# MODELLING OF COMMUTATION PROCESS OF DIODE RECTIFIER BOTH IN CURRENT AND VOLTAGE MODES 

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#### Abstract

The paper shows selected results analysis of diode rectifierduring commutation process. The commutation process and the process between commutations of diode rectifierare investigated. The process is modelled in the current and voltage modes. There are used numerical methods because of result equations (e.g. for length of commutation) cannot be solved analytically due to transcendental nature. The resulting waveforms of the commutation process in MATLAB are presented as well.


## 1 Commutation process in diode rectifier

The commutation process in rectifier is defined as all processes due to current commutating from on branch of the rectifier and followingthrough a given valve to the other branch of the rectifier.During commutation two different voltages interact: the "primary" voltage, i.e. phase voltage of appropriate transformer winding (supply voltage) and self-induced voltage of the anode circuit [2].


a)

b)

Figure 1: Commutation between two phases caused by internal inductances of the rectifier in current mode $a$ ) and voltage mode $b$ )
Fig. 1 shows the basicschematicrepresentation of diode rectifier involvement in both modes. During the commutation the load current floating through the one pair of the diodes $\left(D_{n}, D_{6}\right)$ is exchanged by other pair ones $\left(D_{n+1}, D_{6}\right)$, [1]. It begins when voltage of incoming phase is higher than voltage of actual phase, and it is over when the commutating current $i_{k}$ is reaching the load current $i_{z}$.

$$
\begin{equation*}
\mathrm{i}_{\mathrm{k}}\left(\mathrm{t}_{\mathrm{k}}\right)=\mathrm{i}_{\mathrm{z}}\left(\mathrm{t}_{\mathrm{k}}\right) \tag{1}
\end{equation*}
$$

So, it depend both on the time constant of the commutating circuit $\tau_{k}$ and of the load circuit $\tau_{z}$

$$
\begin{equation*}
\tau_{\mathrm{k}}=\frac{21}{2 \mathrm{r}} ; \tau_{\mathrm{z}}=\frac{2 \mathrm{l}+\mathrm{L}}{2 \mathrm{r}+\mathrm{R}} \tag{2}
\end{equation*}
$$

where $l, r$ are resistance and inductance (parameters) of diodes and $R, L$ are resistance and inductance of load.

## 2 Commutation process of rectifier in current mode

In first step the currentis conducted by diodes $D_{1}$ and $D_{6}$. The voltagesources are notconsidered what is caused by superposition of thecurrent tosource. In the case of commutation the current $i_{\mathrm{k}}$ flows through the diodes $D_{\mathrm{n}}$ and $D_{\mathrm{n}+1}$ is

$$
\begin{equation*}
\mathrm{i}_{\mathrm{k}}(\mathrm{t})=\frac{\mathrm{U}_{\mathrm{m}}^{\mathrm{ba}}}{\left\lfloor\mathrm{Z}_{\mathrm{k}}\right\rfloor}\left[\sin (\omega \mathrm{t}-\gamma)+\sin (\gamma) \cdot \mathrm{e}^{-\frac{\mathrm{t}}{\tau_{\mathrm{k}}}}\right]+\mathrm{I}_{0} \cdot \mathrm{e}^{-\frac{\mathrm{t}}{\tau_{\mathrm{k}}}} \tag{4}
\end{equation*}
$$

where $U \mathrm{~m}^{\text {ba }}$ is the maximal value of line voltage, $\left\lfloor Z_{k}\right\rfloor$ is the value of impedance during commutation, $\omega$ is angle velocity $\tau$ is time constant of the circuit , $\gamma$ is commutation angle.


