MODELING OF STRESS FIELD DISTRIBUTION

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Abstract

The paper is aimed to the proposal for modeling of the stress field distribution in the ferromagnetic steel plate. Computer modelling of various physical phenomena has become an essential tool for persons who work with complicated systems or who desire to understand the behaviour of such systems. Nowadays the development of the numerical methods and abilities of the computer art enabled modelling of lots of problems. Modelling of field problems has a specific attribute because such problems can not be described by a few discrete variables, but they are better understandable if continuous functions of appropriate quantities over some region of space are used. For solving field problems it can be used various professional packages, but MATLAB is useful too.

1 Principles of the Boundary-value Problem Formulation for Plane Stress

The determination of planar stress field distribution in a ferromagnetic steel plate, that has squared shape with a circle hole in the middle, can be formulated as a boundary-value problem in terms of the displacement components. The boundary-value problem formulation requires determining the domain, in which the planar stress field will be solved, with the boundary conditions and formulating the correct partial differential equation (PDE).

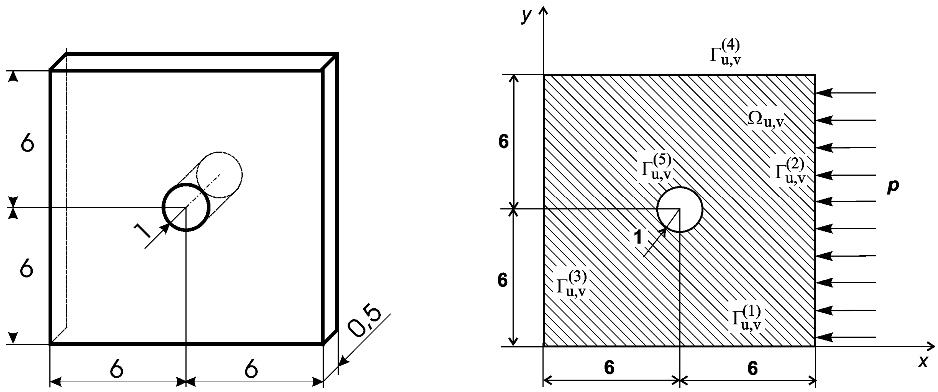

Figure 1: A steel plate (left) and the domain $\Omega_{u,v}$ (right)

The first of all it is necessary to describe the domain $\Omega_{u,v}$, in which the stress field will be solved. The steel plate can be considered to be a planar body (Fig. 1 – left) with the following properties: dimension 12-by-12 mm, thickness 0,5 mm and the radius of the circle hole is 1 mm. The plate is made of ferromagnetic transformer sheet, whose parameters are: $E = 1{,}86.10^5$ MN/m^2, $\nu = 0{,}3$. The plate is subjected to a constant continuous pressure stress 8,23 MPa in the direction of x-axis.

The domain $\Omega_{u,v}$ is surrounded by five boundaries $\Gamma^{(1)}_{u,v}$, $\Gamma^{(2)}_{u,v}$, $\Gamma^{(3)}_{u,v}$, $\Gamma^{(4)}_{u,v}$ a $\Gamma^{(5)}_{u,v}$ (see Fig. 1 – right).

The boundary-value problem formulation also requires formulate the PDE system for balance of force in terms of the displacement components with the boundary conditions.

For plane stress, in the body made of the homogeneous isotropic material, the force balance equations are [1]:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0\,, \tag{1}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0\,, \tag{2}$$

where σ_x is the x-direction stress, σ_y being the y-direction stress, τ_{xy} being the shear stress, X, Y being the volume forces.

The stress components are closely related to the strain components and these relations are defined by Hooke law [2]:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{3}$$

where E is Young's modulus, ν being Poisson's ratio, ε_x being the x – direction strain, ε_y being the y – direction strain, γ_{xy} being the shear strain.

