

# PI-CONTROLLERS DETERMINATION FOR VECTOR CONTROL MOTION

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## Abstract

This paper presents determination of PI-controller parameters for vector control of the drive with permanent magnet synchronous motor. Vector control is widely used and popular control algorithm of the electric drives. Proper calculation of PI-controller parameters ensures correct operation of vector controller and high precision of the drive. Inputs to the control algorithms for calculation of proportional gain (P component) and integral time constant (I component) are motor parameters together with settling times of current, speed and position control loops, which can be chosen by the designer. Settling time of each control loop can be changed to achieve the best results of the motion control (speed or position control). Compensation of an external load torque acting on the shaft of the motor is included.

## 1 VECTOR CONTROL SYSTEM DEVELOPMENT

### A) Current PI-Controllers Design

For the design of PI-controllers it is necessary to know the closed loop transfer function. Block diagrams of permanent magnet synchronous motor (PMSM) in  $d$  and  $q$ -axis are shown in Fig.1.

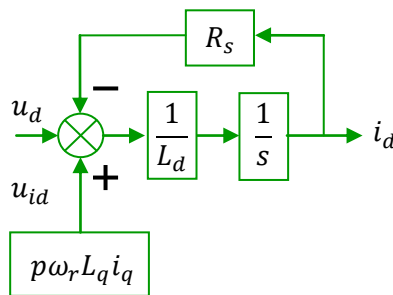


Fig.1 - a) Block diagram of PMSM in  $d$ -axis

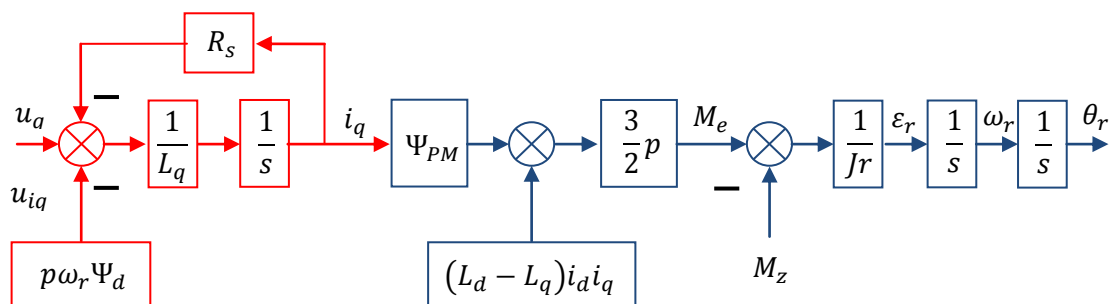


Fig.1 - b) Block diagram of PMSM in  $q$ -axis

To describe appropriate model of PMSM  $d$ - $q$  coordinate system fixed to machine rotor is used. Model is divided into two orthogonal axis enabling independent control of flux and torque component. If Mason's formula is used the transfer function between flux component,  $i_d$  and input voltage,  $u_d$  in  $d$ -axis is:

$$\frac{i_d(s)}{u_d(s)} = \frac{\frac{1}{L_d} \frac{1}{s}}{1 + \frac{R_s}{L_d} \frac{1}{s}} = \frac{\frac{1}{L_d s}}{\frac{L_d s + R_s}{L_d s}} = \frac{1}{L_d s + R_s} = \frac{1}{\frac{L_d}{R_s} s + 1} = \frac{K_a}{1 + s T_{a-d}} \quad (1)$$

Similar way the transfer function between torque component,  $i_q$  and input voltage,  $u_q$  in  $q$ -axis is:

$$\frac{i_q(s)}{u_q(s)} = \frac{\frac{1}{L_q} \frac{1}{s}}{1 + \frac{R_s}{L_q} \frac{1}{s}} = \frac{\frac{1}{L_q s}}{\frac{L_q s + R_s}{L_q s}} = \frac{1}{L_q s + R_s} = \frac{1}{\frac{L_q}{R_s} s + 1} = \frac{K_a}{1 + s T_{a-q}} \quad (2)$$

Both transfer functions in  $d$ - $q$  coordinate system can be replaced by the first order transfer functions with gain  $K_a$  and with electrical time constants  $T_{a-d}$  and  $T_{a-q}$ .

$$K_a = \frac{1}{R_s}, \quad T_{a-q} = \frac{L_q}{R_s}, \quad T_{a-d} = \frac{L_d}{R_s} \quad (3a, b, c)$$

For the design of PI-controller of magnetic flux control in  $d$ -axis following block diagram is used:

