

CONSTITUTIVE MATERIAL MODEL OF FIBER-REINFORCED COMPOSITES AT FINITE STRAINS IN COMSOL MULTIPHYSICS

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Constitutive equations of anisotropic hyperelasticity

The free energy for the material with one family of fiber is:

$$\Psi = \Psi(\mathbf{C}, \mathbf{A}_0) \quad \mathbf{A}_0 = \mathbf{a}_0 \otimes \mathbf{a}_0, \quad |\mathbf{a}_0| = 1.$$

$$\Psi = \Psi \left[I_1(\mathbf{C}), I_2(\mathbf{C}), I_3(\mathbf{C}), I_4(\mathbf{C}, \mathbf{a}_0), I_5(\mathbf{C}, \mathbf{a}_0) \right]$$

For the two families of fibers, the free energy is

$$\Psi = \Psi(\mathbf{C}, \mathbf{A}_0, \mathbf{B}_0) \quad \mathbf{A}_0 = \mathbf{a}_0 \otimes \mathbf{a}_0, \quad \mathbf{B}_0 = \mathbf{b}_0 \otimes \mathbf{b}_0.$$

$$\Psi = \Psi \left[I_1(\mathbf{C}), I_2(\mathbf{C}), I_3(\mathbf{C}), I_4(\mathbf{C}, \mathbf{a}_0), I_5(\mathbf{C}, \mathbf{a}_0), \right. \\ \left. I_6(\mathbf{C}, \mathbf{b}_0), I_7(\mathbf{C}, \mathbf{b}_0), I_8(\mathbf{C}, \mathbf{a}_0, \mathbf{b}_0) \right]$$

Constitutive equations of anisotropic hyperelasticity

$$\Psi = \Psi_{vol}(J) + \Psi_{iso}(\bar{I}_1, \bar{I}_2) + \Psi_{ani}(\bar{I}_\alpha)$$

$$\bar{I}_1 = J^{-2/3} I_1 \quad \bar{I}_2 = J^{-4/3} I_2$$

$$\bar{I}_a = J^{-2/3} I_a \quad \text{for } a=4,6,8; \quad \bar{I}_a = J^{-4/3} I_a \quad \text{for } a=5,7.$$

$$\mathbf{S} = 2 \frac{\partial \Psi}{\partial \mathbf{C}} = \mathbf{S}_{vol} + \mathbf{S}_{iso} + \mathbf{S}_{ani}$$

Some forms of the free energy function

Volumetric part

$$\Psi_{vol}(J) = \frac{\kappa}{2}(J-1)^2$$

$$\Psi_{vol}(J) = \kappa \mathcal{G}(J)$$

$$\mathcal{G}(J) = \beta^{-2} (\beta \ln J + J^{-\beta} - 1)$$

$$\mathcal{G} = \frac{1}{4} (J^2 - 1 - 2 \ln J) \quad \text{for } \beta = -2.$$

Some forms of the free energy function

Isotropic isochoric part

Neo-Hookean:
$$\Psi_{iso} = \frac{\mu}{2} (\bar{I}_1 - 3)$$

Mooney-Rivlin:
$$\Psi_{iso} = c_{10} (\bar{I}_1 - 3) + c_{01} (\bar{I}_2 - 3)$$

Ogden:
$$\Psi_{iso} = \Psi(\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3) = \sum_{a=1}^N \frac{\mu_a}{\alpha_a} (\bar{\lambda}_1^{\alpha_a} + \bar{\lambda}_2^{\alpha_a} + \bar{\lambda}_3^{\alpha_a} - 3)$$

Some forms of the free energy function

Anisotropic isochoric part

$$\Psi_{ani} = \sum_{i=2}^6 a_i (\bar{I}_4 - 1)^{2i} + \sum_{j=2}^6 b_j (\bar{I}_5 - 1)^{2j} + \sum_{k=2}^6 c_k (\bar{I}_6 - 1)^{2k} \\ + \sum_{l=2}^6 d_l (\bar{I}_7 - 1)^{2l} + \sum_{m=2}^6 e_m (\bar{I}_8 - \zeta)^{2m}$$

$$\Psi_{ani} = \frac{k_1}{2k_2} \left\{ \exp \left[k_2 (\bar{I}_4 - 1)^2 \right] - 1 \right\} + \frac{k_1}{2k_2} \left\{ \exp \left[k_2 (\bar{I}_6 - 1)^2 \right] - 1 \right\}$$