Figure 2: The circuit diagramsin the aftermath of the commutation current $i_{\mathrm{k}}$ a) and the load current $i_{\mathrm{z}}$ b) during commutation

The equation describes the current $i_{k}$ according to situation of the circuit on Fig. 2 b ) will be created

$$
\begin{equation*}
\frac{d i_{k}}{d t}=-\frac{r}{l} i_{k}(t)+\frac{1}{2 l} u_{b a}(t) \tag{5}
\end{equation*}
$$

where $u_{b a}$ is the line voltage between phase $a$ phase $b$.
Using Euler's explicit methodthe numerical solution is:

$$
\begin{equation*}
i_{k n+1}=\left(1-h \frac{r}{1}\right) i_{k n}+h \frac{1}{21} u_{b a n} \tag{6}
\end{equation*}
$$

where $n$ is interpolation, $h$ is the size of interpolation step.
Forth is circuit applies:

$$
\begin{equation*}
\mathrm{i}_{\mathrm{z}}(\mathrm{t})=\mathrm{I}_{0} \mathrm{e}^{-\mathrm{t} / \tau_{\mathrm{z}}} \tag{7}
\end{equation*}
$$

where $I_{0}$ is the current through the inductance which is formed by r,l. (2). In spite of due to transcendental nature of the equation (1). Then it is necessary to use numerical (or graphical) solution.

$$
\begin{equation*}
\mathrm{L} \frac{\mathrm{~d} \mathrm{i}_{\mathrm{L}}}{\mathrm{dt}}=-\mathrm{i}_{\mathrm{T}} \mathrm{R} \rightarrow \frac{\mathrm{di} \mathrm{i}_{\mathrm{L}}}{\mathrm{dt}}=-\frac{\mathrm{R}}{\mathrm{~L}} \mathrm{i}_{\mathrm{T}} \tag{8}
\end{equation*}
$$

The differential equations describing current mode, Fig. 2, is necessary to transform to the discrete form. Equations have been transformed by the Euler's explicit method to the discrete form

$$
\begin{equation*}
\frac{\mathrm{di}_{\mathrm{Ln}+1}-\mathrm{di}_{\mathrm{Ln}}}{\mathrm{~h}}=\frac{\mathrm{d} \mathrm{i}_{\mathrm{L}}}{\mathrm{dt}} \tag{9}
\end{equation*}
$$

Than the equation describes circuit of current mode in discrete form is:

$$
\begin{equation*}
\mathrm{i}_{\mathrm{Ln}+1}=-\mathrm{i}_{\mathrm{Ln}}-\mathrm{h} \frac{\mathrm{R}}{\mathrm{~L}} \mathrm{i}_{\mathrm{Ln}}=\left(1-\mathrm{h} \frac{\mathrm{R}}{\mathrm{~L}}\right) \mathrm{i}_{\mathrm{Ln}} \tag{10}
\end{equation*}
$$

where value $h$ is size of every step and setting $i_{L n}=i_{0}+n . h$. So, one step of the Euler method is from $i_{L n}$ to $i_{L n+1}$.
Then as a discrete form has been obtain the equals can be used for create graphical waveforms of commutation process of current mode in the MATLAB environment.


Figure 3: The dependenceof length of commutation on time constants $\tau_{z}$ in current mode

As an example on Fig. 4, the current waveforms measured in current mode.


Figure 4:The current waveform (top) of 3-phase rectifier with commutation angle $\gamma \cong 20^{\circ} \mathrm{el}$. [3]
The commutation drop in rectified voltage depends directly on the reactance of commutation circuit and on rectified current (instantaneous value at the beginning of commutation).


Figure 5: The commutation drop of rectifier voltage
The commutation voltage $u_{\mathrm{k}}$ is defined as

$$
\begin{equation*}
u_{k}=\frac{U_{\mathrm{AV}}^{\mathrm{a}}+U_{\mathrm{AV}}^{\mathrm{b}}}{2} \tag{11}
\end{equation*}
$$

where $U_{\mathrm{AV}}^{\mathrm{a}}$ is average value of voltage phase $a$ and $U_{\mathrm{AV}}^{\mathrm{b}}$ is average value of voltage phase $b$ and then

The average value of the output voltage $U_{\mathrm{AVk}}$ is [6]

$$
\begin{equation*}
\mathrm{U}_{\mathrm{AVk}}^{*}=\frac{3}{\pi} \mathrm{U}_{\max }^{\mathrm{ba}}-\Delta \mathrm{U}_{\mathrm{AVk}} \tag{12}
\end{equation*}
$$

