POSSIBILITY OF NUMERICAL SOLUTION SUPPRESSION VIBRATION

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Abstract

Numerical solution use MATLAB and Simulink to create a comprehensive, dynamic model of the mechanical system with two degree of freedom to suppression of vibration. A concept for active vibration suppression with an absorber is presented. Use the model to produce optimized linear transfer functions based on the step response of the system. The major concepts of multiple degrees of can be understood by looking at just a 2 degree of freedom model as shown in the Fig. 1.

1 Introduction

One possibility of suppression of mechanical vibration different machine systems is to use a vibration absorber. The absorber consists of secondary mass-spring combination added to the primary device to protect it from vibrating. Essentially this converts one-degree of freedom system into two-degrees of freedom system. Consider a system consisting of a main mass m_1 suspended on a spring of stiffness k_1 and damping b_1 on which a force varying harmonically in time with frequency ω and maximum amplitude F is acting. The vibration absorber consists of a second mass m_2 a spring of stiffness k_2 and damping b_2 (Fig. 1).

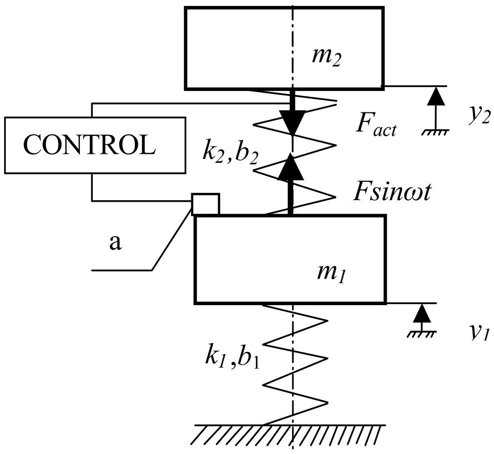

Figure 1: Model of the machine 1 (mass 1) with affiliate absorber 2 (mass 2)

The concept of the vibration absorber is that we want to reduce the motion of the main mass m_1 to zero. To do this, first let us modify the system with one-degree of freedom to make it a two-degree of freedom system, as shown (Fig. 1). The equations of motion for the masses m_1, m_2 are:

$$\begin{aligned} m_1\ddot{y}_1 &= -k_1 y_1 + k_2(y_2 - y_1) - b_1\dot{y}_1 + b_2(\dot{y}_2 - \dot{y}_1) + F\sin\omega t, \\ m_2\ddot{y}_2 &= -k_2(y_2 - y_1) - b_2(\dot{y}_2 - \dot{y}_1). \end{aligned} \tag{1}$$

Solution of equations (1) is (reason of the influence of the damping *b1, b2* of the springs is very small):

$$Y_1 = \frac{(k_1 - m_2\omega^2)}{a}F, \quad Y_2 = \frac{k_2}{a}F, \quad a = (k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2. \tag{2}$$

. Ideally, we completely want to stop the vibration of main mass m_1. We can do this by setting $Y_1=0$ in the first equation (2). These yields:

$$\omega = \sqrt{\frac{k_2}{m_2}}, \text{ or } \omega = \omega_2. \quad (3)$$

That is, if the natural frequency of the added mass-spring system itself is the same as the excitation frequency, the main mass will stop moving.

The Matlab solution of the equations (1) different mechanical system with design absorber will demonstrate in the paper. Detail algorithms in MATLAB are also presented as well.

2 Undamped system

For simplification we want to bring these into a dimensionless form and that purpose we Interduce the symbols:

$Y_{st} = \frac{F}{k_1}$ is static deflection of main mass m_1,

$\omega_1^2 = \frac{k_1}{m_1}$ is natural frequency of main mass m_1,

$\omega_2^2 = \frac{k_2}{m_2}$ is natural frequency of absorber (mass m_2),

$\mu = \frac{m_2}{m_1}$ is mass ratio.

The equations (1) becomes

$$Y_1(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}) - Y_2\frac{k_2}{k_1} = Y_{st}, \qquad Y_1 = Y_2(1 - \frac{\omega^2}{\omega_2^2}), \quad (4)$$

or, solving Y_1, Y_2 :

$$\frac{Y_1}{Y_{st}} = \frac{1 - \frac{\omega^2}{\omega_2}}{d}, \quad \frac{Y_2}{Y_{st}} = \frac{1}{d}, \qquad \text{where } d = (1 - \frac{\omega^2}{\omega_2})(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}) - \frac{k_2}{k_1}. \quad (5)$$

The amplitude Y_1 of the main mass m_1 is zero (3) when

$$1 - \frac{\omega^2}{\omega_2^2} = 0.$$

Let us now examine the second equation (5) for the case that $\omega = \omega_2$:

$$Y_2 = -\frac{k_1}{k_2}Y_{st} = -\frac{F}{k_2}. \quad (6)$$

With the main mass m_1 standing still and the absorber mass m_2 having a motion $-F/k_2 \sin \omega t$ the force in the absorber spring varies as $-F \sin \omega t$ which is actually equal and opposite of the external force.

