathbf{a_0}\right), I_6\left(\mathbf{C}, \mathbf{b_0}\right), I_7\left(\mathbf{C}, \mathbf{b_0}\right), I_8\left(\mathbf{C}, \mathbf{a_0}, \mathbf{b_0}\right)\right]$$

Constitutive equations of anisotropic hyperelasticity

$$\Psi = \Psi_{vol}\left(J\right) + \Psi_{iso}\left(\overline{I_1}, \overline{I_2}\right) + \Psi_{ani}\left(\overline{I_\alpha}\right)$$

$$\overline{I_1} = J^{-2/3} I_1 \qquad \overline{I_2} = J^{-4/3} I_2$$

$\overline{I_a} = J^{-2/3} I_a$ for a=4,6,8; $\overline{I_a} = J^{-4/3} I_a$ for a=5,7.

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}} = \mathbf{S}_{vol} + \mathbf{S}_{iso} + \mathbf{S}_{ani}$$

Some forms of the free energy function

Volumetric part

$$\Psi_{vol}(J) = \frac{\kappa}{2}(J-1)^2$$

$$\Psi_{vol}(J) = \kappa \mathcal{G}(J)$$

$$\mathcal{G}(J) = \beta^{-2}\left(\beta \ln J + J^{-\beta} - 1\right)$$

$$\mathcal{G} = \frac{1}{4}\left(J^2 - 1 - 2\ln J\right)$$ for β=-2.

Some forms of the free energy function

Isotropic isochoric part

Neo-Hookean: $$\Psi_{iso} = \frac{\mu}{2}\left(\overline{I}_1 - 3\right)$$

Mooney-Rivlin: $$\Psi_{iso} = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right)$$

Ogden: $$\Psi_{iso} = \Psi\left(\overline{\lambda}_1, \overline{\lambda}_2, \overline{\lambda}_3\right) = \sum_{a=1}^{N} \frac{\mu_a}{\alpha_a}\left(\overline{\lambda}_1^{\alpha_a} + \overline{\lambda}_2^{\alpha_a} + \overline{\lambda}_3^{\alpha_a} - 3\right)$$

