# INTRODUCING THE DISCRETE ZOLOTAREV COSINE TRANSFORM

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**Abstract**

**The discrete Zolotarev cosine transform (DZCT) represents a new time-frequency transform related to the discrete cosine transform. It enhances a detection of a non-stationary signal. The heart of DZCT is selective cosine functions *zcos* that are able to adapt according to a non-stationarity in a signal. DZCT provides both frequency information and information about non-stationarity.**

## 1 Introduction

A great development has taken place in digital signal processing (DSP) in recent decades. Several time-frequency transforms have been introduced that claim better performance in the field of signal classification and compression. One of them is the discrete cosine transform (DCT), which has become a basic tool for speech and image signal processing. The abilities of DCT are limited by the cosine basis functions, which do not achieve adequate results in detecting a non-stationary signal. We have developed and introduced the discrete Zolotarev cosine transform (DZCT). It is intended to overcome this major drawback of DCT. DZCT extends the operation of DCT by substituting the cosine functions by selective cosine basis functions.

## 2 The Discrete Cosine Transform and its Basis Functions

The discrete cosine transform is an orthogonal transform related to the discrete Fourier transform (DFT). The purpose of DCT is to decorrelate a signal to the cosine series; it therefore gives an N-point real spectrum for an N-point real signal. This property has been employed in compression algorithms which prefer DCT to DFT. DCT has become a powerful tool for the JPEG image format.

The most common definition of the one-dimensional DCT transform is (1), which is derived from the DFT of a 2N-point even extension of series x(n).

$$C(k) = \varepsilon(k) \sum_{n=0}^{N-1} x(n) \cdot \cos\left(\frac{2n-1}{2N} k\pi\right) \qquad (1)$$

$$\varepsilon(k) = \begin{cases} \sqrt{1/N}; & k = 0 \\ \sqrt{2/N}; & else \end{cases}$$

On the other hand, the DCT definition can be understood as the mutual energy or correlation of a signal and the basis cosine function. For this reason, we define the even and odd cosine function satisfying particular symmetry relations. Both cosines are calculated from formula (2). The even cosine is composed of any even multiple of one half of the standard cosine period. Likewise, the odd cosine is composed of any odd multiple of one half of the standard cosine period. For the two cosine functions, see Figure 1.

$$\cos\left(\frac{2n-1}{2N} k\pi\right) \qquad (2)$$

We should note here that the basis cosine vectors are actually a solution of the Chebyshev approximation problem - an approximation of the zero by a polynomial on the interval $\langle -1, 1 \rangle$ represented by the Chebyshev polynomial of the first kind $T_n(x)$.

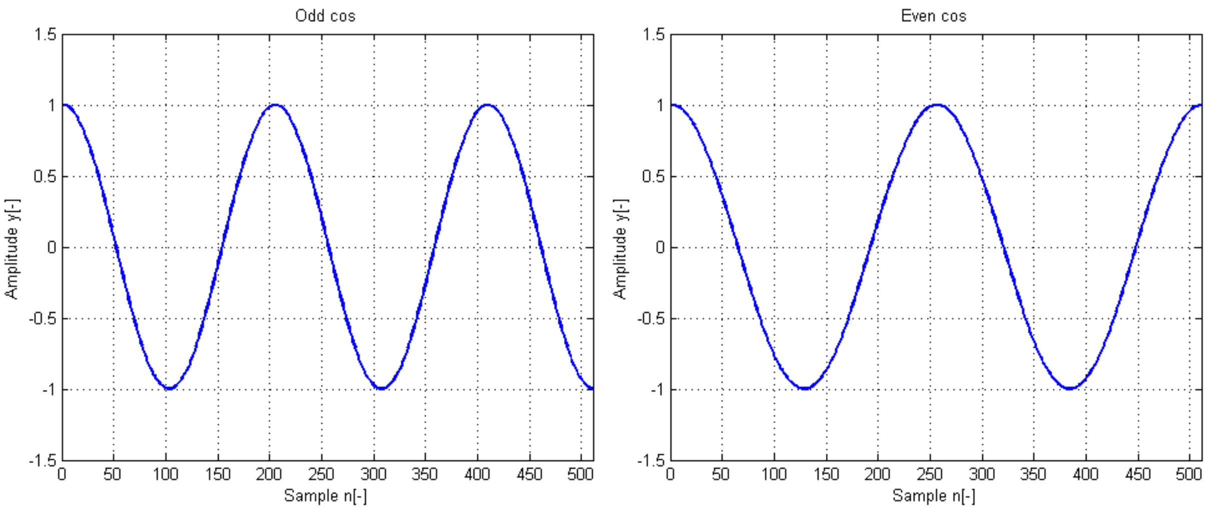


Figure 1: Odd and even cosine function $\cos\left(\frac{2n-1}{2N}k\pi\right)$ for $k_{odd} = 5$, $k_{even} = 4$.