The addition of an absorber has not much reason unless the original system is in resonance or at least near it.

For case $\omega_1 = \omega_2$, equations (5) becomes

$$\frac{Y_1}{Y_{st}} = \frac{1-\dfrac{\omega^2}{\omega_2^2}}{e}\sin\omega t, \qquad \frac{Y_2}{Y_{st}} = \frac{1}{e}\sin\omega t, \quad \text{where } e = (1-\frac{\omega^2}{\omega_2^2})(1+\mu-\frac{\omega^2}{\omega_2^2}). \qquad (7)$$

The result (7) is show in Fig. 2 and Fig. 3 for $\mu = 0.05$ and $\omega_1 = \omega_2$, i.e. $\omega/\omega_1 = \omega/\omega_2$.

3 Damped system

Solution of equations (1) of the model (Fig. 1) in case when the damping $b_1 = 0$ and $b_2 = b$ is

$$\begin{aligned} &-m_1\omega^2 Y_1 + k_1 Y_1 + k_2(Y_1 - Y_2) + i\omega b(Y_1 - Y_2) = F, \\ &-m_2\omega^2 Y_2 + k_2(Y_2 - Y_1) + i\omega b(Y_2 - Y_1) = 0. \end{aligned} \qquad (8)$$

We express Y_2 in terms of Y_1:

$$Y_1 = \frac{(k_2 - m_2\omega^2) + i\omega b}{[(-m_1\omega^2 + k_1)(-m_2\omega^2 + k_2) - m_2\omega^2 k_2] + i\omega b(-m_1\omega^2 + k_1 - m_2\omega^2)} F. \qquad (9)$$

The complex equation (9) can be reduced to the form

$$Y_1 = \frac{A + iB}{C + iD} F, \qquad (10)$$

this can be transformed

$$\frac{Y_1}{F} = \sqrt{\frac{A^2 + B^2}{C^2 + D^2}}. \qquad (11)$$

Applying now this to equation (9) we obtain the amplitude of the mass m_1:

$$\frac{Y_1^2}{F^2} = \frac{(k_2 - m_2\omega^2)_2 + \omega^2 b^2}{[(-m_1\omega^2 + k_1)(-m_2\omega^2 + k_2) - m_2\omega^2 k_2]^2 + \omega^2 b^2(-m_1\omega^2 + k_1 - m_2\omega^2)^2}. \qquad (12)$$

We Interduce the next symbols:

$f_\omega = \dfrac{\omega_2}{\omega_1}$ is frequency ratio,

$g = \dfrac{\omega}{\omega_1}$ is forced frequency ratio,

$b_b = 2m_2\omega_1$ is critical damping.

The equation (12) is transformed into

$$\frac{Y_1}{Y_{st}} = \sqrt{\frac{(2\dfrac{b}{b_b} g f_\omega)^2 + (g^2 - f_\omega^2)}{(2\dfrac{b}{b_b} g f_\omega)^2 (g^2 - 1 + \mu g^2)^2 + [\mu g^2 f_\omega^2 - (g^2-1)(g^2 - f_\omega^2)]^2}}. \qquad (13)$$

The amplitude ratio $\dfrac{Y_1}{Y_{st}}$ of the main mass m_1 is function of the four variables $\mu, \dfrac{b}{b_b}, f_\omega, g$.

4 Result

The amplitude ratio $\frac{Y_1}{Y_{st}}$ of the main mass m_1 is function of the four variables $\mu, \frac{b}{b_b}, f_\omega, g$.

Fig. 2 shows a plot of dimensionless $\frac{Y_1}{Y_{st}}$ main mass m_1 as a function of the forced frequency ratio $\frac{\omega}{\omega_2}$ for definite system $f_\omega = 1, \mu = 0{,}05$ [1] and various values of the damping $\frac{b}{b_b}$.