Some forms of the free energy function

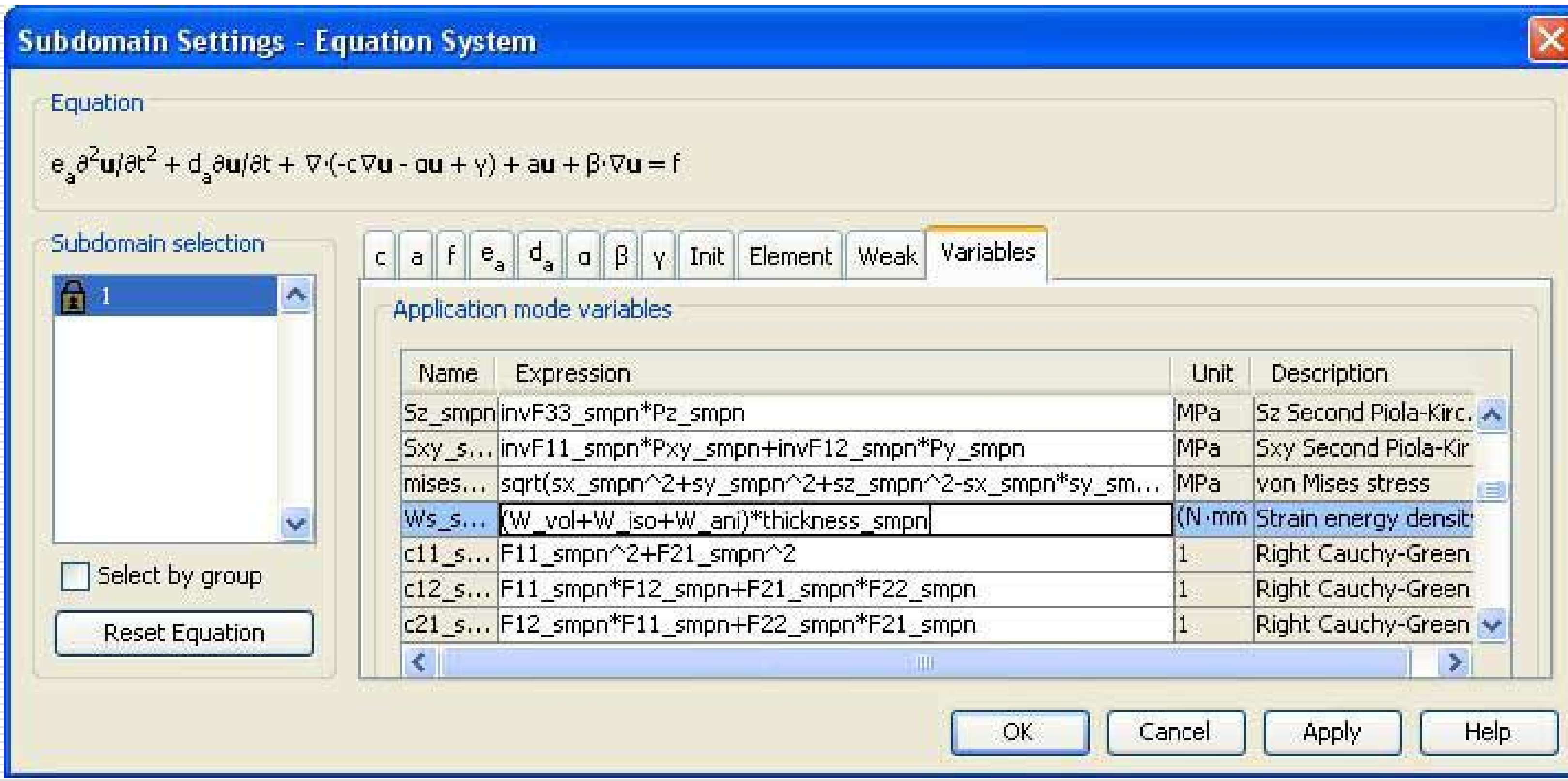
Anisotropic isochoric part

$$\Psi_{ani} = \sum_{i=2}^{6} a_i \left(\overline{I_4} - 1\right)^{2i} + \sum_{j=2}^{6} b_j \left(\overline{I_5} - 1\right)^{2j} + \sum_{k=2}^{6} c_k \left(\overline{I_6} - 1\right)^{2k}$$

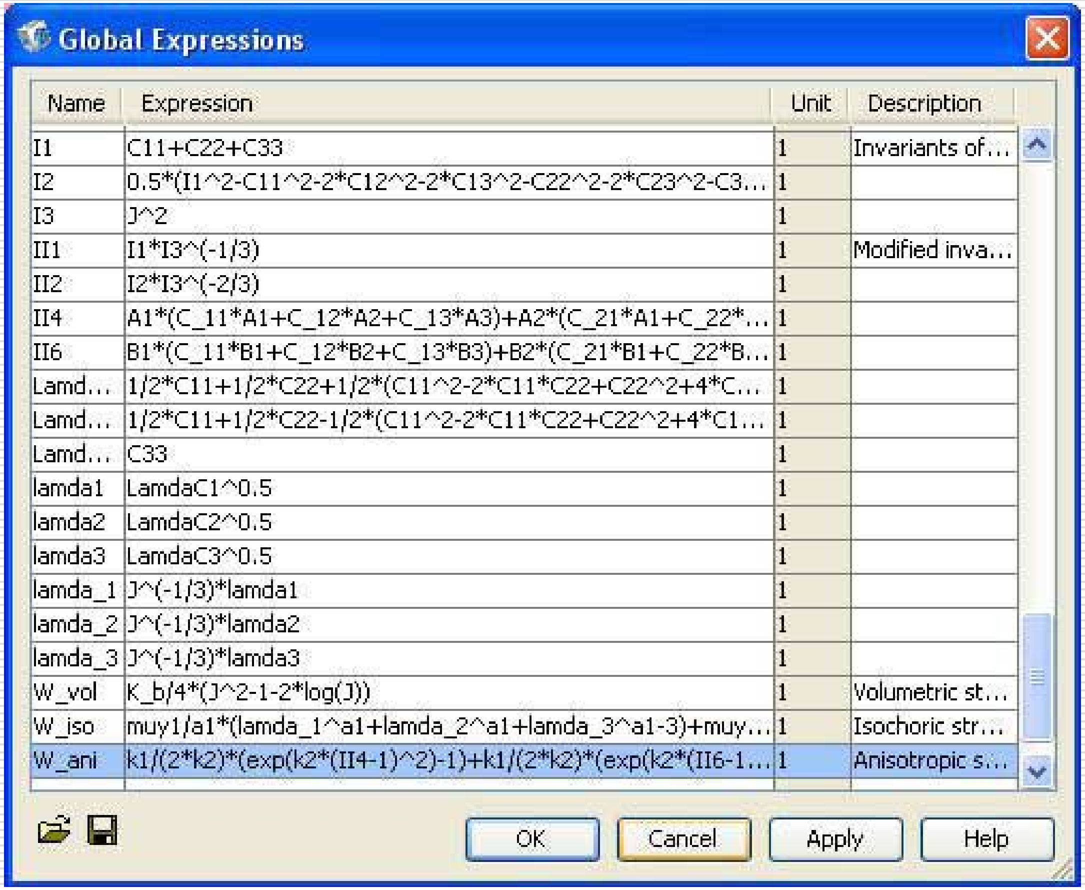
$$+ \sum_{l=2}^{6} d_l \left(\overline{I_7} - 1\right)^{2l} + \sum_{m=2}^{6} e_m \left(\overline{I_8} - \varsigma\right)^{2m}$$

$$\Psi_{ani} = \frac{k_1}{2k_2} \left\{ \exp\left[k_2 \left(\overline{I_4} - 1\right)^2 \right] - 1 \right\} + \frac{k_1}{2k_2} \left\{ \exp\left[k_2 \left(\overline{I_6} - 1\right)^2 \right] - 1 \right\}$$