## 3 The Zolotarev Cosine Functions

Egor Ivanovich Zolotarev stated and solved four problems in approximation theory, two concerned with polynomials and the other two with rational functions. The first and second problems concern polynomials which deviate least from zero in two disjoint intervals $\langle -1, -w_s \rangle$ and $\langle w_s, 1 \rangle$. Solutions of the first and second Zolotarev problem are selective cosine and sine functions. This paper introduces only two functions, the even and odd selective cosine *zcos*. The algebraic solutions which led to recursive algorithms for these two cosines were developed by Miroslav Vlček.

The selective cosines are determined by the degree and the lobe parameters. For even *zcos*, parameters $w_p$ and $-w_p$ fully determine the width and height of the central lobe, see Figure 2. In cases when the parameters are equal, the *zcos* function is equal to the *cos* function and the DZCT spectral bin is identical to the DCT spectral bin.

For odd *zcos*, parameters $w_p$, $w_s$, $w_m$ were introduced for the left lobe and the same parameters but with negative values for the right lobe. Parameter $w_m$ represents the position of the maximum of the lobe, and parameters $w_p$ and $w_s$ are related to the width and height of the lobe, again see Figure 2. In cases when the difference $w_s$ - $w_p$ tends to zero, the *zcos* function is equal to the *cos* function and, as in the case of even *zcos*, the DZCT spectral bin is identical to the DCT spectral bin.

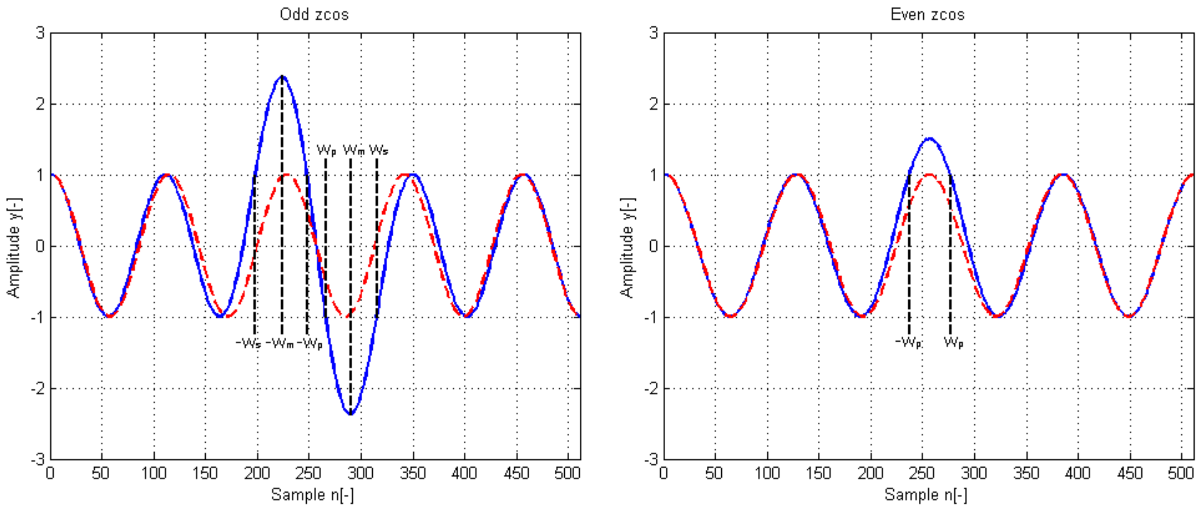


Figure 2: Odd and even selective cosine function *zcos*.

## 4 Results

Transforms utilizing harmonic basis functions, such as DCT or DFT, generally do not achieve adequate results in detecting a non-stationary signal. The discrete Zolotarev transform (DZT) extends the options of the DFT and improves the detection of non-stationary signals [1]. However many applications require a transform using only real numbers, such as DCT.

Our intention is to substitute the cosine *cos* function of the DCT for the selective cosine *zcos* function, which will lead to the new Discrete Zolotarev Cosine Transform. DZCT is based on two types of polynomial functions, the even and odd Zolotarev cosines, which provide both frequency information and information about non-stationarity.

## 5 Acknowledgements

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