This paper deals with two mathematical models to optimization of a continuous casting process control. The first model is our original numerical model of temperature field, while the second one is represent a black-box type of optimization algorithm. The aim of optimization and control of the steel slabs production is to achieve both the maximum possible savings and product quality. The main focus is on water spray control in a secondary cooling zone. The continuous casting process is described by a three-dimensional mathematical model, containing a Fourier-Kirchhoff equation together with boundary conditions. From a material perspective, presence of phase and structural changes is modeled by an enthalpy method, where the enthalpy is computed from the chemical composition of the steel by using solidification analysis package IDS. The optimization part is performed by our heuristic self-regulating algorithm based on the idea of simulated annealing. Software implementation for the mathematical model of the temperature field was executed in MATLAB and regulating model in Python. Final results from both models are discussed. Future development of this research, which considers more factors, and which aims to come very close to the underlying real process, is presented at the end of this paper.

1 Introduction

Nowadays, continuous casting is the predominant way of producing steel in the world. Every year, steel industry processes millions of tons of liquid steel into semi-finished products such as slabs, blooms, and billets. Steel production ratio via continuous casting process in Czech Republic is comparable with industrialized European countries. The natural effort to reach high quality products require production innovations and an application of new approaches supported by technological development [1]. Industrial trials are very expensive and time-consuming, thus the more economical way is to use numerical simulations of the casting process.

Schematic representation of the continuous slab casting installation is shown in Figure 1. Molten steel (roughly 1830 K) is poured down from a tundish into a water cooled mould (primary cooling zone), where the steel gains a solid shell. Afterwards, the steel is transported by rollers and cooled down by water sprays (secondary cooling zone). Groups of nozzle of sprays divide the secondary cooling zone into several coolant circuits. In the last zone, the steel surface is cooled down by free convection and radiation only (tertiary cooling zone).
Previous works were generally based on simplified 1D or 2D temperature field models and were optimized by mathematical programming [2] or heuristic methods as neural networks [3], genetic algorithm [4], and firefly algorithm [5]. Many of these models was based on simplify assumptions and describe the casting process very roughly. Therefore their usage in the real casters is not satisfactory. Our original numerical model of the temperature field is designed for the real caster geometry and its results are validated with real temperature measurements by pyrometers installed on real caster. A self-regulating method based on self-regulating algorithm algorithm is used to obtain a set of optimal control parameters. The goal of the optimization is to improve material properties of the final slab and increase its production. In order to achieve this goal we modify the casting process by controlling casting speed and cooling rates. The productivity is defined as the amount of cast material per unit time, thus we maximize casting speed under certain metallurgical criteria. Metallurgical criteria used in optimization are formulated as a series of constraints that represent quality of the slab products and process feasibility. The criteria which must be met are completeness of solidification before unbending point (metallurgical length) and reaching prescribed temperature in the exit area. Quality of the final material is influenced by the course of the surfaces and core temperatures. The courses have to decrease in the whole profile and the temperature in the straightening area must be in the given range. The values for these constraints depend on the grade of employed steel.

2 Mathematical model of temperature field

The temperature distribution through the casting process is described by the mathematical equation in differential form. There are three basic mechanisms of heat transfer, the conduction mechanism plays the dominant role inside the body of cast steel, whereas convection and radiation take place only in the secondary and tertiary cooling zone, where they form boundary conditions. The temperature field of the slab is described by Fourier-Kirchhoff equation [6], [7]

\[ \frac{\partial}{\partial \tau} \left( \rho(T) c(T) \, T \right) = \nabla \left( \lambda(T) \nabla T \right) + \frac{\partial}{\partial z} \left( v_z \, \rho(T) c(T) \, T \right) + Q, \]

where the velocity component \( v_z \) [m/s] is considered only in the direction of casting, \( \tau \) is time [s], \( T \) is the temperature [K], \( \rho \) is the density [kg/m\(^3\)], \( c \) is the specific heat capacity [J/kgK], \( \lambda \) is thermal conductivity [W/mK] and \( Q \) represents the term associated to internal heat generation due to the phase change [W/m\(^3\)]. Phase and structural changes are included in the model by the use of a thermo-dynamical function of volume enthalpy \( H \) [J/m\(^3\)]. The method is called the Latent heat accumulation [8] or sometimes the enthalpy method [7]. The enthalpy is used as the primary variable and the temperature is calculated from a defined enthalpy-temperature (E-T) relationship

\[ H(T) = \int_0^T \left( \rho(\xi) c(\xi) - \rho(\xi) \Delta H \frac{\partial f_s}{\partial T} \right) d\xi, \]

where \( \Delta H \) is the latent heat [J/kg], and \( f_s \) is the solid fraction [-]. Figure 2 shows the relation between temperature and enthalpy for three grades of steel. Substituting equation (2) to equation (1) we have

\[ \frac{\partial H}{\partial \tau} = \nabla \left( \lambda(T) \nabla T \right) + v_z \frac{\partial H}{\partial z}. \]