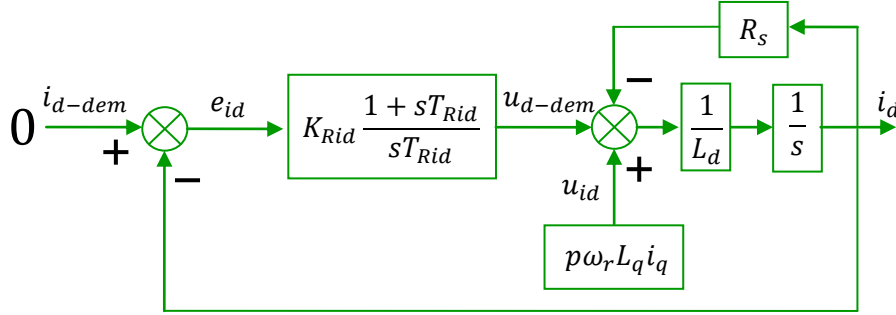


Fig.2 – Current loop of PMSM in  $d$ -axis

To achieve best results of the motion control and full flux of the machine condition  $i_{d-dem} = 0$  must be secured. If current PI-controller in  $d$ -axis secures such condition, then the only magnetic flux created will be flux of permanent magnets. Closed loop transfer function with PI-controller can be written in following way:

$$\frac{i_d(s)}{i_{d-dem}(s)} = \frac{K_{Rid} \frac{1 + sT_{Rid}}{sT_{Rid}} \frac{K_a}{1 + sT_{a-d}}}{1 + K_{Rid} \frac{1 + sT_{Rid}}{sT_{Rid}} \frac{K_a}{1 + sT_{a-d}}} = \frac{1}{\frac{T_{Rid}}{K_{Rid} K_a} s + 1} \quad (4)$$

For the design of PI-controller by "pole placement method", the demanded transfer function (5) of corresponding order is compared with denominator of closed loop transfer function:

$$(s + \omega_0)^n \quad (5)$$

Where  $n$  is order of the system and  $\omega_0$  is minimal natural frequency obtained from Dodd's formula (6), which respects prescribed settling time  $T_u$  of corresponding loop.

$$T_u = 1,5 \cdot (1 + n) \cdot \frac{1}{\omega_0} \quad (6)$$

The comparison of the ideal first order transfer function with denominator of the closed-loop transfer function of current control loop results in values of proportional gain  $K_{Rid}$  and time integral constant  $T_{Rid}$  of PI-controller in  $d$ -axis while respecting prescribed settling time:

$$\frac{T_{ud}}{3}s + 1 \leftrightarrow \frac{T_{Rid}}{K_{Rid}K_a}s + 1 \quad (7)$$

$$K_{Rid} = \frac{3L_d}{T_{ud}}, \quad T_{Rid} = \frac{L_d}{R_s} \quad (8 a, b)$$

Similar way, constants of current PI-controller in  $q$ -axis can be calculated. For the design of PI-controller of current loop in  $q$ -axis for control of electromagnetic PMSM torque following block diagram is used:

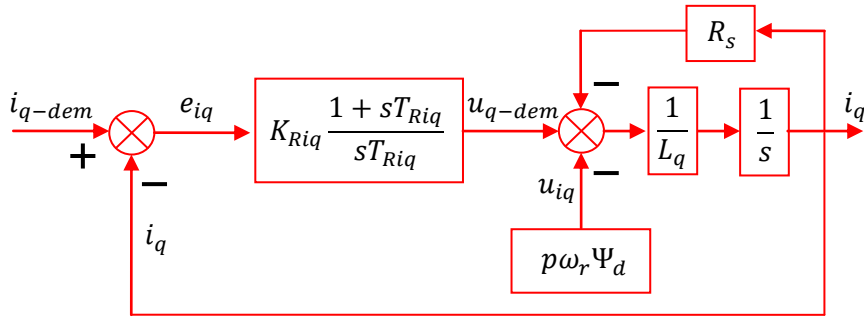


Fig.3 – Current loop of PMSM in  $q$ -axis

Values of current PI-controller in  $q$ -axis are calculated similar way as it was described for  $d$ -axis and the results are as follows:

$$K_{Riq} = \frac{3L_q}{T_{uq}}, \quad T_{Riq} = \frac{L_q}{R_s} \quad (9 a, b)$$

where  $T_{uq}$  is settling time of current loop in  $q$ -axis,  $K_{Riq}$  is proportional gain and  $T_{Riq}$  is time integral constant of PI-controller in  $q$ -axis.

