THE AIR CHANGE RATE CONTROL BY LOCAL MEAN AGE OF AIR IN VENTILATED SPACES

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Abstract

The indoor air quality parameters in naturally and/or mechanically ventilated buildings expressed in many standards on national and international level require appropriate control methods of air supply equipment adequate to specific requirements. On one of them focuses this work aiming on the local mean age of air (LMA) as an effective quantity on a micro-climate level able to manage the supply and exhaust conditions at the room level (the macro-climate) on the side of air supply equipment.

1 Carbon Dioxide as Microclimate Parameter

There are several definitions of thermal comfort, however in this work will be retained the broader standard definition in [1] (ISO 7730), which it describes as that condition of mind, which expresses satisfaction with the thermal environment. Similarly, with ASHRAE 55-1992 [2] could be defined as the absence of thermal discomfort and conditions in which 80 % of people don’t express dissatisfaction. It should be noted that these two normative documents are widely accepted and used in praxis when it comes to tackling thermal environment methods (of thermal comfort evaluation) and practices. The first one based on Fanger work [3], 1982, gave hints and tips for other procedures, some of them developed in connection with problems, including air contamination, occupants’ health related issues, etc. Basically, procedures that could decisively quantify thermal comfort felt by occupants with a different attitude, clothing and level of energy activity, are potential parts in advanced decision-making steps of controllers set for maintaining the thermal environment. Taking into consideration human sensuality, some of them with psychological background underpin the creation of a large database of thermal comfort field research observation [3].

The other possibility of microclimate evaluation proposed Boerstra et al (2003) in Holland. It is an alternative to the above mentioned Fanger’s PMV/PPD model uses the momentary comfort performance and the ‘over time’ comfort performance. In this point the possible odors content as natural product of some building materials, the room content, technology devices and/or processes, human clothing, man-made smell and smoking as well as indoor/outdoor-born other odors affect the human organism variously. The time-dependent effect through the man’s olfactory center connected to the part of brain observes Weber-Fechner principle which relates the degree of response or sensation of a sense organ and the intensity of the stimulus. On the same principle acts on human activity the contaminant produced by man breathing, i.e. carbon dioxide. Simultaneously, man’s body odor level depends on the level of his physical activity. There are several studies, investigating the health effects while elevated CO₂ level would be of concern (hospital bedrooms, schoolrooms, theatre halls etc.). From those studies were formed recommendations [4] of its acceptable level: the 9 000 mg.m⁻³ is a time-weighted average threshold limit value based on an 8 hours exposure a day within a 40 hour-long work week. The other, the short-term exposure limit, amounts to the 54 000 mg.m⁻³ in 15 minutes exposure [5].

2 Case Study

The reference space was the office room occupied during work hours by one adult person with sedentary activity equipped with air-conditioning device. The air change rate (ACH) could be set in a range from 0.5 to 10 and natural infiltration estimated using AHRAE Standard 136 (ASHRAE, 1993)
and by means of a mathematical model developed at the Lawrence Berkeley National Laboratory (LBNL) (Sherman and Modera, 1986). The space with the occupied zone \( OZ \) dimensions \( L_z \times B_z \times H_z \) shows Fig. 1 below:

1. **Reference room:**
   - Model scale: 1 : 6
   - Dimensions:
     - \( L \times B \times H \): 7250 x 4550 x 3000 (mm)
     - Model: 1 : 6
     - \( L \times B \times H \): 1200 x 760 x 500 (mm)

2. **Occupied zone:**
   - \( L_z \times B_z \times H_z \): 6050 x 3350 x 1900 (mm)
   - Model: 1 : 6
     - \( H \) - (reference room) height (2850 mm)
     - \( H_z \) - occupied zone height (checkpoint for air velocity), (1900 mm)
     - \( H_2 \) - mean radiant temperature checkpoint (ISO 7721)
     - \( H/L \) - aperture of 0.42.

The mean radiant temperature checkpoint at height: \( H_2 = 1 \ 100 \) mm.

Occupied zone length \( L_z = L - 2 \times 0.6 \) m  
Occupied zone width \( B_z = B - 2 \times 0.6 \) m

Figure 1: Occupied zone in tested room in side view. \( H = 3000 \) mm; \( H_2 = 1100 \) mm, distance from surrounding vertical walls 600 mm, \( H/L \) ratio: 0.42.

The boundary layers are adjacent to surfaces of inner walls, where were also placed thermometers; the globe thermometer was placed in the middle of the occupied zone at height 1,1 m above the floor. The thermal sensor locations were placed on the occupied zone boundary (dashed line on shadow area in Fig. 1).