Material model implementation into Comsol Multiphysics

Subdomain Settings - Equation System

Equation

$$e_a \partial^2 \mathbf{u} / \partial t^2 + d_a \partial \mathbf{u} / \partial t + \nabla \cdot (-c \nabla \mathbf{u} - \alpha \mathbf{u} + \gamma) + \mathbf{a} \mathbf{u} + \beta \cdot \nabla \mathbf{u} = f$$

Subdomain selection

1

Select by group

Reset Equation

Application mode variables

Name	Expression	Unit	Description
Sz_smpn	invF33_smpn*Pz_smpn	MPa	Sz Second Piola-Kirc.
Sxy_s...	invF11_smpn*Pxy_smpn+invF12_smpn*Py_smpn	MPa	Sxy Second Piola-Kir
mises...	sqrt(sx_smpn^2+sy_smpn^2+sz_smpn^2-sx_smpn*sy_sm...	MPa	von Mises stress
Ws_s...	(W_vol+W_iso+W_ani)*thickness_smpn	(N·mm)	Strain energy densit
c11_s...	F11_smpn^2+F21_smpn^2	1	Right Cauchy-Green
c12_s...	F11_smpn*F12_smpn+F21_smpn*F22_smpn	1	Right Cauchy-Green
c21_s...	F12_smpn*F11_smpn+F22_smpn*F21_smpn	1	Right Cauchy-Green

OK Cancel Apply Help

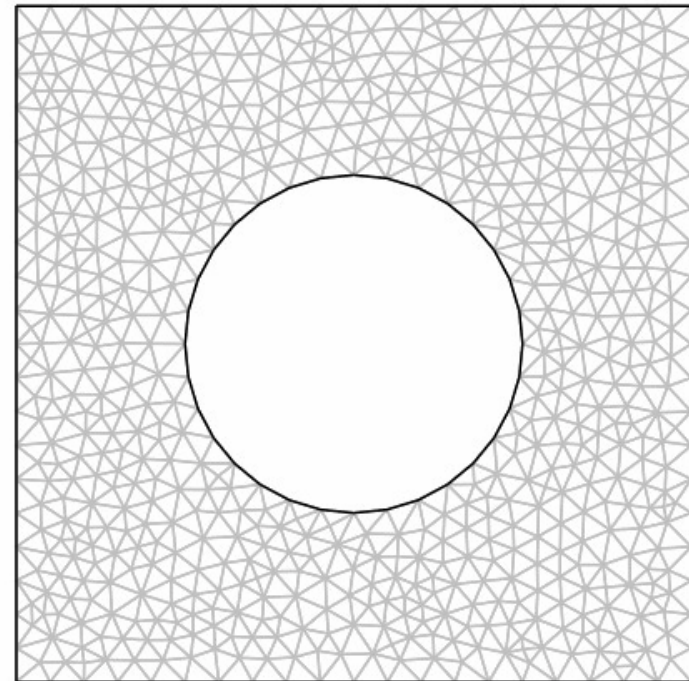
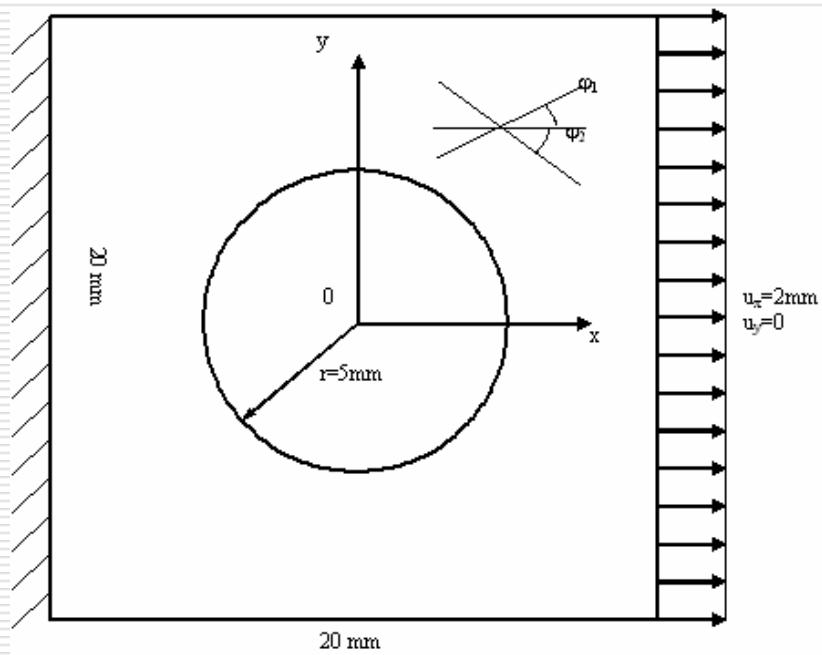
Material model implementation into Comsol Multiphysics

