Abstract

Scanning Probe Microscopy, a large family of microscopies, is a sensitive technique with unprecedented resolution. However, the accuracy of the measurement is strongly influenced by the setting and calibration of the apparatus. In this article, we present system analysis approach to the problem of the feedback adjustment and probe’s stiffness selection.

1 Scanning Probe Microscopy

The principle of Scanning Probe Microscopy [1, 3, 5–7] resides in the force/field interaction between very small probe (typically with the radius of curvature in the order of ones or tens of nanometers) and the sample. Types of interactions are namely:

- Atomic forces, such as van der Waals forces, Pauli repulsion, etc.
- Tunneling current
- Electrostatic force
- Optical force
- Electromagnetic reflection
- Chemical forces
- and many others

Since the Scanning Probe Microscopy is able to measure only a local interaction, the topography is reconstructed by scanning the surface of the sample. Scanning Probe Microscopy is also widely used as a non-imaging technique enabling measurement of the local properties of a sample, such as Young’s modulus, permittivity, charge density, adhesion, etc.

To maintain desired precision of measurement, feedback is used in various operation modes of the Scanning Probe Microscopy. In this article, we present system analysis approach to the problem of the feedback adjustment and probe’s stiffness selection, which is based on our previous work [4].

2 Linear Model

Considering linear model based on damped oscillator [2] (see Fig. 1), we may represent probe-sample system by following equation of the motion

\[ m \frac{d^2 z}{dt^2} + \beta \frac{dz}{dt} + (k + \kappa)z = \kappa z_0, \]  

(1)

where \( m \) is the effective mass, \( k \) is the spring constant, and \( \beta \) is the drag (damping) of the probe, \( z_0 \) is the topography of unloaded sample, \( \kappa \) is the surface stiffness of the sample, and \( z \) is the sample-probe distance. To be accurate, we must note that \( \kappa \) contains also the stiffness of the interaction; however, this not important in the linear regime.

Feedback loop connected to a certain type of actuator is used in order to keep the setpoint and enhance the sensitivity in case of high \( k/\kappa \) ratio. The proportional-integral regulation is
usually used to do so. We may introduce the feedback into equation 1 in following way

\[
m \frac{d^2z}{dt^2} + \beta \frac{dz}{dt} + (k + \kappa)z = \kappa \left( z_o - Pz - I \int_0^t z(\tau)d\tau \right),
\]

where \( P \) and \( I \) is proportional and integral gain of the feedback, respectively.

For transfer function of feedback controlled probe-sample system we may write as follows

\[
H(s) = \frac{s \kappa}{s^3m + s^2\beta + s(k + \kappa + P\kappa) + I\kappa},
\]

where \( s \) is the complex Laplace parameter. In ideal case \( L^{-1}H \to 0 \) and the information about topography is kept in the feedback loop with transfer function

\[
H_h(s) = \frac{\kappa(sP + I)}{s^3m + s^2\beta + s(k + \kappa + P\kappa) + I\kappa},
\]

and corresponding magnitude of the frequency response

\[
|H_h(j\omega)| = \sqrt{\frac{\kappa^2 (P^2\omega^2 + I^2)}{(I\kappa - \omega^2\beta)^2 + (\omega (k + \kappa + P\kappa) - \omega^3m)^2}},
\]

where \( \omega \) is the angular frequency.

### 3 Nonlinear Model

The force acting between the probe and the sample can not be linearized for some operation modes and branches of the Scanning Probe Microscopy. Usually, the term of driving harmonic force must be added and non-linear behavior of the spring constant \( \kappa = \kappa(z) \) must be considered. Since the interaction force follows typically the derivative of the Lennard-Jones potential (see Fig. 2), the stiffness of the interaction is then

\[
\kappa(z) = \kappa_o + \frac{4\varepsilon}{z} \nabla \left[ \left( \frac{\zeta}{z} \right)^{12} - \left( \frac{\zeta}{z} \right)^6 \right],
\]

where \( \varepsilon \) is a constant (depth of the potential well), \( \zeta \) represents finite (!) distance of the potential of zero, and \( \kappa_o \) is the stiffness of the sample. There are, indeed, other nonlinearities in the system. Resulting equation is to be solved numerically.

For the most precise analysis of the Atomic Force Microscopy, the state-space model proposed by Stark et al. [8] is to be employed. This model uses cantilever beam equation rather than damped oscillator approach.
4 Results & Conclusion

We employed MATLAB and Simulink in analysis of linear and simple non-linear model with attention to feedback adjustment (see Figs. 3 and 4).

Figure 2: The Lennard-Jones force versus sample-probe distance, $z$. Arbitrary units.

Figure 3: Effect of the feedback on the magnitude of the frequency response at given pass-band frequency versus $\kappa/k$ ratio is depicted below. Blue curve represents case with feedback $P = 6$, $I = 4$. Gray lines represent cases with feedback with parameters logarithmically distributed between 0.1 and 100. Red curve shows case without feedback.

Figure 4: Effect of the feedback on the frequency response of loaded probe (parameters: $m = 3.909 \times 10^{-12}$ kg, $k = 0.05$ N/m, and $\beta = 1.418 \times 10^{-8}$ Ns/m) is shown in following. Solid and dashed curves represent cases with and without feedback ($P = 6, I = 4$), respectively. Three different values of sample stiffness are presented: $k = \kappa = 0.05$ N/m green line, $k > \kappa = 0.005$ N/m blue line, $k < \kappa = 5000$ N/m red line. Note, that solid curves are based on actuator movement, not the probe displacement.
Properly adjusted feedback enhance the sensitivity of Scanning Probe Microscopy. However, sensitivity may be by contrast significantly worsen by wrong feedback setting, especially for high $k/k$ ratio. The value of $|H_h(j\omega)|$ (Eq. 5) expressed in percents represents the vertical accuracy of the measurement. The goal of setting of the feedback is to adjust $P$ and $I$ gains in a way to obtain the highest sensitivity and not to bring the system into oscillations caused by time delay of z-piezo actuator. Improper setting leads to suppressed vertical information in the image and eventually to loss of details. The probe will then more or less indent into the sample and may mechanically disrupt it, or exhaust the process under test. This is of the great significance primarily for biological experiments. We thus suggest to include information about parameters of the feedback in to standardly published experimental parameters of Scanning Probe Microscopy topographical and functional imaging.

5 Acknowledgments

OK was supported by the Czech Science Foundation, GA CR, grant no. 102/08/H008, and the Grant Agency of the Czech Technical University in Prague, grant no. SGS10/179/OHK3/2T/13.

References


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