THE ZOLOTAREV TRANSFORM AND SELECTED APPLICATION

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Abstract

We present a new time-frequency Zolotarev transform (ZT) which uses a new data adaptive basis. Several applications of ZT in analysing of non-stationary signals and signal compression are discussed.

1 Introduction

Signal processing is often applied for analysing non-stationary signals, e.g. for analysing speech, biological signals, or for diagnosing machinery. Many types of transform, such as the Short Time Fourier Transform (STFT), the Wavelet Transform (WT), the Hilbert-Huang Transform (HHT), and Principal Component Analysis (PCA), are used for this purpose.

The paper introduces a new time-frequency transform (the Discrete Zolotarev Transform - DZT) together with its selected application in spectral analysis of non-stationary signals and reconstruction of signal. DZT may appear to be similar to the Discrete Fourier Transform (DFT), but there is one important difference between them. DZT uses a data-adaptive basis adjusting to the degree of non-stationarity of the signal under analysis. This basis consists of two selective Zolotarev polynomials of the first and second kind

$$Z_N^{\ell}(n) = \sum_{m=-\ell}^{\ell} \alpha_{2m} \cos\left(\frac{2\pi\ell n}{N}\right) + i \sum_{m=-\ell}^{\ell} \beta_{2m} \sin\left(\frac{2\pi\ell n}{N}\right) \ , \quad \ell \in Z, . \tag{1}$$

where α_{2m} and β_{2m} are coefficients of these polynomials [1], [2].

The basis of DFT can be express through the complex exponentials

$$W_N^l(n) = \frac{1}{\sqrt{N}} \exp\left(\frac{i2\pi \ln}{N}\right), \quad l, n = 0,1 \ldots N-1. \tag{2}$$

Using new basis (Eq. 1), DZT is able to improve some properties of DFT. For example, DZT very effectively suppresses spectral leakage, leading to better time-frequency resolution (see section 2.2. and 2.3). This property allows a better description of a non-stationary signal by the Short Time Discrete Zolotarev Transform (STDZT), which uses the new basis (Eq. 1), than by the widely-used Short Time Discrete Fourier Transform (STDFT) using DFT basis (Eq. 2). The equation of STFT is presented in [3] and STDZT is gained by replacing the DFT basis by the basis of DZT.

2 Selected application

2.1 Lossy signal compression

The figure (Fig. 1a) depicts the part of speech (Czech word “osum”) and his reconstruction from the one coefficient of Fourier and Zolotarev spectrum with the highest energy. The reconstructed signal from spectral coefficient of Fourier spectrum is the harmonic waves and the frequency is accordance to spectral coefficient (see Fig. 1b). The course of reconstructed signal from spectral coefficient of Zolotarev spectrum is almost identical to the original signal (see Fig.1c).