Global Expressions

Name	Expression	Unit	Description
I1	$C_{11} + C_{22} + C_{33}$	1	Invariants of...
I2	$0.5 * (I_1^2 - C_{11}^2 - 2 * C_{12}^2 - 2 * C_{13}^2 - C_{22}^2 - 2 * C_{23}^2 - C_{33}^2)$	1	
I3	J^2	1	
II1	$I_1 * I_3^{-1/3}$	1	Modified inva...
II2	$I_2 * I_3^{-2/3}$	1	
II4	$A_1 * (C_{11} * A_1 + C_{12} * A_2 + C_{13} * A_3) + A_2 * (C_{21} * A_1 + C_{22} * A_2 + C_{23} * A_3) + A_3 * (C_{31} * A_1 + C_{32} * A_2 + C_{33} * A_3)$	1	
II6	$B_1 * (C_{11} * B_1 + C_{12} * B_2 + C_{13} * B_3) + B_2 * (C_{21} * B_1 + C_{22} * B_2 + C_{23} * B_3) + B_3 * (C_{31} * B_1 + C_{32} * B_2 + C_{33} * B_3)$	1	
Lamda...	$1/2 * C_{11} + 1/2 * C_{22} + 1/2 * (C_{11}^2 - 2 * C_{11} * C_{22} + C_{22}^2 + 4 * C_{12}^2)$	1	
Lamda...	$1/2 * C_{11} + 1/2 * C_{22} - 1/2 * (C_{11}^2 - 2 * C_{11} * C_{22} + C_{22}^2 + 4 * C_{12}^2)$	1	
Lamda...	C_{33}	1	
lamda1	$Lamda C_1^{0.5}$	1	
lamda2	$Lamda C_2^{0.5}$	1	
lamda3	$Lamda C_3^{0.5}$	1	
lamda_1	$J^{-1/3} * lamda_1$	1	
lamda_2	$J^{-1/3} * lamda_2$	1	
lamda_3	$J^{-1/3} * lamda_3$	1	
W_vol	$K_b / 4 * (J^2 - 1 - 2 * \log(J))$	1	Volumetric st...
W_iso	$muy_1 / a_1 * (lamda_1^{a_1} + lamda_2^{a_1} + lamda_3^{a_1} - 3) + muy_2$	1	Isochoric str...
W_ani	$k_1 / (2 * k_2) * (\exp(k_2 * (II_4 - 1)^2) - 1) + k_1 / (2 * k_2) * (\exp(k_2 * (II_6 - 1)^2) - 1)$	1	Anisotropic s...