## B) Speed PI-Controller Design

For the design of speed PI-controller block diagram shown as Fig.4 is used. Inner current control loop is replaced by its first order transfer function respecting settling time of current control loop  $T_{uq}$ , where  $T_p$  is time constant of current loop. Following relation between these two time constants is valid:

$$T_p = \frac{T_{uq}}{3} \quad (10)$$

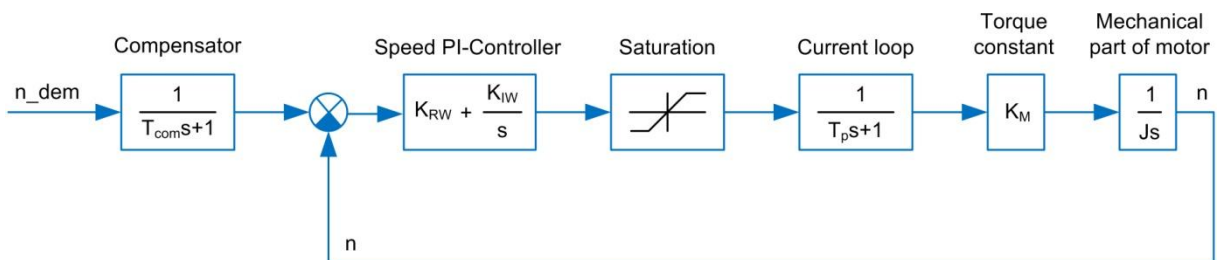


Fig.4 – Speed loop of PMSM in  $q$ -axis

The closed loop diagram for speed control of PMSM includes filtering block on the input of demanded speed (*the first order transfer function*), which compensates overshooting of speed. Time constant of compensator  $T_{com}$  is then equal:

$$T_{com} = \frac{K_{R\omega}}{K_{I\omega}} \quad (11)$$

Closed loop transfer function of speed control loop with PI-controller is as follows:

$$\begin{aligned} \frac{n(s)}{n_{-dem}(s)} &= \frac{\left(K_{R\omega} + \frac{K_{I\omega}}{s}\right) \left(\frac{1}{T_p s + 1}\right) \left(\frac{K_M}{J s}\right)}{1 + \left(K_{R\omega} + \frac{K_{I\omega}}{s}\right) \left(\frac{1}{T_p s + 1}\right) \left(\frac{K_M}{J s}\right) T_{com} s + 1} = \frac{1}{\frac{T_p J}{K_{I\omega} K_M} s^3 + \frac{J}{K_{I\omega} K_M} s^2 + \frac{K_{R\omega}}{K_{I\omega}} s + 1} \\ &= \frac{\frac{K_{I\omega} K_M}{J T_p}}{s^3 + \frac{1}{T_p} s^2 + \frac{K_{R\omega} K_M}{J T_p} s + \frac{K_{I\omega} K_M}{J T_p}} = \frac{\frac{K_M}{J T_p T_{I\omega}}}{s^3 + \frac{1}{T_p} s^2 + \frac{K_{R\omega} K_M}{J T_p} s + \frac{K_M}{J T_p T_{I\omega}}} \end{aligned} \quad (12)$$

For designing speed PI-controller by pole placement method, denominator of closed-loop speed transfer function is compared with ideal transfer function of the same order, which for order  $n = 3$  is as follows:

$$s^3 + 3\omega_0 s^2 + 3\omega_0^2 s + \omega_0^3 \quad (13)$$

For calculation of proportional gain  $K_{R\omega}$  and integral time constant  $T_{I\omega}$  of speed PI-controller, ideal transfer function (13) is designed exploiting Dodd's formula (6) which respects settling time of speed control loop. Then the ideal transfer function respecting settling time of speed control loop is as follows:

$$s^3 + \frac{18}{T_{u\omega}} s^2 + \frac{108}{T_{u\omega}^2} s + \frac{216}{T_{u\omega}^3} \quad (14)$$

Comparing (12) and (14) speed PI-controller parameters are obtained:

$$s^3 + \frac{1}{T_p} s^2 + \frac{K_{R\omega} K_M}{J T_p} s + \frac{K_M}{J T_p T_{I\omega}} \leftrightarrow s^3 + \frac{18}{T_{u\omega}} s^2 + \frac{108}{T_{u\omega}^2} s + \frac{216}{T_{u\omega}^3} \quad (15)$$

$$K_{R\omega} = \frac{108 J T_p}{K_M T_{u\omega}^2} \quad T_{I\omega} = \frac{K_M T_{u\omega}^3}{216 J T_p} \quad (16 a, b)$$

For the proper function of speed control loop with chosen settling time  $T_{u\omega}$ , settling time of current loop in  $q$ -axis should be adjusted to satisfy following condition (*where equation (10) is used*):

$$\frac{1}{T_p} = \frac{18}{T_{u\omega}} \rightarrow T_{uq} = \frac{T_{u\omega}}{6} \quad (17)$$

If condition (17) is respected, speed loop for chosen settling time will work with high precision. On the other hand, if settling time of current control loop in  $q$ -axis will not fit condition (17) dynamics of speed control loop will differ from the demanded.