In order to have a well-posed problem, initial and boundary conditions must be provided. The boundary conditions include the heat flux in the mould and under the rollers, forced convection
under the nozzles, and free convection and radiation in tertiary cooling zone.

\[
T = T_{\text{casting}} \quad \text{the level of steel,} \tag{4}
\]

\[- \frac{\partial T}{\partial n} = 0 \quad \text{the plane of symmetry and exit area,} \tag{5}\]

\[- \frac{\partial T}{\partial n} = \dot{q} \quad \text{in the mould and beneath the rollers,} \tag{6}\]

\[- \frac{\partial T}{\partial n} = htc (T_\infty - T_{\text{surf}}) + \sigma \varepsilon \left( T_4^4 - T_{\text{surf}}^4 \right) \quad \text{within the secondary and tertiary zones,} \tag{7}\]

where \(T_\infty\) is water cooling temperature or ambient temperature [K], \(\varepsilon\) is average emissivity [-], \(\sigma\) is Stefan-Boltzmann constant \([\text{W/m}^2\text{K}^4]\) and \(htc\) is heat transfer coefficient \([\text{W/m}^2\text{K}]\) for certain cooling circuit. The primary cooling zone (mould) is considered only as heat flux depends on casting speed \(\dot{q} = K_m \tau_m^{-0.5} 10^6\, \text{m}\,\text{s}^{-1}\), where \(K_m\) is mould constant [-] between values 7-10 and \(\tau_m\) is time for slab remaining in the mould [s]. The model discussed in this paper obtains its heat-transfer coefficients from measurements of the spraying characteristics of all nozzles used by the caster on a so-called hot plate in an experimental laboratory, according to [9] and for a sufficient range of operational pressures of water and a sufficient range of casting speeds of the blank (i.e. casting speed).

The equation (3) is discretized by the finite difference method [10] using an explicit formula for the time derivative.

\[
H_{i,j,k}^{n+1} = H_{i,j,k}^n + \Delta \tau \lambda_i,j,k(T_{i,j,k}^n) [QX + QY + QZ] + v_z \Delta \tau \frac{H_{i,j,k}^n - H_{i,j,k}^{n-1}}{\Delta z_{k-1}}, \tag{8}\]

where

\[
QX = \frac{T_{i+1,j,k}^n - T_{i,j,k}^n - T_{i-1,j,k}^n}{\Delta x_i + \Delta x_{i-1}} \tag{9}
\]

\[
QY = \frac{T_{i,j+1,k}^n - T_{i,j,k}^n - T_{i,j-1,k}^n}{\Delta y_j + \Delta y_{j-1}} \tag{10}
\]

\[
QZ = \frac{T_{i,j,k+1}^n - T_{i,j,k}^n - T_{i,j,k-1}^n}{\Delta z_k + \Delta z_{k-1}} \tag{11}
\]

The mesh for the finite difference scheme is non-equidistant in all directions and its nodes are adapted to the real rollers and nozzles positions. Stability criteria for inside nodes and boundary nodes are follows [10]

\[
\Delta \tau_{\text{inside}} \leq \frac{1}{2\lambda \left( \frac{1}{c_p(\Delta x)^2} + \frac{1}{c_p(\Delta y)^2} + \frac{1}{c_p(\Delta z)^2} + \frac{c_p v_z}{\Delta z} \right)}, \tag{12}\]

\[
\Delta \tau_{\text{boundary}} \leq \frac{1}{2htc \left( \frac{1}{c_p(\Delta x)^2} + \frac{1}{c_p(\Delta y)^2} + \frac{1}{c_p(\Delta z)^2} + \frac{c_p v_z}{\Delta z} \right)} \tag{13}\]

Time step can be chosen as their minimum \(\Delta \tau = \min \{\Delta \tau_{\text{inside}}, \Delta \tau_{\text{boundary}}\}\).

The way how to calculate the temperature from the actual enthalpy is to use a search technique [11]. In our model the binary search algorithm is used. We have a table of temperatures \((T_1, \ldots, T_n)\) and corresponding enthalpies \((H_1, \ldots, H_n)\). Thus, we are looking for the position of known enthalpy which directly represents the corresponding temperature. The algorithm is shown below. The symbol \(H\) represents the actual enthalpy corresponding to the temperature we looking for. When searching is finished the desired temperature \(T = i\).
Algorithm 1

1. [Initialize.] Set $l \leftarrow 1, u \leftarrow n$.
2. [Get midpoint.] if $u < l$ algorithm terminates unsuccessfully.
   Otherwise, set $i \leftarrow [(l + u)/2]$.
3. [Compare.] if $H < H_i$, go to 4; if $H > H_i$ go to 5; and if $H = H_i$, the algorithm terminates successfully.
4. [Adjust $u$.] Set $u \leftarrow i - 1$ and return to 2.
5. [Adjust $l$.] Set $l \leftarrow i + 1$ and return to 2.