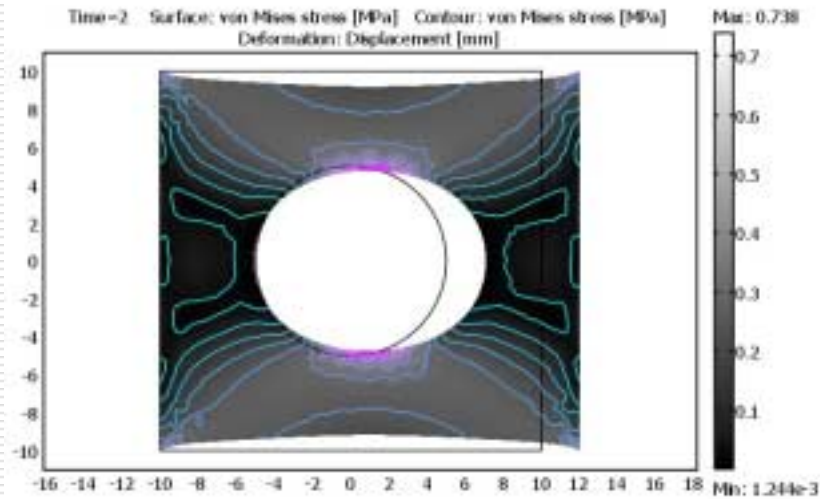
OK Cancel Apply Help

Applications and examples

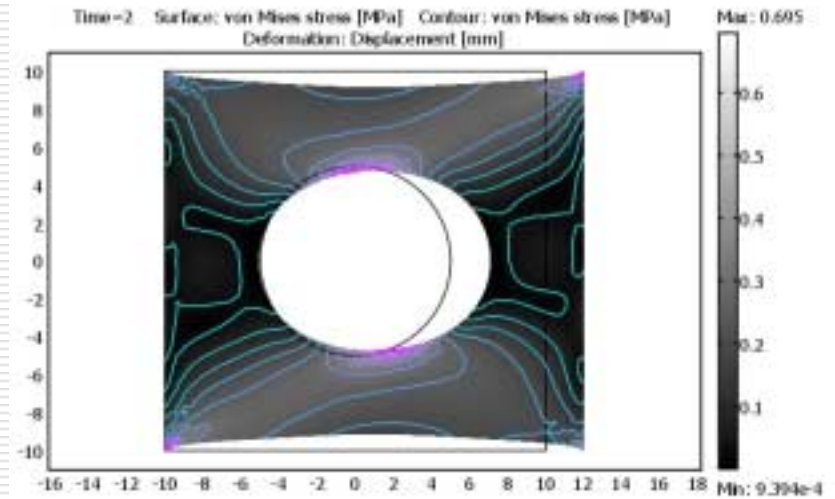


Applications and examples

One family of fiber



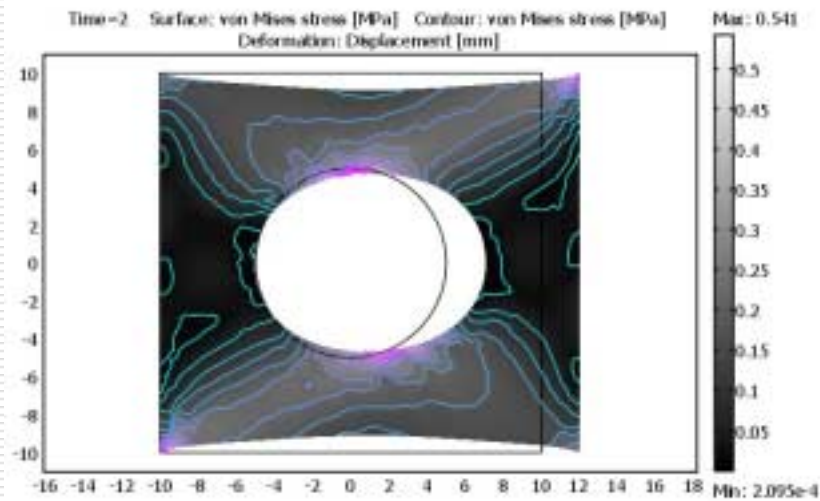