Material model implementation into Comsol Multiphysics



Material model implementation into Comsol Multiphysics



Global Expressions

Name	Expression	Unit	Description
I1	C11+C22+C33	1	Invariants of...
I2	0.5*(I1^2-C11^2-2*C12^2-2*C13^2-C22^2-2*C23^2-C3...	1	
I3	J^2	1	
II1	I1*I3^(-1/3)	1	Modified inva...
II2	I2*I3^(-2/3)	1	
II4	A1*(C_11*A1+C_12*A2+C_13*A3)+A2*(C_21*A1+C_22*...	1	
II6	B1*(C_11*B1+C_12*B2+C_13*B3)+B2*(C_21*B1+C_22*B...	1	
Lamd...	1/2*C11+1/2*C22+1/2*(C11^2-2*C11*C22+C22^2+4*C...	1	
Lamd...	1/2*C11+1/2*C22-1/2*(C11^2-2*C11*C22+C22^2+4*C1...	1	
Lamd...	C33	1	
lamda1	LamdaC1^0.5	1	
lamda2	LamdaC2^0.5	1	
lamda3	LamdaC3^0.5	1	
lamda_1	J^(-1/3)*lamda1	1	
lamda_2	J^(-1/3)*lamda2	1	
lamda_3	J^(-1/3)*lamda3	1	
W_vol	K_b/4*(J^2-1-2*log(J))	1	Volumetric st...
W_iso	muy1/a1*(lamda_1^a1+lamda_2^a1+lamda_3^a1-3)+muy...	1	Isochoric str...
W_ani	k1/(2*k2)*(exp(k2*(II4-1)^2)-1)+k1/(2*k2)*(exp(k2*(II6-1...	1	Anisotropic s...

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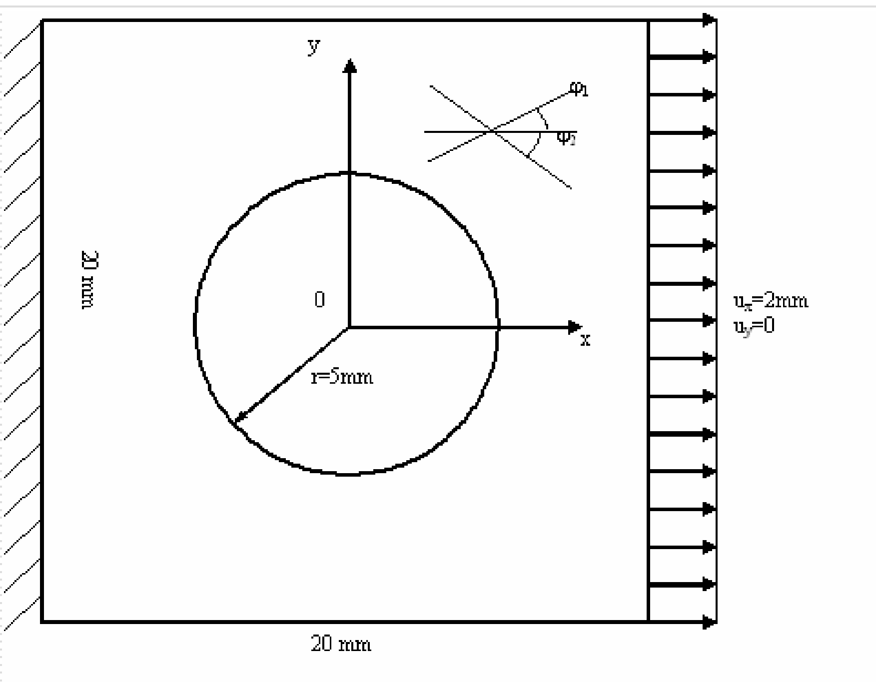
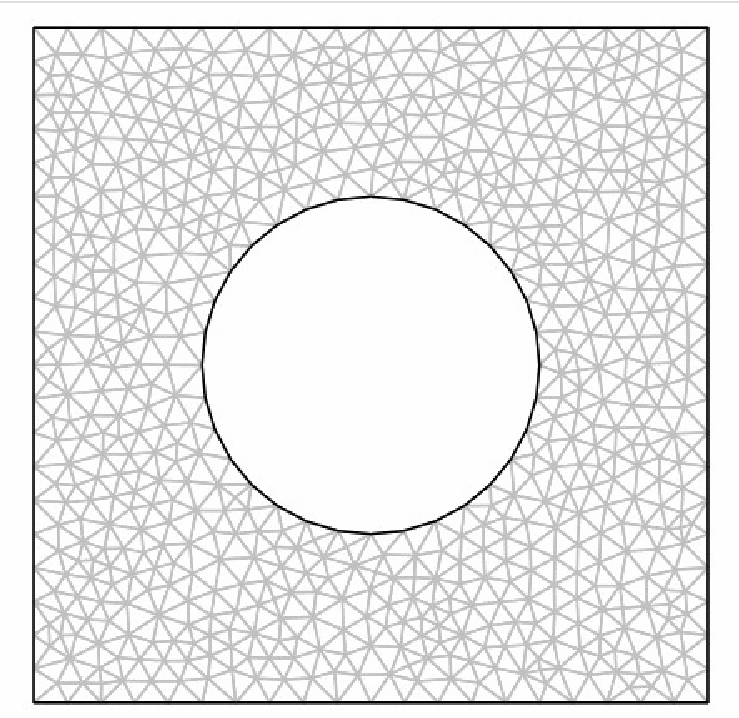
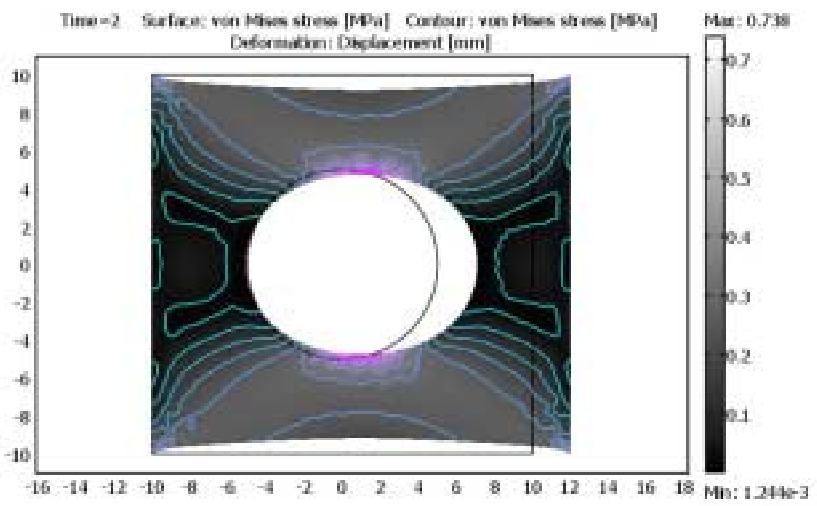
Applications and examples