### 3 Simulation Model

#### 3.1 CFD (Computer Airflow Dynamics) Airflow Model

The objective of the decreasing of contaminants (carbon dioxide) from an occupied zone of a room prescribes quantitatively standard [4] (or similar national recommendations). Technically, the non-relocatable HVAC system through its air inlets/outlets should ensure the acceptable level of carbon dioxide \((CO_2)\). The same HVAC equipment meets in order other indoor thermal conditions [2]:

- indoor air temperature,
- rel./abs. air humidity,
- basic air filtration.

To ensure in the enclosed space on-line control of all of air quality parameters the MIMO (multi-input/output)-controller is required. The direct monitoring of \( CO_2 \) level is often one of controller’s inputs for higher indoor thermal comfort requirements. Another way to achieve the same objective without need to install the \( CO_2 \) sensors in occupied space comes out with assumption of
known outdoor air CO$_2$ concentration value and the estimate of indoor airflow regime. This part was done by means of CFD-calculation with code Fortran on the room’s main meridian (in vertical direction), Fig. 2 below:

**Figure 2:** 2D-velocity field of airflow at the meridian plane of non-uniformly cooled space; ACH=0.5.

Central-fan-integrated supply ventilation (on the right side in figure) is implemented by an outside air intake into the room and simple return openings on opposite wall. The local velocity vectors in main, resp. secondary airflows (in the meridian), lead to the estimates of local mean age of air (LMA). The user-defined functions (UDF) in code Fortran calculated the distribution of horizontal velocities in the reference room’s meridian. They were plotted dimensionless in Fig. 3 at the height of occupied zone $H_z$ (checkpoint for air velocity, 1900 mm):

**Figure 3:** 2D-velocity profiles in vertical (series 1) and horizontal (series 2) velocities in the reference room’s cross-cut plane (meridian) at occupied zone (OZ) height $H_z = 1900$ mm.

From the plotted curve of distribution of vertical and horizontal velocities is further calculated the maximal velocity on the upper boundary of occupied zone OZ. This is one of quantities which should lay within the acceptable air velocity range. If it is higher, then the maximal velocity of the ventilation device has to decrease or the direction divert more from OZ boundaries. The CFD-calculations of meridian velocities $v_x$, resp. $v_y$ yields the velocity matrix, which is further effectively handled in MATLAB program. The velocity matrix holds no other information about the airflow condition within the broader area even if the tested space shows some kind of geometrical symmetry with the meridian. Actually, there exist secondary airflows and more stagnation areas. Beside the calculus of airflow velocity distribution the LMA calculation involves one more additional convection-diffusion differential equation. The equation (so called ‘passive scalar’ equation) is derived from the concentration equation with the assumption that the production of carbon dioxide (and other contaminants) throughout the enclosed space is uniform (Sandberg and Sjöberg 1983, Davidson and...
The LMA is generally defined as time needed for air to travel from a supply inlet area to the specific location within a ventilated room (Sandberg and Sjöberg, 1983):

\[
\tau_i = \int_0^\infty \left[ 1 - \frac{C_i(t)}{C_i(\infty)} \right] dt 
\]  

(1)

where:

- \( \tau_i \) - the local mean age of air (LMA) (s)
- \( C_i(t), C_i(\infty) \) - the contaminated concentration at sampling position \( i \) at time \( t \) and infinite time (kg contaminant/kg mixture)
- \( t \) - time (s).

There are two ways to determine LMA-value: 1. an experiment by tracer gas methods (pulse, step-up, and step-down (decay), or, 2. employment of CFD techniques. The later one involves the above mentioned one or more additional convection-diffusion differential equations. However, if focused on the areas of primary airflow areas close to the air supply inlet(s), i.e. areas where the inlet air contaminant concentration prevails, the rough assessment of LMA would be less affected by absence of diffusional therm in code Fortran calculus. Except the design of floor-perforated air supply, in many cases in occupied mechanically ventilated rooms, the primary airflows reach the OZ in their higher third, not far from range of checkpoints for sedentary activity (1.1 m) and its boundary (1.9 m). From that reason we attempted to estimate the ‘contaminant-free’ LMA as a pattern, which may control air rate, resp. ACH-value.