$$\varphi = 0^\circ$$



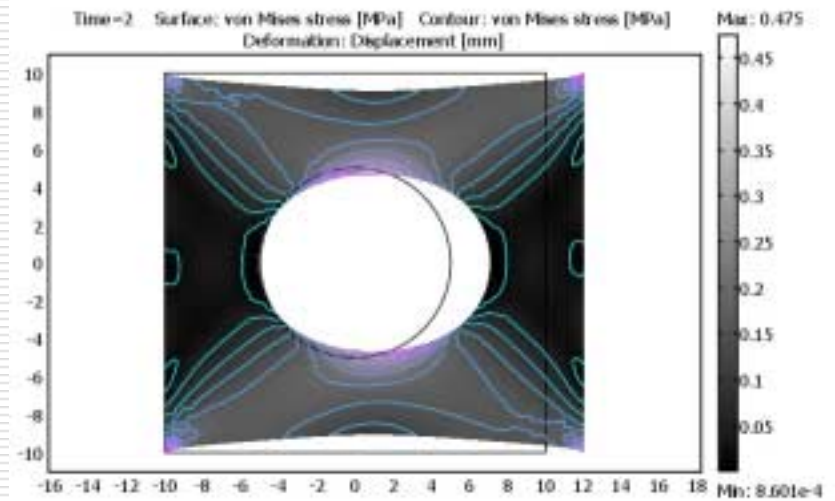
$$\varphi = 15^\circ$$

Applications and examples

One family of fiber



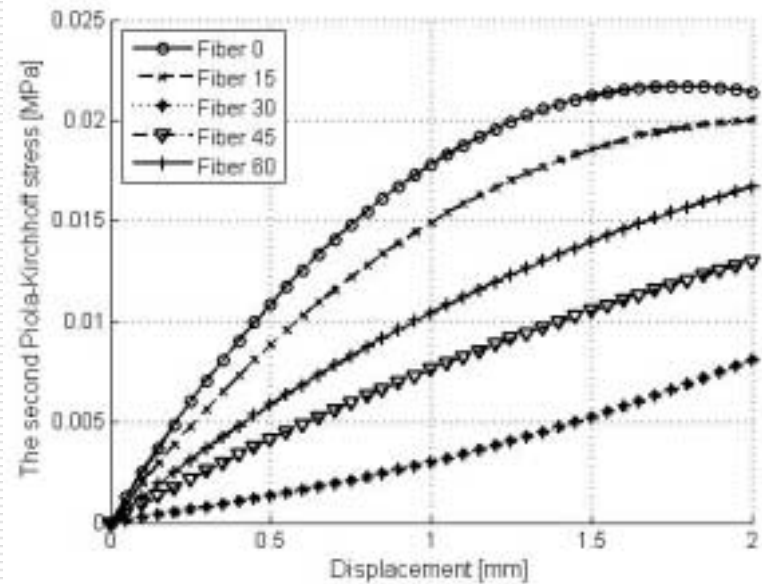
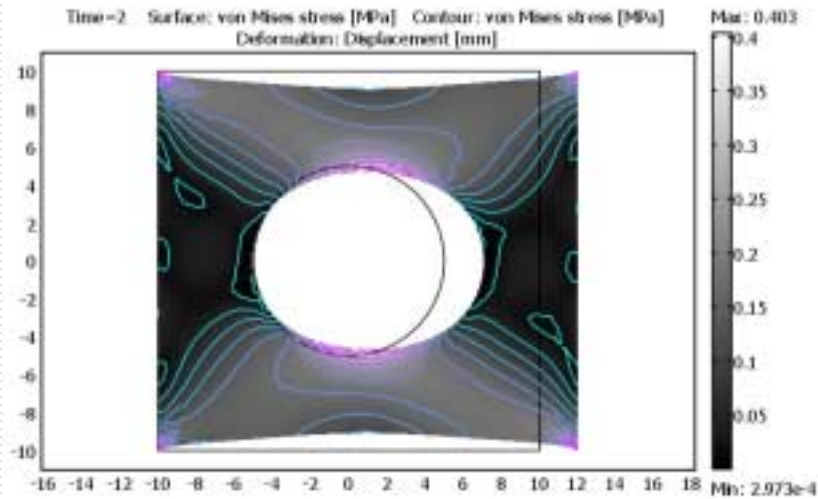
$$\varphi = 30^\circ$$