Then commutation drop $\Delta U_{\mathrm{AVk}}$

$$
\begin{equation*}
\Delta \mathrm{U}_{\mathrm{AVk}}=\mathrm{U}_{\mathrm{AV}}^{\mathrm{ba}}-\frac{\mathrm{U}_{\mathrm{AV}}^{\mathrm{a}}+\mathrm{U}_{\mathrm{AV}}^{\mathrm{b}}}{2} \tag{13}
\end{equation*}
$$

and consequently average value of rectifier current $I_{A V}$ is

$$
\begin{equation*}
\mathrm{I}_{\mathrm{AV}}=\frac{\mathrm{U}_{\mathrm{AVk}}{ }^{*}}{\mathrm{R}}=\frac{3}{\pi} \frac{\mathrm{U}_{\max }^{\mathrm{ba}}}{\mathrm{R}}-\Delta \mathrm{U}_{\mathrm{AVk}} \tag{14}
\end{equation*}
$$

Average value of the rectified current doesn't depend on load inductance, just on load resistance and commutation voltage drop. The commutation drop is, for negligible commutation time, equal zero.

Then

$$
\begin{equation*}
\mathrm{U}_{\mathrm{AV}}=\mathrm{U}_{\mathrm{AV}}{ }^{*}-\Delta \mathrm{U}_{\mathrm{AVk}}=\frac{3}{\pi} \mathrm{U}_{\mathrm{m}}\left\{1-\frac{\sqrt{3}}{2}\lfloor 1-\cos (\gamma)\rfloor\right\} \tag{15}
\end{equation*}
$$

So,

$$
\begin{equation*}
\mathrm{I}_{\mathrm{AV}}=\frac{\mathrm{U}_{\mathrm{AV}}}{\mathrm{R}} \tag{16}
\end{equation*}
$$

## 3 Commutation process of rectifier in voltage mode

The schematic representation of the circuit in voltage mode is shown on the Figure 6. Compared to circuit of current mode the voltage mode circuit contains capacitor C (includes $r_{C}$ ) parallel connected to the load.


Figure 6: The circuitdiagramin the aftermath ofthe loadcurrentduringcommutation in voltage mode

The inductor $L_{z}$ is the source of current $I_{0}$. Resistance of the inductor $\left(r_{z}\right)$ and capacitor $\left(r_{\mathrm{C}}\right)$ may or may not bounder consideration. The current $i_{z}$ is closing inside the circuit. By the circuit, fig. 4, the differential equations of the system may be created.

$$
\frac{\mathrm{d}}{\mathrm{dt}}\binom{\mathrm{i}_{\mathrm{L}}}{\mathrm{u}_{\mathrm{C}}}=\left(\begin{array}{cc}
-\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{~L}_{\mathrm{z}}} & -\frac{1}{\mathrm{~L}_{\mathrm{z}}}  \tag{17}\\
1 / \mathrm{C} & -\left(\frac{1}{\mathrm{r}_{\mathrm{C}}}+\frac{1}{\mathrm{R}}\right) \frac{1}{\mathrm{C}}
\end{array}\right)\binom{\mathrm{i}_{\mathrm{L}}}{\mathrm{u}_{\mathrm{C}}}
$$

Using numerical methods the discrete form may be obtained.

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{dt}} \overline{\mathrm{x}}(\mathrm{t})=\mathrm{A} \overline{\mathrm{x}}(\mathrm{t})+\mathrm{B} \overline{\mathrm{u}}(\mathrm{t}) \rightarrow \frac{\overline{\mathrm{x}}_{\mathrm{n}+1}-\overline{\mathrm{x}}_{\mathrm{n}}}{\mathrm{~h}}=\mathrm{A} \overline{\mathrm{x}}(\mathrm{t})+\mathrm{B} \overline{\mathrm{u}}(\mathrm{t}) \tag{18}
\end{equation*}
$$

Where $A$ is the matrix of the system and $B$ is the excitation matrix. The superposition of thecurrent tosourcecauses that the sources are notconsidered in circuit. So, the excitation matrix is not considered.