### C) Position P-Controller Design

P-controller of position control loop can be calculated similar way as it was for current or speed control loop. For the design of position P-controller block diagram shown in Fig.5 is used. For proper function of position controller the settling time of current control loop in  $q$ -axis is respected and parameters of PI speed controller and must be calculated as describe equations (22a and 22b).

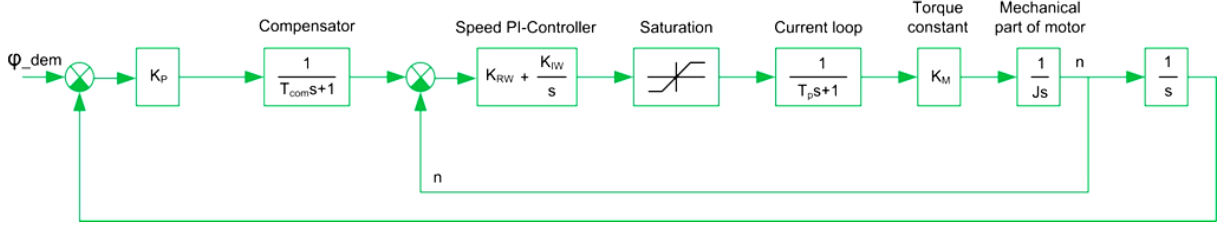


Fig.5 – Position loop of PMSM in  $q$ -axis

Closed loop transfer function of position control loop with P-controller is as follows:

$$\frac{\varphi(s)}{\varphi_{dem}(s)} = \frac{\frac{K_P}{s} \frac{1}{\frac{T_p J}{K_{I\omega} k_M} s^3 + \frac{J}{K_{I\omega} k_M} s^2 + \frac{K_{R\omega}}{K_{I\omega}} s + 1}}{1 + \frac{K_P}{s} \frac{1}{\frac{T_p J}{K_{I\omega} k_M} s^3 + \frac{J}{K_{I\omega} k_M} s^2 + \frac{K_{R\omega}}{K_{I\omega}} s + 1}} = \frac{\frac{K_P K_M}{J T_p T_{I\omega}}}{s^4 + \frac{1}{T_p} s^3 + \frac{K_{R\omega} K_M}{J T_p} s^2 + \frac{K_M}{J T_p T_{I\omega}} s + \frac{K_P K_M}{J T_p T_{I\omega}}} \quad (18)$$

To design P-controller for position control and speed PI-controller by pole placement method, denominator of closed speed loop transfer function is compared with ideal transfer function of the same order. Demanded transfer function now of order  $n = 4$  is as follows:

$$s^4 + 4\omega_0 s^3 + 6\omega_0^2 s^2 + 4\omega_0^3 s + \omega_0^4 \quad (19)$$

Using Dodd's formula (6) the ideal transfer function (19) is adjusted to fit settling time for position control loop as:

$$s^4 + \frac{30}{T_{up}} s^3 + \frac{675}{2T_{up}^2} s^2 + \frac{3375}{2T_{up}^3} s + \frac{50625}{16T_{up}^4} \quad (20)$$

Comparing (18) and (20) position P-controller and speed PI-controller parameters are obtained:

$$s^4 + \frac{30}{T_{up}} s^3 + \frac{675}{2T_{up}^2} s^2 + \frac{3375}{2T_{up}^3} s + \frac{50625}{16T_{up}^4} \leftrightarrow s^4 + \frac{1}{T_p} s^3 + \frac{K_{R\omega} K_M}{J T_p} s^2 + \frac{K_M}{J T_p T_{I\omega}} s + \frac{K_P K_M}{J T_p T_{I\omega}} \quad (21)$$

$$K_{R\omega} = \frac{675 J T_p}{2 K_M T_{up}^2}, \quad T_{I\omega} = \frac{2 K_M T_{up}^3}{3375 J T_p}, \quad K_P = \frac{50625 J T_p T_{I\omega}}{16 K_M T_{up}^4} \quad (22 a, b, c)$$

To secure proper function of position control loop with chosen settling time  $T_{up}$ , the settling time of current control loop in  $q$ -axis must be adjusted to satisfy following condition:

$$\frac{1}{T_p} = \frac{30}{T_{up}} \rightarrow T_{uq} = \frac{T_{up}}{10} \quad (23)$$