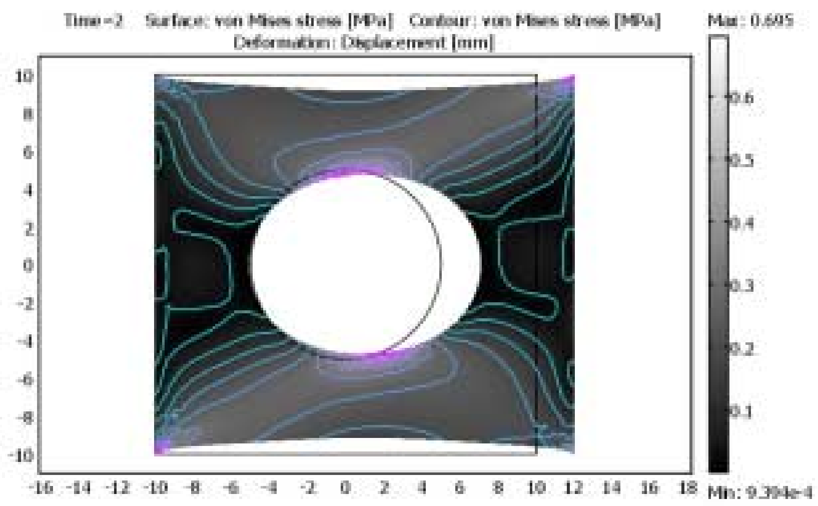


Applications and examples

One family of fiber



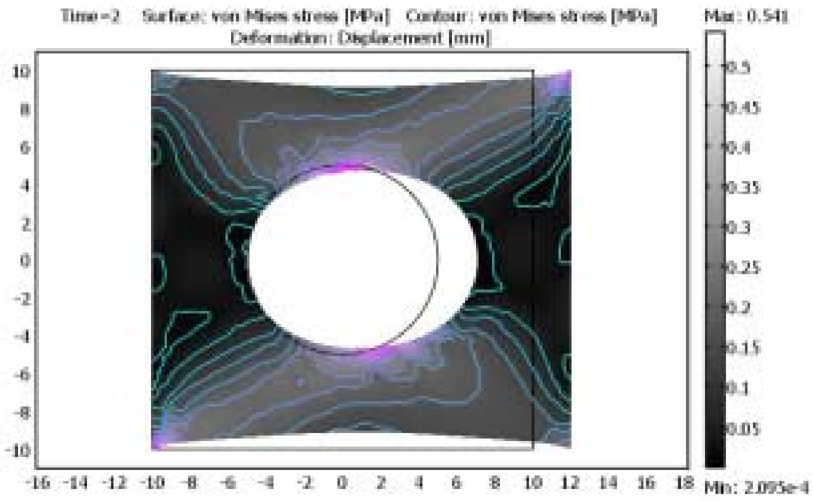
$\varphi = 0^0$



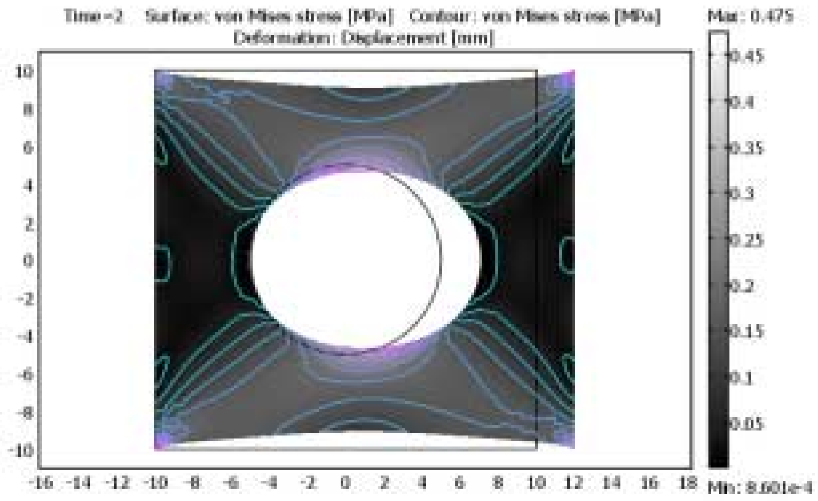
$\varphi = 15^0$

Applications and examples

One family of fiber



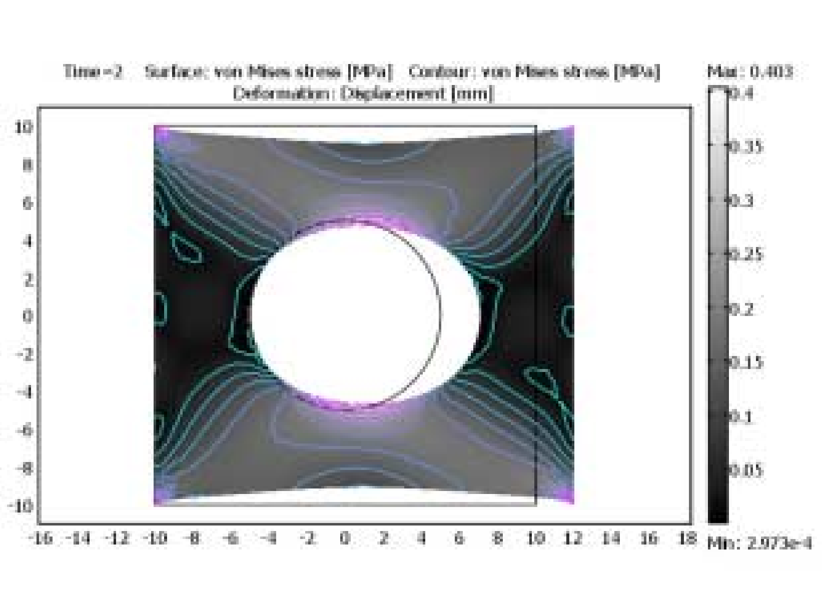
φ=30⁰