One can say that this algorithm can be replaced by some more intelligent searching method. But for example in the temperature interval 274-1873 K the temperature is found roughly in eight iterations. Moreover, in order to speed this algorithm, the user can use the knowledge from the previous time iteration and set the upper ($u$) and the lower ($l$) limits closer to the desired value. Although it can speed up the algorithm, the user has to check whether the desired value still in search range. Thus the total time savings are not so significant in comparison to the number of searching through the simulation. With increasing number of nodes and time steps, the numerical accuracy increases but the solution time grows dramatically. In one-dimensional heat transfer the execution time is not a problem, but in heat transfer in three-dimensional space the time required by the E-T calculation is significant comparison to the rest of calculation. For instance in a mesh with 20x40x1200 nodes, we have to search for almost one million values in each time step. One way is to use the parallel calculation. Our numerical model was created in mathematical environment MATLAB, which is the most widely used tool in scientific and technical computing. MATLAB Parallel Computing Toolbox 5 is potentially useful for parallel programming [12], [13]. In this case of E-T search when the loop goes over the nodes from 1 to $n$, where $n$ is equal to the number of nodes, the parfor function is used.

<table>
<thead>
<tr>
<th>Table 1: Technical parameters of the real caster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mould length</td>
</tr>
<tr>
<td>Straight part of mould</td>
</tr>
<tr>
<td>The level of steel in mould</td>
</tr>
<tr>
<td>Slab thickness</td>
</tr>
<tr>
<td>Slab width</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Straight part of cage</td>
</tr>
<tr>
<td>Distance between end of the cage to exit area</td>
</tr>
<tr>
<td>The number of coolant circuits</td>
</tr>
</tbody>
</table>

Accuracy of the numerical model has to be verify by experimental measurements on a real caster. Our team collaborate with EVRAZ VTKOVICE STEEL, a.s. This allowes compare the numerical model and historical company data. Technical parameters of the caster are shown in Table 1 and their are taken as a imput for numerical model.

Temperature field computed by numerical model for casting spees 0.8 m/min and water flows [139, 60, 171, 128, 148, 112, 112, 112, 112, 94, 94, 148, 148] l/min can be seen in Figure 3 and temperature profiles in Figure 4. There we can see temperature of liquidus (red straight line) and temperature of solidus (blue straight
line) for steel S355J0H. Temperature sinuosity on surface is caused by alternating nozzles with rollers. Each nozzle is next to a roller and therefore the cooling effect is much smaller under the roller.

Figure 4: Temperature profiles

3 Optimization algorithm

Many important physical features of the cast material are closely coupled with its chemical composition, but some other are strongly related on particular casting technique. Among others, the crucial influenced properties are surface roughness, material texture or stress distribution. Poor structure with many defects is not accepted by the final customers. During its passage through the caster, the solidifying slab is subjected to varying thermal conditions and mechanical loading, both of which contribute to the generation of stresses and strains, what are the main sources of the defects. Serious defects might be removed by the quality control simulation and optimal controlling of the casting process. The only controlled physical property of the casting process is temperature (controlled by casting speed and cooling intensity), therefore we need to express the final material quality in terms of the temperature course. Due to experts’ knowledges several conditions were found.

The aforementioned numerical model describes the temperature field in the entirely material body, but we focus only on the temperatures on the top side, bottom side and in the middle of the body. The course of the temperatures on the top and bottom sides in uences the roughness of the metal, whereas the temperature in the middle describes the highest temperature in the given cross section. The other temperatures run between them and therefore they do not give much of additional information.

Very important feature is decreasing trend of the all courses. Significant alternation in the trends (upwards and downwards) can cause non-uniform material texture and therefore inconvenient stress concentration. The longer and more gradual decreasing trends are, the better and more uniform the final material is. It is also very advantageous to keep the temperature in the ”curved part of the caster” above some lower limit. It helps to balance the material structure and avoids creating cracks and disorders. On the end of the caster there is performed a division of the material and therefore the surface temperatures have to be in the given range and the whole body has to finish its complete solidification (the temperature of the core has to be below the temperature of the phase transformation). Some of these expert advices can look variously between each other, but all of them can be reformulated as a special case of one general condition. The condition is only to keep the temperature in defined point of the defined ranges. Thus for each grade of steel there have to be defined a set of points and their associated
temperature ranges which have to be fulfilled.