$$\varphi = 45^\circ$$

Applications and examples

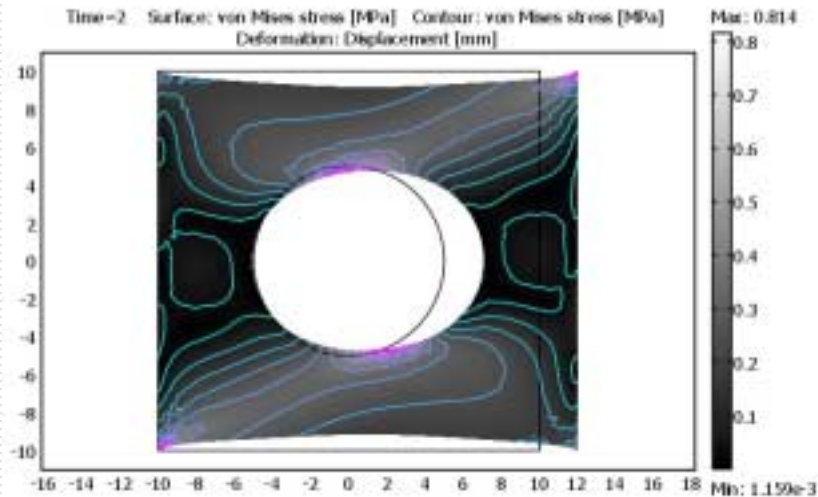
One family of fiber



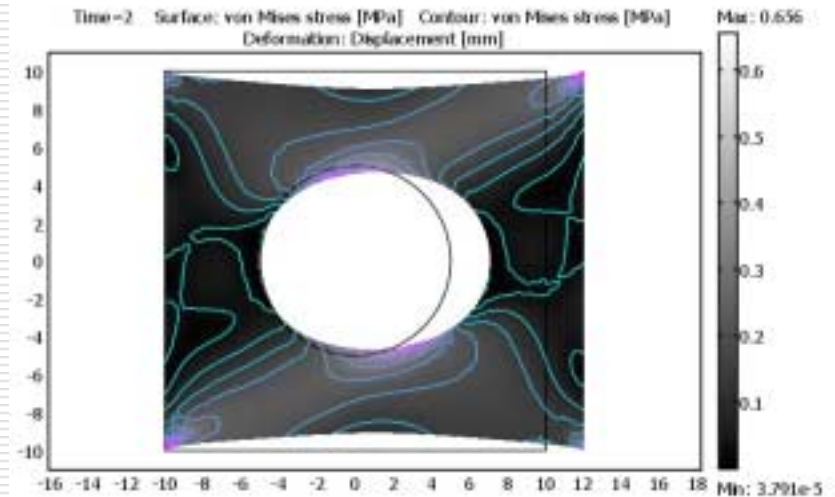
$$\varphi = 60^\circ$$

Applications and examples

Two perpendicular families of fibers



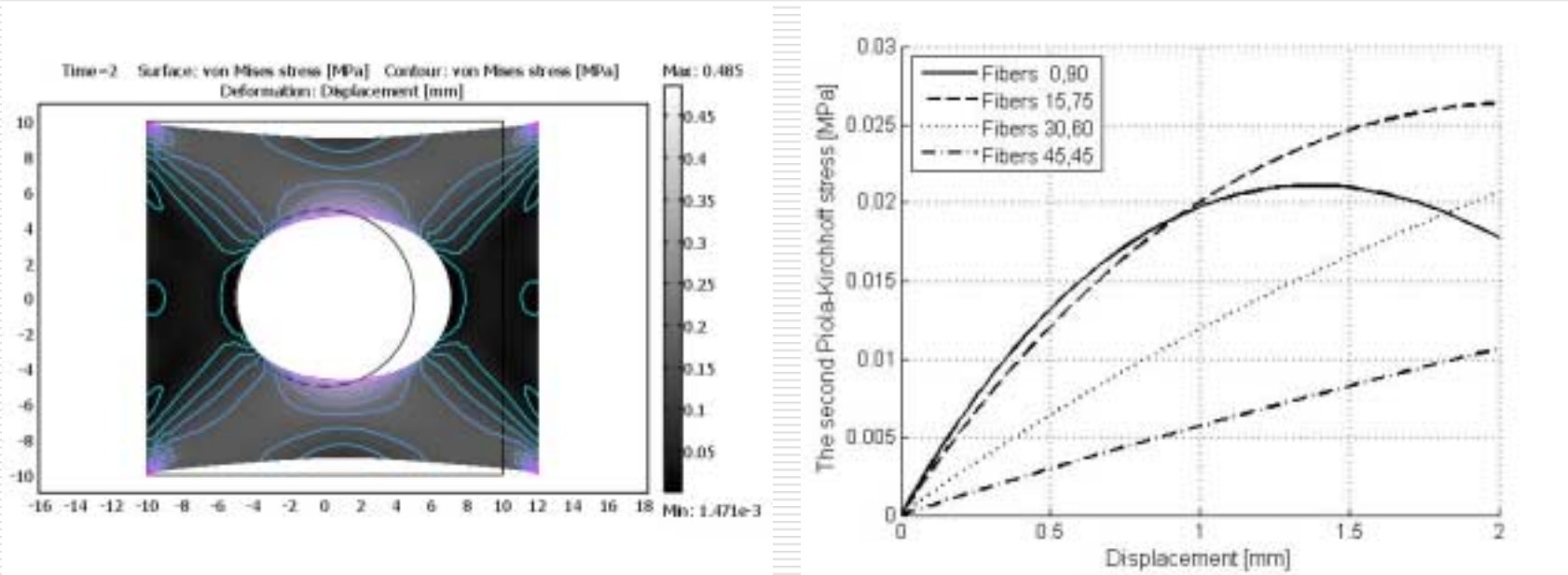
$$\varphi = 15^\circ$$



$$\varphi = 30^\circ$$

Applications and examples