The strain components can be expressed by the displacements, u the displacement in the x-direction and v the displacement in the y-direction [2]:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\,. \tag{4}$$

Combining the equations (1) – (4) and assuming that there are no volume forces, the PDE system for balance of force in terms of the displacement components is obtained. The system takes form [2]:

$$\frac{E}{1-\nu^2}\frac{\partial^2 u}{\partial x^2} + \frac{E}{2(1+\nu)}\frac{\partial^2 u}{\partial y^2} + \nu\frac{E}{1-\nu^2}\frac{\partial^2 v}{\partial x \partial y} + \frac{E}{2(1+\nu)}\frac{\partial^2 v}{\partial y \partial x} = 0 \tag{5}$$

$$\frac{E}{2(1+\nu)}\frac{\partial^2 v}{\partial x^2} + \frac{E}{1-\nu^2}\frac{\partial^2 v}{\partial y^2} + \frac{E}{2(1+\nu)}\frac{\partial^2 u}{\partial x \partial y} + \nu\frac{E}{1-\nu^2}\frac{\partial^2 u}{\partial y \partial x} = 0 \tag{6}$$

The following boundary conditions for the system PDE can be given:

- For each point on the boundary is specified the value of the displacement [2]:

$$u = g_1, \quad v = g_2\,. \tag{7}$$

  This boundary condition is called Dirichlet boundary condition.

- For each point on the boundary is known the surface load (the derivative of the displacement by the outward normal vector) [2]:

$$\mathbf{e}_n \left(\begin{bmatrix} \frac{E}{1-\nu^2} & 0 \\ 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \nabla u \right) + \mathbf{e}_n \left(\begin{bmatrix} 0 & \frac{\nu E}{1-\nu^2} \\ \frac{E}{2(1+\nu)} & 0 \end{bmatrix} \nabla v \right) = p_x\,, \tag{8}$$

$$\mathbf{e}_n \left(\begin{bmatrix} 0 & \frac{E}{2(1+\nu)} \\ \frac{\nu E}{1-\nu^2} & 0 \end{bmatrix} \nabla u \right) + \mathbf{e}_n \left(\begin{bmatrix} \frac{E}{2(1+\nu)} & 0 \\ 0 & \frac{E}{1-\nu^2} \end{bmatrix} \nabla v \right) = p_y\,, \tag{9}$$

  where $\mathbf{e}_n$ is the outward normal vector of the boundary, ∇ being Hamilton operator, p_x being the pressure in the direction of x-axis, p_y being the pressure in the direction of y-axis.

  These boundary conditions are called Neumann boundary condition.

The boundary conditions for the displacement components in domain $\Omega_{\mathrm{u,v}}$, in which the stress field will be solved, are following ones:

- the boundary $\Gamma^{(2)}_{\mathrm{u,v}}$ is subjected to a pressure $\boldsymbol{p} = -p_{\mathrm{x}}\mathbf{e}_{\mathrm{x}}$:

$$\frac{E}{1-\nu^2}\frac{\partial u}{\partial x} + \frac{\nu E}{1-\nu^2}\frac{\partial v}{\partial y} = -p_{\mathrm{x}}\,, \tag{10}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0\,. \tag{11}$$

- there is no displacement on the boundary $\Gamma^{(3)}_{\mathrm{u,v}}$:

$$u = 0, \quad v = 0\,, \tag{12}$$

- the remaining boundaries $\Gamma^{(1)}_{\mathrm{u,v}}$, $\Gamma^{(4)}_{\mathrm{u,v}}$ a $\Gamma^{(5)}_{\mathrm{u,v}}$ are free (no normal stress):

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0\,, \tag{13}$$

$$\nu\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,. \tag{14}$$

2 Results

The investigated boundary-value problem was solved by using professional code PDE Toolbox. The toolbox is based on finite element method (FEM). The solution was made by using the adaptive mode option, which enables to refine the mesh in areas where the gradient of the solution is large in order to increase the accuracy of the solution [7]. The obtained results (for generated triangular mesh consisting of 42 112 nodes and 83 456 triangles) at the pressure force $p_{\mathrm{x}} = 8{,}23$ MPa are depicted in Fig. 2 – 4.

In these figures are plotted the contour lines, e.g. the lines connected points of equal value of the depicted quantity in the domain $\Omega_{\mathrm{u,v}}$.

In Fig. 2 are visualised the displacements $u(x, y)$ in the x – direction (left) and $v(x, y)$ in the y – direction (right). The x – direction strain ε_{x} and the y – direction strain ε_{y} are depicted in Fig. 3. The x – direction stress σ_{x} and the y – direction stress σ_{y} are shown in Fig. 4.