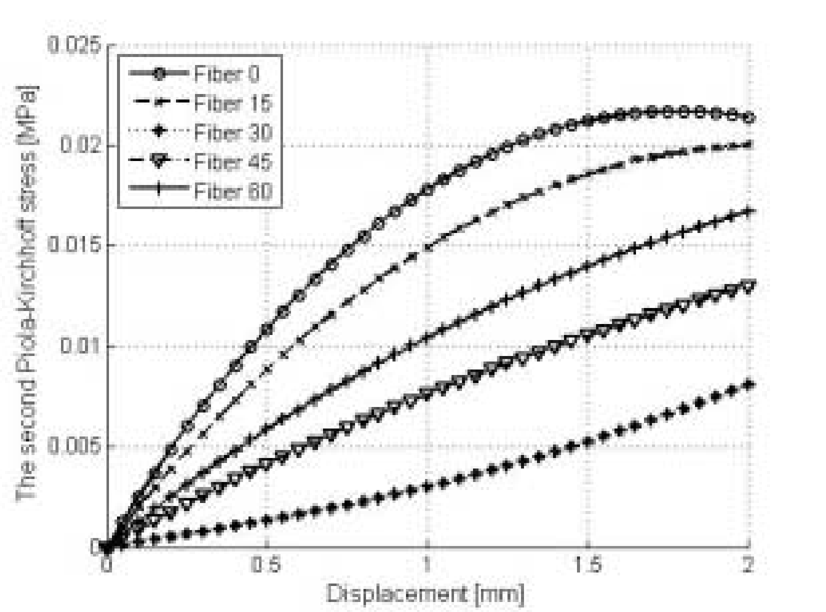


φ=45⁰

Applications and examples

One family of fiber

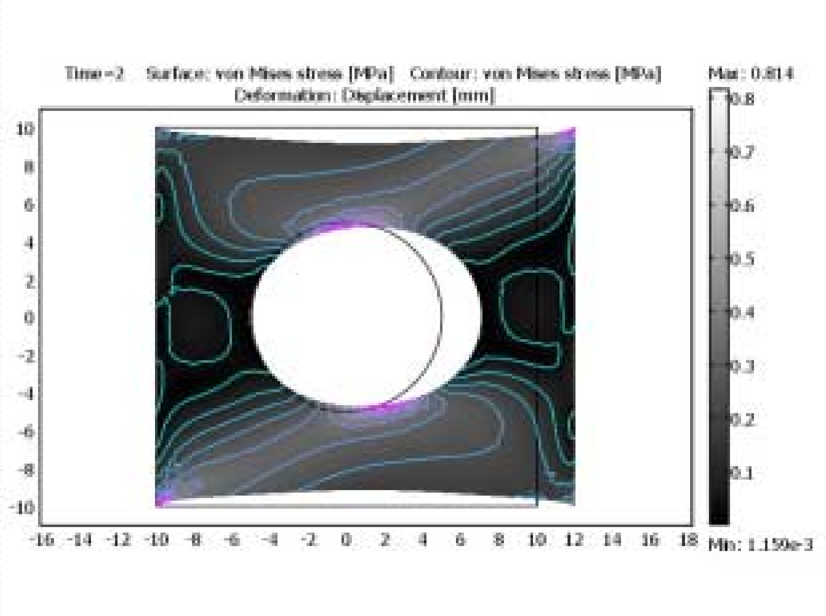




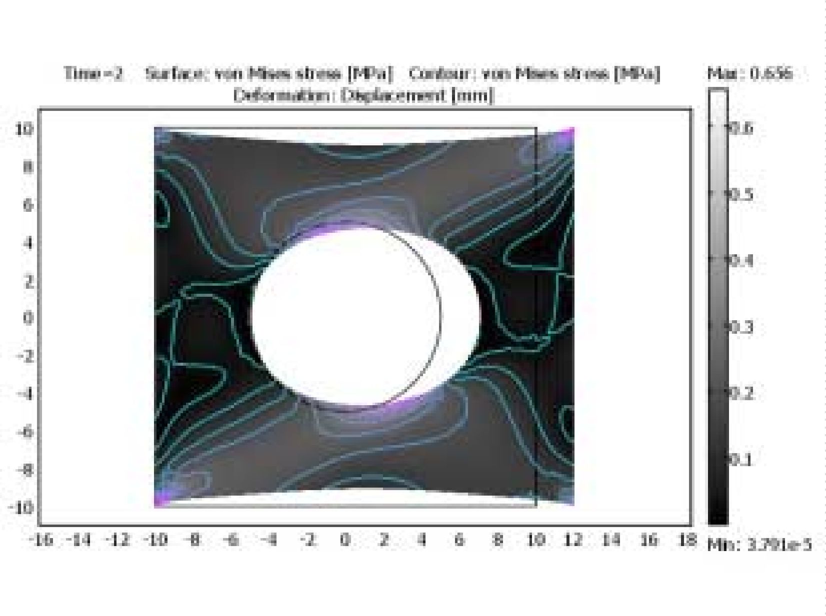
$\varphi = 60^0$

Applications and examples

Two perpendicular families of fibers



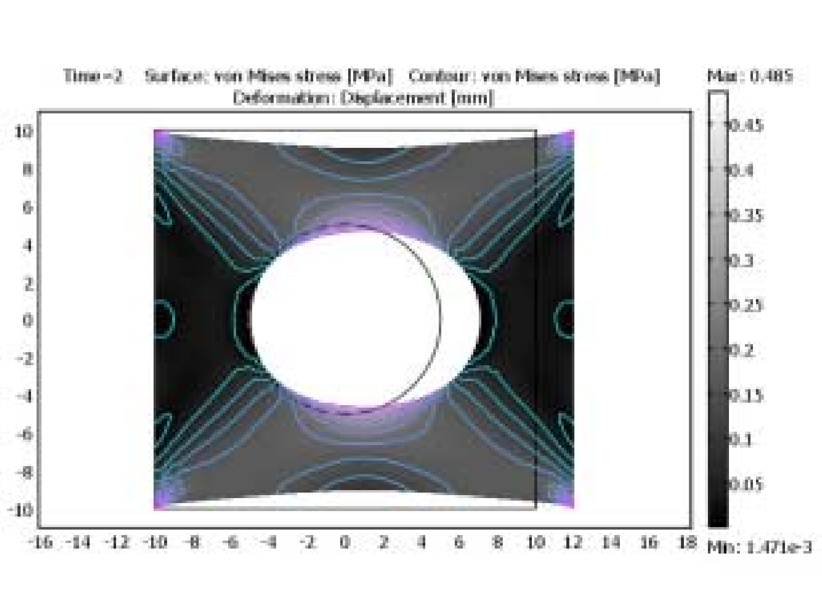
$\varphi=15^0$