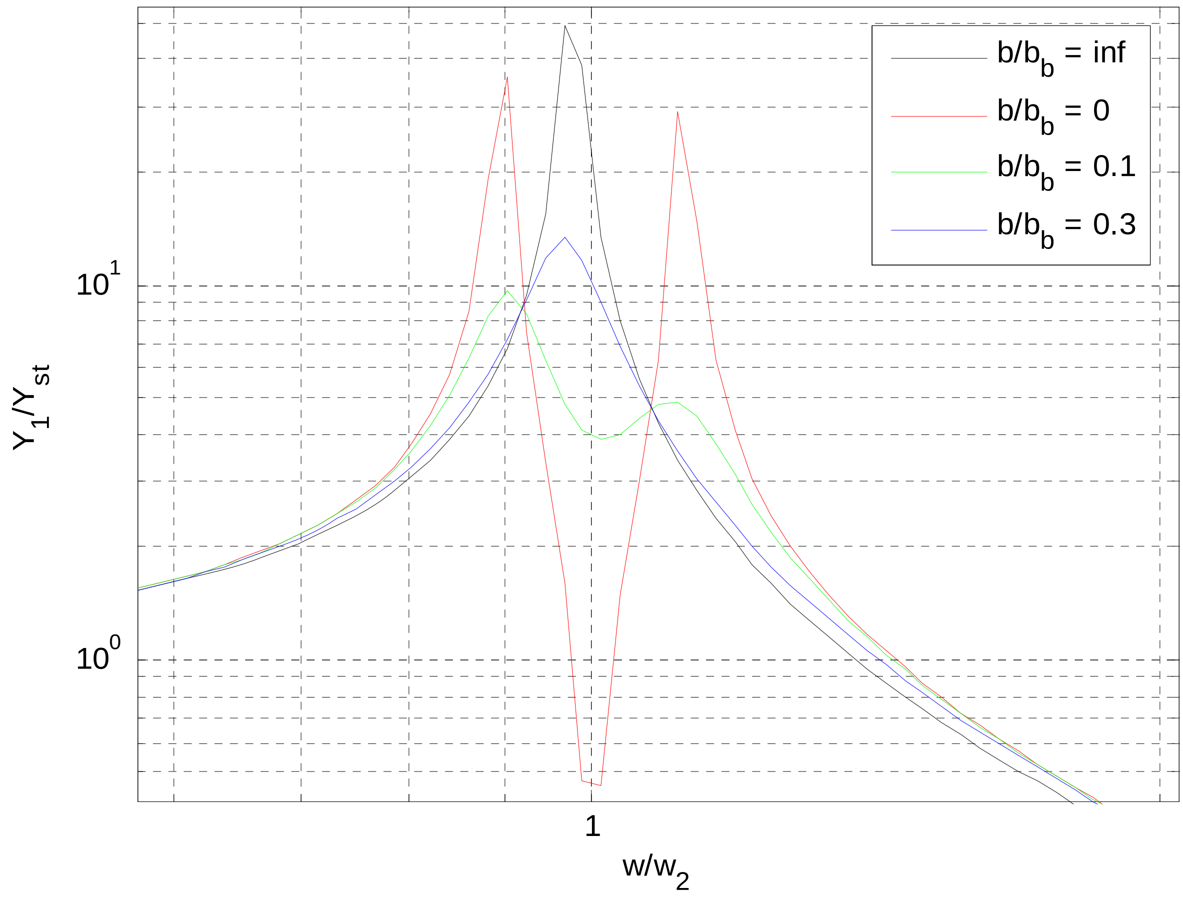

Figure 2: Dimensionless amplitudes Y_1/Y_{st} of the mass m_1 for $f_\omega = 1, \mu = 0{,}05$ and various values of the damping b/b_b (0 0,1 0,3 ∞) for various disturbing dimensionless frequencies ω/ω_2

Fig. 3 shows m-file the solution equations (1):

```
mi = 0.05;

fw = 1;

b_bb = 0.3;
m1 = 1e3;
w1 = 10;
w2 = fw*w1;
m2 = mi*m1;
k1 = w1^2 * m1;
k2 = w2^2 * m2;
bb = 2*m2*w1;
b2 = b_bb*bb;
b1 = 0;
m = [m1  0;  0  m2];
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b = [b1+b2  -b2;  -b2  b2];
k = [k1+k2  -k2;  -k2  k2];
m_1 = inv(m);
A = [ -m_1*b   -m_1*k; eye(2) zeros(2) ];
B = [ m_1; zeros(2) ];
C = [ zeros(2) eye(2) ];
D = zeros(2);

sys = ss(A,B,C,D);
w = logspace(0,2,200);
[mag, phase] = bode(sys, w);
figure(1);
amp = mag(1,1,:)*k1; amp = amp(:); loglog(w,amp,'b'); hold on;
figure(2);
amp = mag(2,1,:)*k1; amp = amp(:); loglog(w,amp,'b'); hold on;
```

Figure 3: m-file solution the equations (1)

Fig. 4 shows a plot of dimensionless amplitude $\frac{Y_2}{Y_{st}}$ of the absorber (mass m_2) as a function of the forced frequency ratio $\frac{\omega}{\omega_2}$ for definite system $f_\omega = 1, \mu = 0{,}05$ and various values of the damping $\frac{b}{b_b}$.