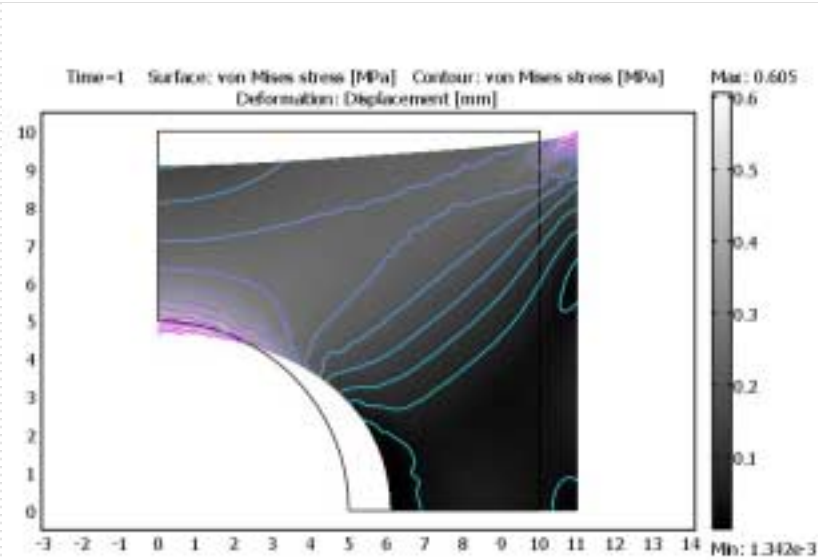
Two perpendicular families of fibers



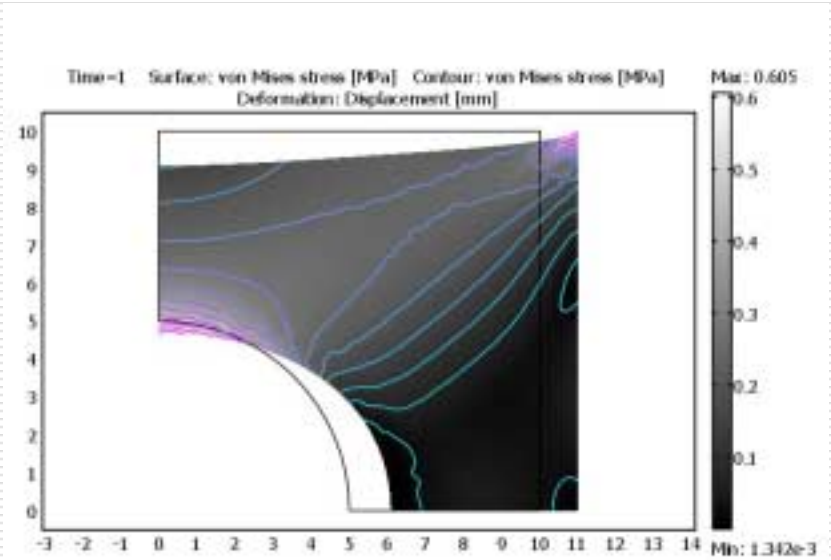
$$\varphi = 45^{\circ}$$

Applications and examples

Two families of fibers arranged symmetrically with respect to the axis of loading



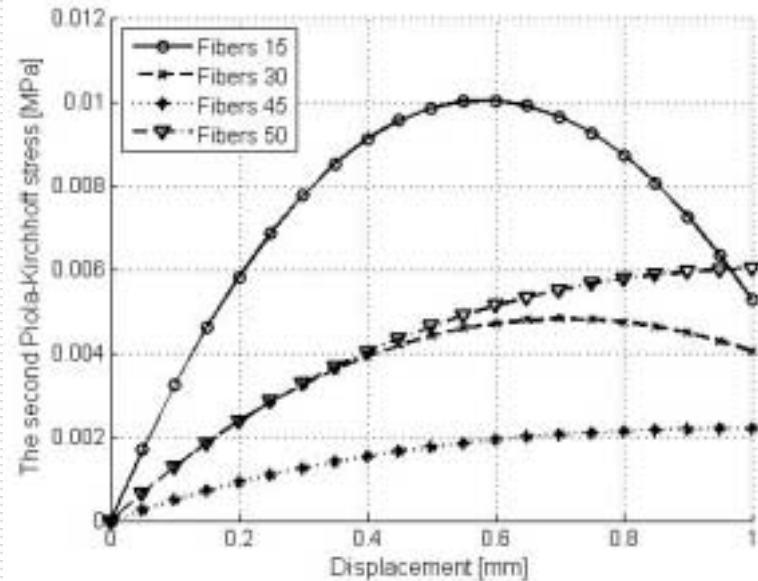
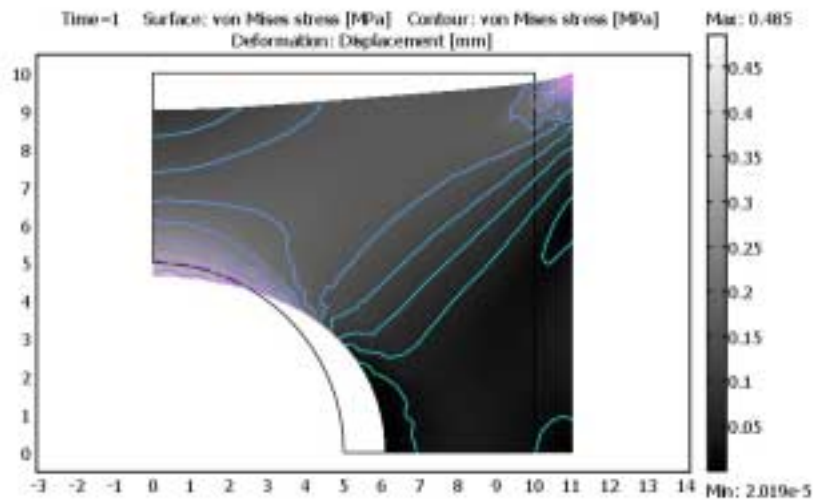
$$\varphi = 15^\circ$$