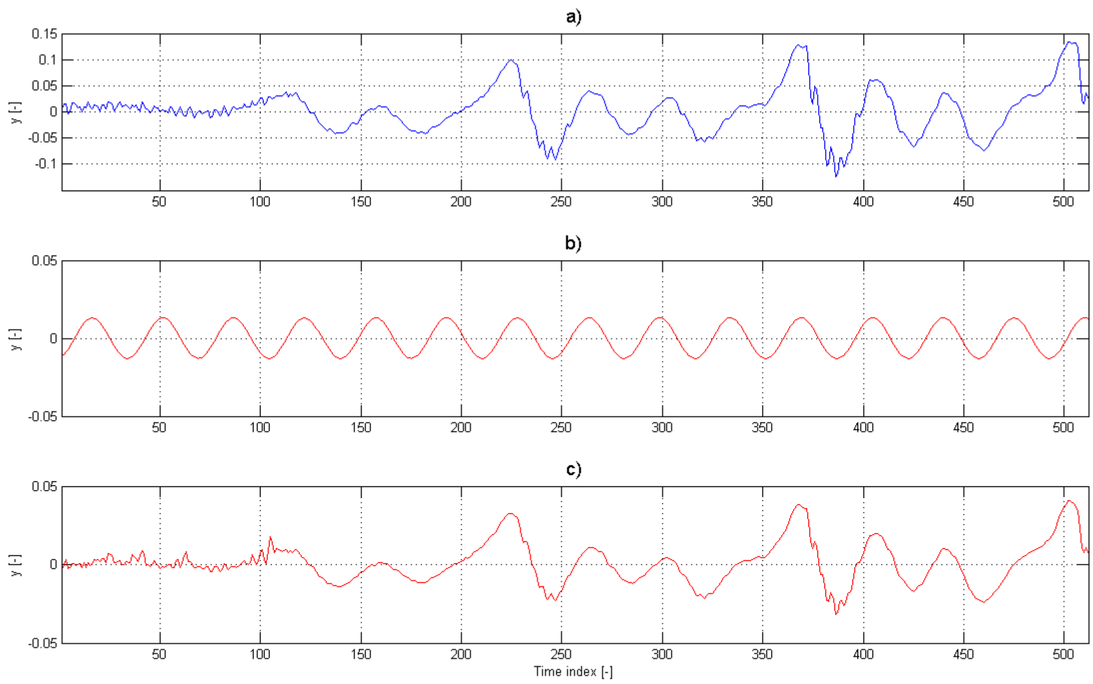

Figure 1: a) Real signal (part of Czech word "osum"), b) Reconstructed signal from one spectral coefficient of Fourier spectrum with the highest energy (Eq. 2), c) Reconstructed signal from one spectral coefficient of Zolotarev spectrum with the highest energy (Eq. 1)

2.2 Spectral analysis – sudden change in frequency

Harmonic signal with a sudden change of frequency is generated according to

$$s(n) = \cos\left(f \frac{2\pi n}{f_S} \right), \quad n = 0 \ldots N-1, \quad N = 2000 \tag{3}$$

and frequency f is changed from 500 Hz to 3500 Hz

$$f = \begin{cases} 600, & n \in \langle 0, N/2 \rangle \\ 3100, & n \in \langle N/2+1, N-1 \rangle \end{cases}. \tag{4}$$

The figure (Fig. 2) depicts an input signal and its spectrogram (STDFT) and zologram (STDZT), which are created by the Hamming window and rectangle window respectively. The step of segmentation is one sample for both. The non-stationarity of signal is created by a sudden change of frequency in the middle of signal length (see Fig. 2a). The spectrogram shows this non-stationarity but the time localization is not accuracy (see Fig. 2b), the length of used window is 512 samples. When we use the shorter window (length 128 samples) to segmentation, the time localization will be better, but the frequency resolution will be lost (see Fig. 2c). The time localization of sudden change of frequency is very accuracy in zologram (see Fig. 2d), which is created by the window with length 512 samples. This non-stationarity could be determined by 3 samples in zologram opposite to 128 samples in spectrogram (see Fig. 2c, 2d).

2.3 Spectral analysis – speech

The figure (Fig. 3) depicts a real signal (the Czech word "osum") and its spectrogram and zologram. The spectrogram is created by Hamming window. The length of window is chosen to 256 samples, because of compromise between time and frequency resolution (see Fig. 3b). The zologram is created by rectangle window with the length 512 samples (see Fig. 3c). The step of segmentation is one sample for both transform. We can see the pulsing power between each formant of speech signal in the zologram. This information is not evidenced in spectrogram.