The target we want to meet is to make the continuous casting as effective as possible. It means that we need to determine the highest feasible casting speed which can be used for producing steel with defined quality. If the casting speed is too high, the nozzles are not able to cool the material down, and if the speed is too low, the whole process is ineffective because of slow production. The crucial factor of the optimization is the preservation of defined material quality, therefore we split the algorithm into two separate parts. The first part is described as algorithm 2 and it determines whether the nozzles are able to handle cooling for fixed casting speed. The second part of the algorithm exploits the first part and finds the highest possible speed which fulfills prescribed criteria.

Algorithm 2

```python
intensity = 1000
while all is not satisfied do:
    for each controlled point do:
        if temperature(point)>boundary:
            request = intensity * weight * rand(0.95, 1.05)
            append request for increasing htc in previous nozzles
        if temperature(point)<boundary:
            request = intensity * weight * rand(0.95, 1.05)
            append request for decreasing htc in previous nozzles
    for each controlled pair do:
        if temperature(pair[0])<temperature(pair[1]):
            request = intensity * weight * rand(0.95, 1.05)
            append requests for increasing and decreasing nozzles
    for each nozzle do:
        modify htc by the highest absolute value of the all appended requests
    end for
    evaluate model with new htc setting
    intensity = 0.8*intensity
end while
```

We have implemented the described heuristic algorithm 2 in Python and the numerical model of the temperature field in MATLAB. The communication between them is provided through COM technology [14].

4 Results and discussion

The mathematical model is verify by measurements from two pyrometers fixed on caster. Due to the use of enthalpy approach the numerical model is very general and therefore we needed to choose particular grade of steel for our experiments. In this paper we chose steel with number S355J0H which is very common in most industries. The setting of the optimization algorithm was following. At the beginning, the energy (the variable iteratively decreasing like in simulated annealing) is set to the value 1000. After each iteration the value is multiplied by 0.8 and it makes it exponentially decreasing. It is very reasonable to restrict the decrease by some lower limit, otherwise the energy would become very small and the convergence would be very slow. We iteratively decrease the energy level down to the value 100 and do not use any smaller. The interval for searching the highest casting speed is bounded by 0.5 from the bottom and by 2.0 from the top. These values are given by physical construction of the caster.

The temperature ranges for checking the quality of the given material are expertly specified in six points distributed along top side and other six points along the bottom side of the final product. The ranges have (in average) width about 10 K. There is also check whether the temperature in the middle is less than 1730 K, otherwise it would not be solidified before cutting point. Temperatures in every of the six points have to be in decreasing order, except the first one and last one (because of the physical nature of the process). Important part is setting up
dependences between checking points and nozzles. We set that each point is influenced by two previous nozzles, the closer one has weight 0.8 and the more distant has 0.2. Finding out whether the actual casting speed is feasible for given coolant circuits is tested through 25 iterations. It means that if the Algorithm 2 does not find any feasible solution during 25 iterations it is consider to be infeasible and the upper boundary in Algorithm 2 is set lower.

One evaluation of the numerical model with arbitrary casting speed and cooling intensities takes about 10 second on our computer and the total number of such evaluations was about 150 (in total about 25 minutes). Figure 4 shows obtained result. The maximal founded casting speed is 1.02 m/min and the obtained values for each of twelve coolant circuits are written in the title of the chart. Three depicted curves represent temperatures in the watched surfaces. Their sinuosity is caused by alternating nozzles with rollers. Each nozzle is next to a roller and therefore the cooling effect is much smaller under the roller. Cooling effect of each nozzle is dramatically decreased in the close neighbourhood of a roller because the stream of liquid is not able to get to all the places.

![Graph showing casting speed and temperature](image_url)

All the prescribed conditions of the material quality were successfully met and therefore the algorithm showed its ability to find the desired optimum.

5 Conclusion

This paper was focused on optimalization-numerical model of temperature field for continuous casting process. Two mathematical models were presented. The first model is the numerical model of temperature field where the phase and structural changes are modeled by an enthalpy method, while the second one represents a heuristic optimization algorithm. The aim of optimization and control of the steel slabs production is to achieve both the maximum possible savings and product quality. Some results are shown on steel grade S355J0H. The whole technique has very general nature and therefore, it can be easily modified for arbitrary grade of steel, quality conditions or specific caster geometry including rollers and nozzles positions. Further research will be focused on making the algorithm more precise, which includes considering more factors and they randomness which influence the casting process (stochastic-based optimalization), more accurate specifications of the final material quality (temperature limits).

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