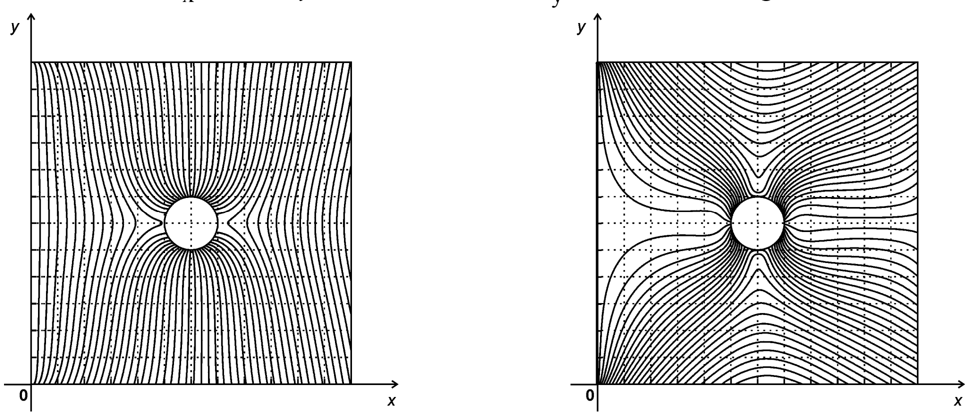

Figure 2: The displacement u in the x – direction (left) and the displacement v in the y – direction (right)

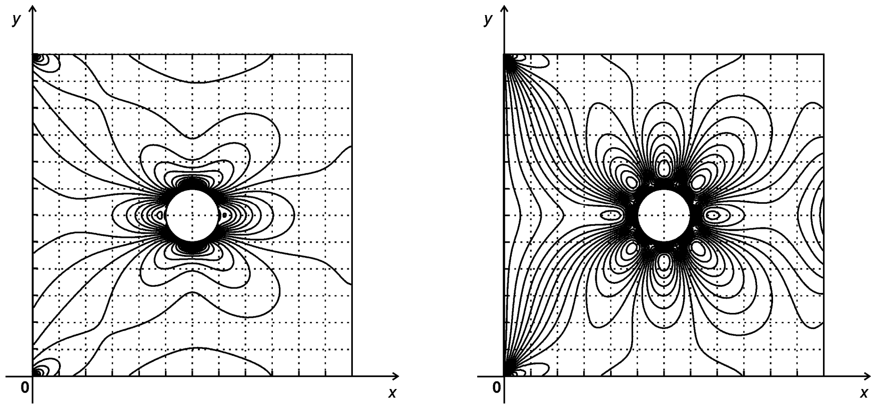

Figure 3: The strain ε_x in the x – direction (left) and the strain ε_y in the y – direction (right)

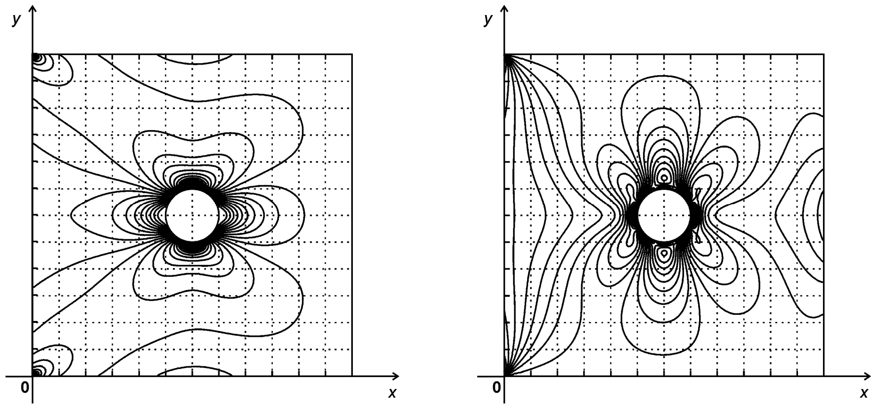

Figure 4: The stress σ_x in the x – direction (left) and the stress σ_y in the y – direction (right)

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