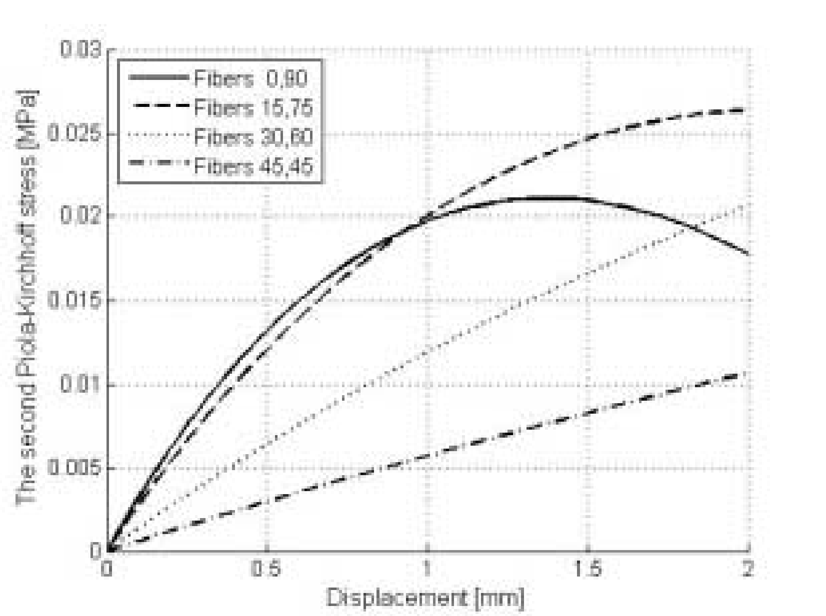


$\varphi=30^0$

Applications and examples

Two perpendicular families of fibers

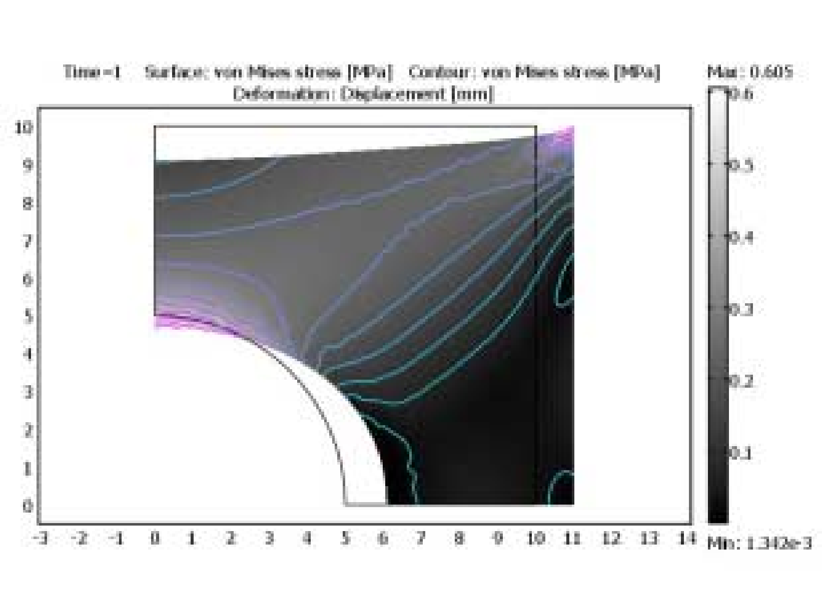




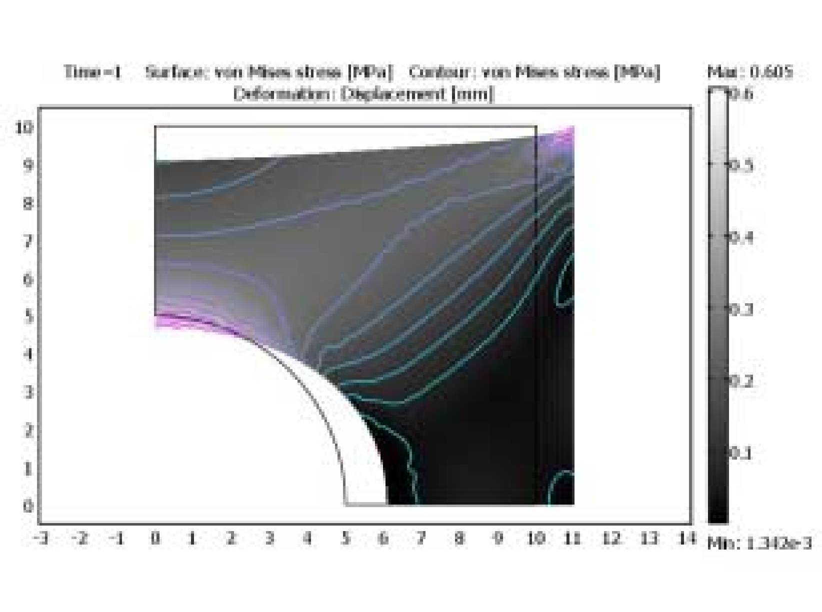
$\varphi = 45^0$

Applications and examples

Two families of fibers arranged symmetrically with respect to the axis of loading