$$
\begin{equation*}
\bar{x}_{n+1}=\bar{x}_{n}+h(A \bar{x}(t)) \rightarrow \bar{x}_{n+1}=(1+h(A \bar{x}(t))) \bar{x}_{n} \tag{19}
\end{equation*}
$$

Than the final matrix of the system is:

$$
\binom{i_{L}}{u_{C}}_{n+1}=\left[\left(\begin{array}{ll}
1 & 0  \tag{20}\\
0 & 1
\end{array}\right)+h\left(\begin{array}{cc}
-\frac{r_{z}}{L_{z}} & -\frac{1}{L_{z}} \\
1 / C & -\left(\frac{1}{r_{C}}+\frac{1}{R}\right) \frac{1}{C}
\end{array}\right)\right]\binom{i_{L}}{u_{C}}_{n}
$$

After the system matrix in discrete form has been obtained the graphical waveform in MATLAB environment may be created.


Figure 7: Depending of length of commutation on time constants $\tau_{z}$ in voltage mode

The average value of the output voltage is

$$
\begin{equation*}
\mathrm{U}_{\mathrm{AVk}}^{*}=\frac{3}{\pi} \mathrm{U}_{\max }^{\mathrm{ba}}-\Delta \mathrm{U}_{\mathrm{AVk}} \tag{21}
\end{equation*}
$$

and consequently average value of rectifier current is

$$
\begin{equation*}
\mathrm{I}_{\mathrm{AV}}=\frac{\mathrm{U}_{\mathrm{AVk}}{ }^{*}}{\mathrm{R}}=\frac{3}{\pi} \frac{\mathrm{U}_{\max }^{\mathrm{ba}}}{\mathrm{R}}-\Delta \mathrm{U}_{\mathrm{AVk}} \tag{22}
\end{equation*}
$$

The commutation of the rectifier underway always between pair of diodes from the same group. The distance between commutations is $60^{\circ}$.

Thecalculation of the commutation drop in voltage mode of the rectifier has the same procedure as in the current mode.

## 4 The discussion of therectifier analysisresultsworked-out

The dependenceof the commutation drop of the time constant of both modes is given in Table 1 and Table 2..

Table 1: THE DEPENDENCE OF THE COMMUTATION DROP AND TIME CONSTANT OF CURRENT MODE

| $\Delta \mathbf{U}_{\mathbf{A V k}} \sim \mathbf{f}\left(\mathbf{t}_{\mathbf{k}}\right)$ | $\mathbf{t}_{\mathbf{k} \mathbf{0}}$ | $\mathbf{t}_{\mathbf{k} \mathbf{1}}$ | $\mathbf{t}_{\mathbf{k} \mathbf{2}}$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{t}_{\mathbf{k}}$ | 0,250 | 0,149 | 0,023 |
| $\Delta \mathbf{U}_{\mathbf{A V k}}$ | 93,717 | 37,002 | 0,928 |

Table 2: THE DEPENDENCE OF THE COMMUTATION DROP AND TIME CONSTANT OF VOLTAGE MODE

| $\Delta \mathbf{U}_{\mathbf{A V k}} \sim \mathbf{f}\left(\mathbf{t}_{\mathbf{k}}\right)$ | $\mathbf{t}_{\mathbf{k} \mathbf{0}}$ | $\mathbf{t}_{\mathbf{k} \mathbf{1}}$ | $\mathbf{t}_{\mathbf{k} \mathbf{2}}$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{t}_{\mathbf{k}}$ | 0,100 | 0,071 | 0,022 |
| $\Delta \mathbf{U}_{\mathbf{A V k}}$ | 17,1448 | 8,509 | 0,703 |

The graphical dependence of the commutation drop to the time constant of both modes is given in Figure 8.


Figure 8: dependence of the commutation drop of the time constant of both modes

The Fig. 8 shows graphical dependence of the commutation drop of the time constant. On the vertical axes it ratio of commutation drop and the maximum voltage and on the horizontal axes is time constant. The waveform can be seen that with increasing commutation time the commutation drop increases.

Note: In the casethat the current value $i_{z}$ is constant and time constant $\tau_{z}$ is "infinitely large "the commutation time analytically may be calculate.

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