## D) GUI Application

The ‘*Graphical User Interface*’ application shown in Fig.6 was created using ‘*switched board programming*’ method. This application enables fast determination of PI-controllers for position and speed vector control of PMSM. At first user must define its own parameters of PMSM, due to fact that some parameters are redefined as initial values. Then settling times of current control loops in  $d$  and  $q$  axis, settling time of speed control loop and settling time of position control loop can be changed by typing new value to the corresponding edit box or slider cursor can be used for changing these settling times. If settling time of one of loop is set then PI-controller parameters of chosen loop are calculated.

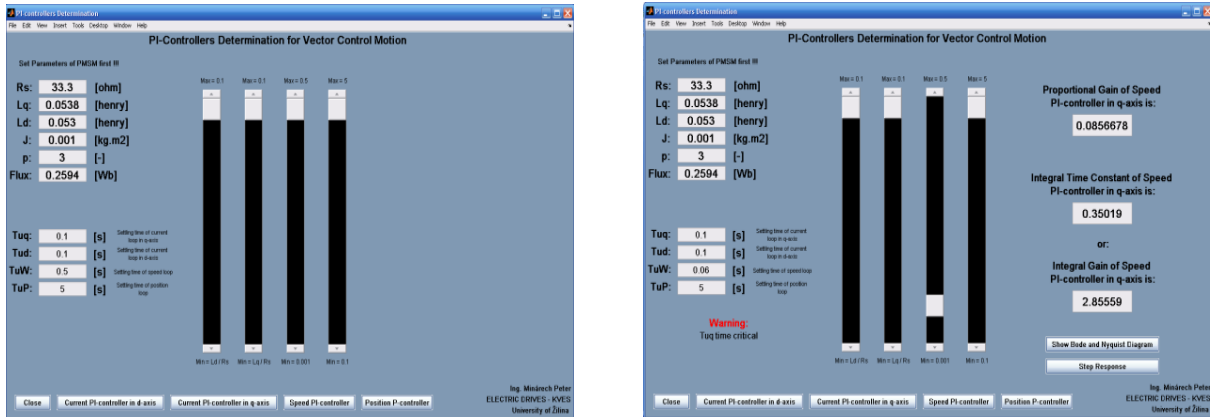


Fig.6 – Basic windows of GUI application

Bode and Nyquist diagrams can be also plotted to consider control quality. Application calculates transfer function of chosen closed-loop control system together with step response to verify correct controller design. User can convince himself that the designed controller fits chosen settling time by displaying step response function. If settling times of control loops are changed independently a new calculation of PI-controller parameters follows and a new motion behavior can be displayed. But for proper and precise function of motion vector controller (*speed or position*) conditions for settling times given by equations (17) and (23) must be satisfied.

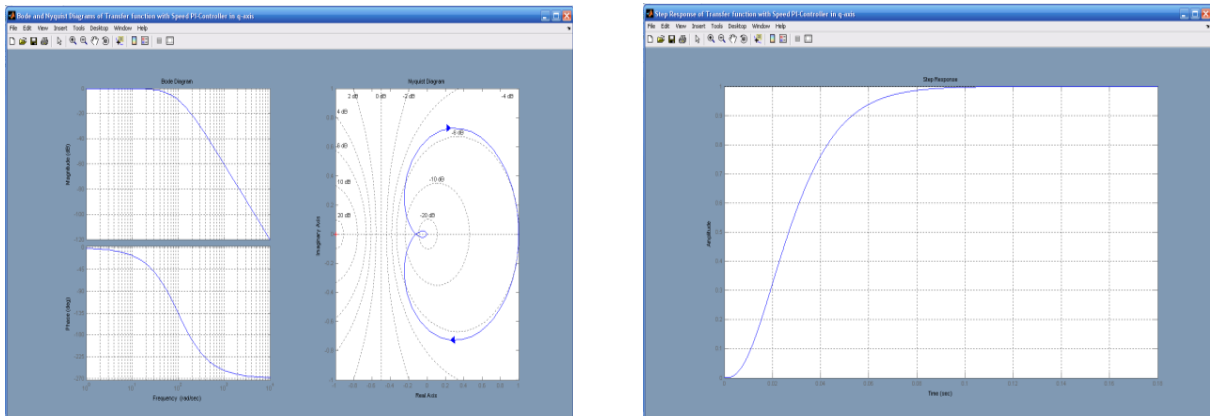


Fig.7 – Bode, Nyquist and step response diagram of GUI application

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