### 3.2 Heat Energy Transfer Modeling

Primary input quantity into the MIMO controller was indoor air temperature \( t_i \). This most used quantity summarizing thermal effect of several internal (external) heat sources influencing the overall indoor thermal comfort. Its components are defined as the radiant \( t_r \) and air \( t_a \) temperatures [3]. The (automated) measurement was carried out by ca 20 contact temperature sensors including Vernon thermometer, all properly positioned and guarded prior direct sunshine and any kind of air draft which may occur (near air inlets/outlets area etc.). Simultaneously was measured relative humidity in several checkpoints. The monitoring 24 hours round of temperature together with the data of relative humidity were collected with a datalogger and served for the validation of the reference space’s mathematical model.

For energy flows through the observed space \((L \times B \times H: 7250 \times 4550 \times 3000 \, \text{mm})\) and thus for model development, basic relations of physical phenomena that govern the thermal energy transfer process, were employed: the law of mass conservation and the first law of thermodynamics. The basic system of equations

\[
dq_h(\tau) = dq(\tau) + dq_i(\tau) + dq_v(\tau) - \sum_i dq_i(\tau) - dq(\tau) + \sum_j m_j c_j \frac{dT_r}{d\tau},
\]  

(2)

where:

- \( Q_h \) - heat rate covered by heater (W)
- \( Q_i \) - heat rate of internal heat sources (W)
- \( Q_h \) - heat amount excerpt of occupant (W)
- \( Q_{s,wf} \) - transmitted solar radiation heat rate on window (W)
- \( Q_{s,wo} \) - transmitted solar radiation heat rate on ext. wall (W)
- \( Q_t \) - heat rate through wall (heat conduction) (W)
- \( Q_v \) - heat rate by infiltration (W)
- \( Q_{vc} \) - heat rate by controlled ventilation (W)
- \( j \) - number of internal heat sources.
- \( i \) - number of internal heat sources,
- \( j \) - number of construction parts referred to accounted stored heat

was built into MATLAB program. The equation (2) contain terms \( dq_h(\tau) \), resp. \( dq_v(\tau) \), i.e. heat exchange between the room of a volume \( V \) and the outdoor environment by infiltration and controlled ventilation proportional to the ventilation rate \( \nu \) [5], [6]:

\[
Q_{v e}(\tau) = Q_{\nu e}(\tau) + Q_{\inf}(\tau) = \rho_a Vc_a \left[ T_{a,e}(\tau) - T_{a,i}(\tau) \right] \frac{\nu}{3600} (W)
\]  

(3)
The heat transfer process was termed as a *quasi-steady-state process* and with respect to spatial heat exchanges it would be seen as a well-stirred system, i.e. without appearance of significant heat rate and/or temperature gradients throughout the whole air space. Further, linearization or at least quasi-linearization of differential equations was reasonably applied, because of the known difficulties with solving even a simple nonlinear system. Quantities in their nature are subject of continua and grouping or gathering them into ‘lump’ places affects local solutions, but enables describing it in the whole as well as maintains entire – macroscopic – solution reliable. The next decision made focused on the order of the model output variables: the classification of the 3rd – 4th order would be maximal if an analytical solution attempted, in other cases numerical methods take place. The feature of model reduction ability corresponds mostly with this point and emerges as a matter of physical process itself.

The described mathematical model of the 1st-order differential equations (2) was set up with Simulink program in electro-thermal analog, Fig. 4:

![Electro-thermal analog of tested room](image)

Figure 4: Electro-thermal analog of tested room with convective heat transfer. Convection transfer elements in shadow filling.

The simulation model’s outputs are ambient and radiant temperatures with direct input into the PS-controller. Beside their values comes signal from LMA-parameter. The MIMO controller may override the maximal inlet supply velocity for limited time interval if necessary ensuring more fresh air supply in lesser time interval, thus increasing the actual air change rate.

### 4 PS-Control Scheme of Air Change Rate

The main output from reference room’s simulation model serves as input signal into the PI (PS)-controller. It maintains the selected thermal control parameters, the indoor air temperature and relative humidity RH in the recommended range [2]. The CO₂-level originated from occupants is bound with their water vapor production increasing the RH-value. The complementary calculation of state equation for air \( (p/\rho T = \text{const}) \) yielded assessment of density fluctuations and water content as well. Thus the acceptable level according to [8] shouldn’t reach workplace safety standards of 10 000 ppm for an 8-hour period and 30 000 ppm of CO₂ for a 15 minute period (parts per million). This means the average concentration over an 8-hour period should not exceed 10 000 ppm and similarly, the average concentration over a 15 minute period should not exceed 30 000 ppm of CO₂.