$$\varphi = 30^\circ$$

Applications and examples

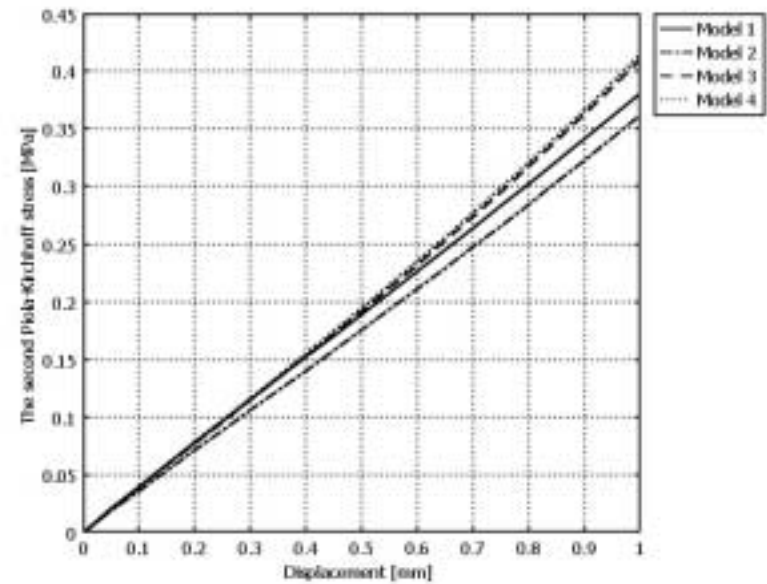
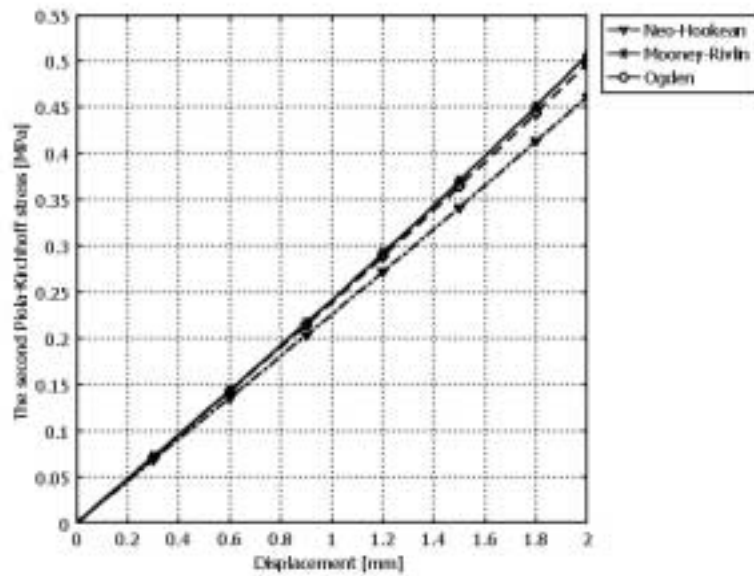
Two families of fibers arranged symmetrically with respect to the axis of loading



$$\varphi = 45^\circ$$

Applications and examples

Comparison of different material models



Conclusion

The new material model of the anisotropic composite material in finite strain and the Ogden model of hyperelastic material in 2-D were implemented into Comsol Multiphysics.

Some simple examples were subsequently presented, namely the extension of a composite block with a central hole. The results are in qualitative agreement with experimental observations.

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