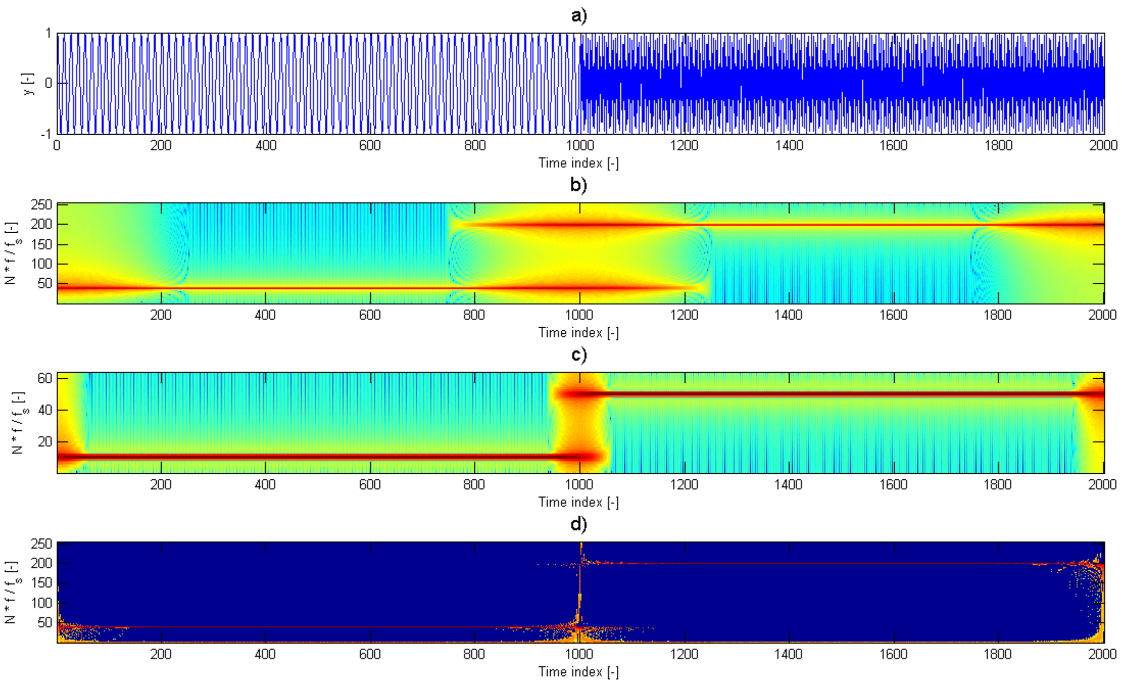

Figure 2: a) Harmonic signal with a sudden change of frequency generated according to Eq. 3, b) Spectrogram with window length 512 samples, c) Spectrogram with window length 128 samples, d) Zologram with window length 512 samples

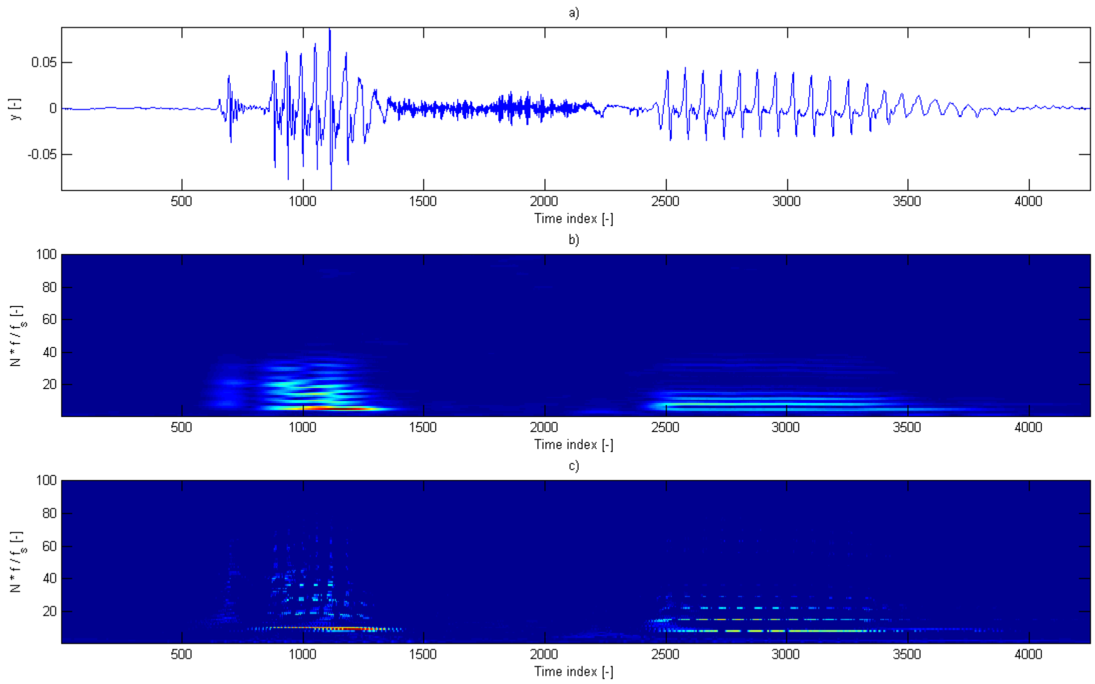

Figure 3: a) Real signal – the Czech word "osum", b) Spectrogram with window length 512 samples, c) Zologram with window length 512 samples

3 Conclusion

This paper has introduced the new time-frequency Zolotarev transform and has shown its several applications in a reconstruction of signal and spectral analysing. The DZT improves the widely-used DFT and achieves very good results in short time analysing of non-stationary signals, e.g. speech signal.

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