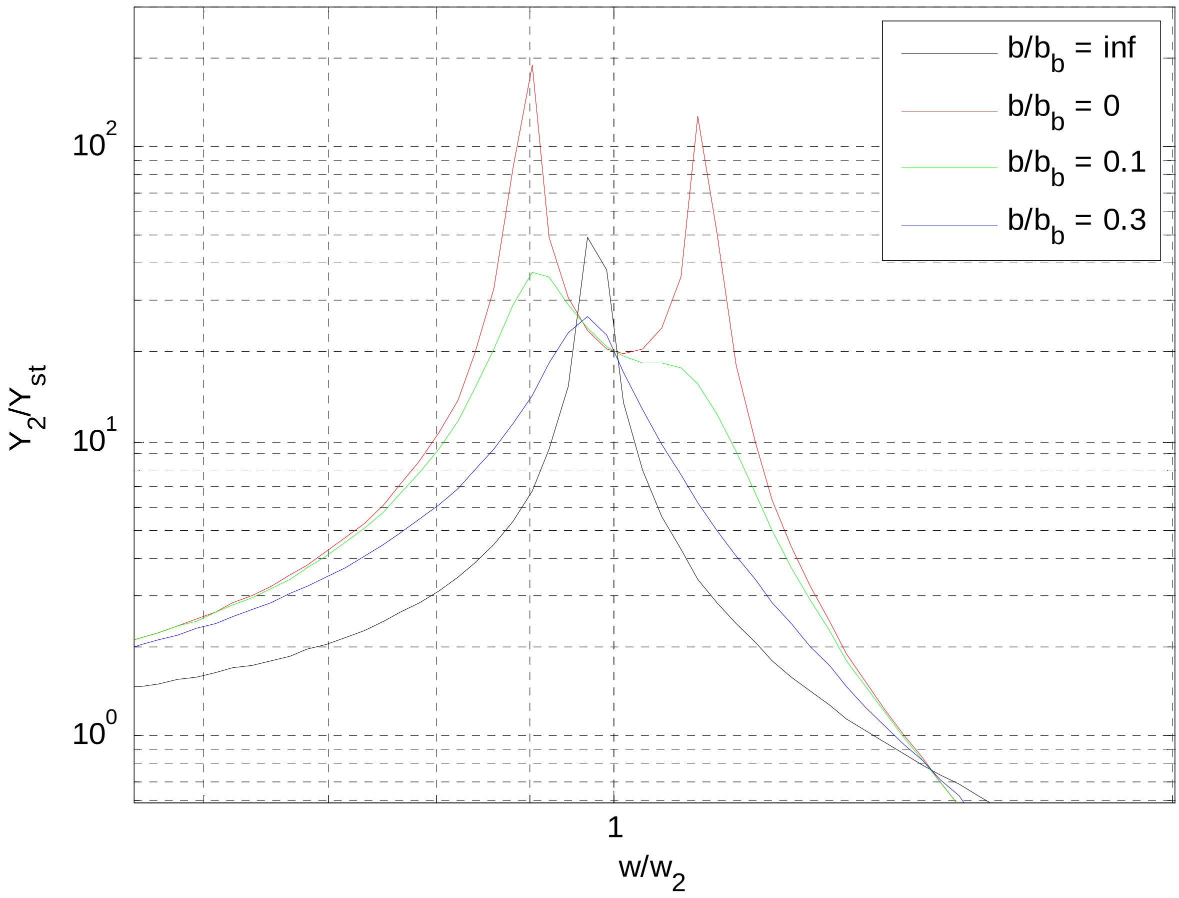

Figure 2: Dimensionless amplitudes Y_2/Y_{st} of the mass m_1 for $f_\omega = 1, \mu = 0{,}05$ and various values of the damping b/b_b (0 0,1 0,3 ∞) for various disturbing dimensionless frequencies ω/ω_2

5 Conclusion

From an inspection of Fig. 2, which represents frequency response the vibrations (course $b/b_b = 0$) of the main mass m_1, shown that the undamped absorber is useful only in cases where the frequency of the acting force is nearly constant. Then we can operate at $\omega/\omega_2 = 1$ with an amplitude $y_1 = 0$. This is the case with all machine coupled to synchronous motors or generator [2]. In variable speed machines, however, such as internal-combustion engines automotive or aeronautical applications, the device is entirely useless, since we merely replace the origin system of one resonant speed (at $\omega/\omega_2 = 1$) by another system with two resonant speeds.

By the damping system when damping is infinite, the two masses are virtually clamped together and we have a single degree of freedom system with a mass 21/20 m_1. Two other curves are drawn in Fig. 2 for b/b_b =0,1 and 0,3.

In adding the absorber to the system, the object is to bring the resonant peak of the amplitude down to its lowest possible value. With damping b=0 the peaks are very big. With b=inf. it is again very big. Somewhere in between there must be a value of b for which the peak becomes a minimum (Fig. 2).

Next examples numerical solution of suppression of dynamical vibration with absorber machines are in publications [3-21].

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