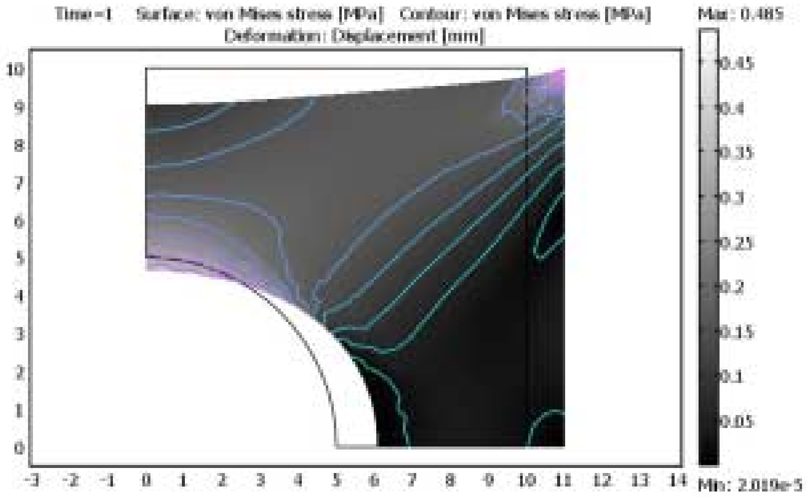
φ=15⁰

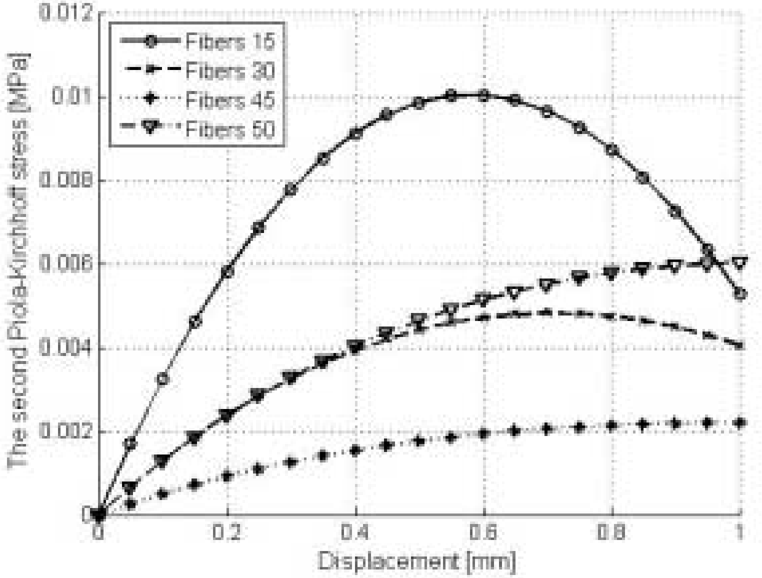


φ=30⁰

Applications and examples

Two families of fibers arranged symmetrically with respect to the axis of loading

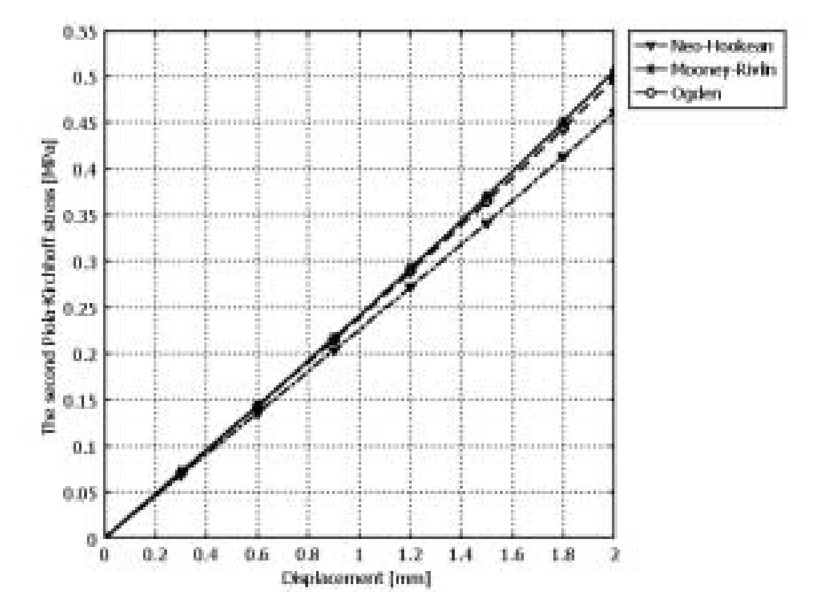


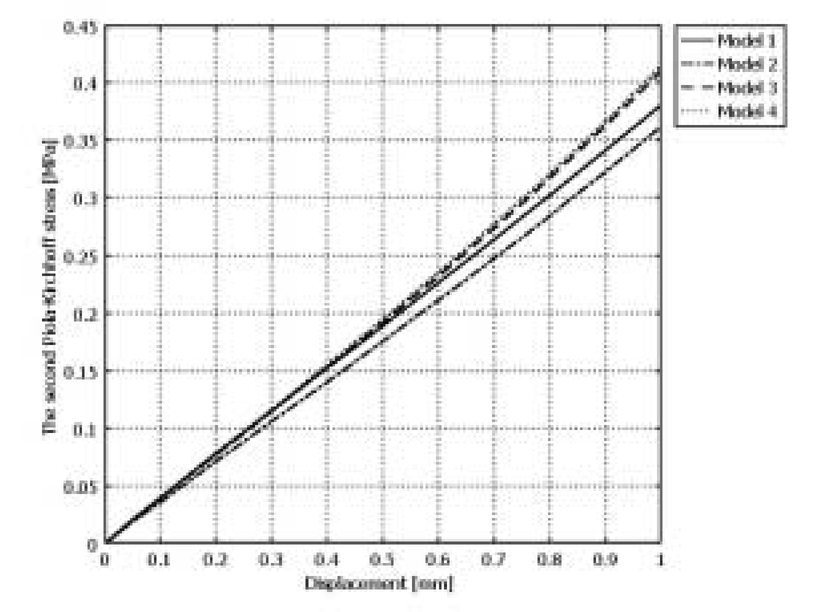


$\varphi = 45^0$

Applications and examples

Comparison of different material models





Conclusion

The new material model of the anisotropic composite material in finite strain and the Ogden model of hyperelastic material in 2-D were implemented into Comsol Multiphysics.

Some simple examples were subsequently presented, namely the extension of a composite block with a central hole. The results are in qualitative agreement with experimental observations.

Acknowledgement:
This work was supported by the subvention from Ministry of Education of the Czech Republic under Contract Code MSM 4674788501