In Fortran code were calculated the LMA-profiles in three different locations in the tested room, the Fig. 5 shows the LMA* profile (i.e. LMA without the diffusion term) in distance of one third of the tested space length from air supply inlet (2.3 m) in the plane of main meridian of the room:
It is shown in Fig. 2 that inlet airstream for ACH<1 becomes less dominant on the space overall airflow regime: its core is positioned well above the OZ boundary at the height approximately 12.5 (2.5 m), Fig. 5. The airflow core itself corresponds in our case with higher LMA* value as there would switch the sign of vertical velocity component \( v_y \). The more distant location from air supply inlet axis means wider gap between LMA and LMA* (i.e. without diffusion term) values, more time-fluctuating and therefore less reliable. This is a good reason for monitoring the LMA* value there and set its maximal value into the relationship with air change quantity (ACH) of the forced ventilation (HVAC equipment):

\[
k_{ACH}^* (\tau) = ACH(\tau - \Delta \tau) \frac{LMA(\tau)}{LMA_{*\text{max}}} \tag{4a}
\]

or, with diffusion term included:

\[
k_{ACH} (\tau) = ACH(\tau - \Delta \tau) \frac{LMA(\tau)}{LMA_{\text{max}}} \tag{4b}
\]

The associated LMA* or ideally LMA parameter yields finally \( k_{ACH} \) parameter capable of air change (ACH) control. The ACH-value which was set in previous controller’s sampling time \( \Delta \tau \) would be kept unchanged, unless the \( k_{ACH}^* \) (\( k_{ACH} \)) parameter changes, i.e. differs from 1 according to eq. 4a, therefore controlling the actual ACH-value. The control action begins if the LMA* value would drop under the LMA*_{max} value as shown in Fig. 5. The estimate function of LMA* as well as the block of ACH-value is set with Simulink library items. The time-dependent ACH-vector enters via D/A interface the controller:

\[
ACH(\tau) = f(LMA*)
\]

Figure 5: LMA*-profile in distance of one third of the tested space length from air supply inlet in the tested room main meridian (2.3m). The height H range is from 1.1 m above floor to 3m (dimensionless (5-15)).

Figure 6: Block scheme of closed control loop with PS controller unit and the system (space) \( f \) with the standard measurement unit of \( t_i \), RH (MU A/D block) and CFD-block with function \( ACH = f(LMA*) \).

The closed loop was completed with time response constants from convective heat transfer block, Fig. 4, and with MATLAB modeled response of ACH value on LMA* increase which corresponds to increase of contaminant concentration (CO_2). Properly set proportional, resp. sum time constant of PS-controller were able to get back the LMA* value to the previous level. The simulation results are in Fig. 7 which shows cases of LMA* near air supply opening and that for average LMA**
value of the tested space. The red curve shows percentual change of initial value $ACH = 0.5$ for PS-controller settings upon the LMA* time in distance $2.3$ m from the air supply inlet. The blue curve shows drop in air change rate supply for the room averaged LMA* value of the tested room in less than $2$ minutes. These profiles indicate considerably fast responses than would be expected for LMA. The third curve records small increase in air density corresponding with lower temperature of supply air entering the room because of simultaneous detection of temperature change with $CO_2$ production from occupant.

![Figure 7](image-url) Figure 7: Air change rate response (dimensionless) to LMA* unit step change at distance of $1/3$ L $(2.3\text{ m})$ from the air supply inlet at height $12.5$ (2.5m)-red profile, and for average LMA* value - blue profile. The indoor air density profile in green.

5 Results

In mechanically ventilated space with internal heat sources were measured basic thermal quantities in order to assess its airflow regime. There were carried out a CFD-calculus with Fortran code and computed 2D-profiles of indoor air thermal state parameters including the air velocity distribution in the tested space’s main meridian. Modified value of LMA stripped of diffusion term was limited to the vicinity of the primary airflow range in the tested room meridian. The LMA* quantity was then associated with air change rate in order to set the $k_{\text{max}}^*$ control parameter guarding the actual $ACH$ volume. The adverse effect of air change increase on $CO_2$ concentration in the room was set with the $k_{\text{max}}^*$ parameter entering the PS-controller. In the closed loop with the controller were simulated 2 cases for the local one and averaged LMA* value for the tested room with Simulink toolbox. As is shown in Fig. 7, it could increase overall air supply rate for a limited time and decrease the $CO_2$ level in the ventilated space while the indoor air temperature and relative humidity would be kept unchanged.

References


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