VERIFICATION OF OBSERVER ALGORITHMS USING MEASURED DATA FILES

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Abstract

Nowadays, the emphasis is given on the efficiency of the control of electrical drives. Parts of the control are observers, which are responsible for estimated values for the control algorithm. The observers are proposed for electrical drive with a flexible coupling using Forced Dynamic Control (FDC) with a position sensor on the rotor shaft. The flexible connection between the load and the motor shaft could be a source of the undesirable torsion vibrations. FDC is used to prevent this possibility. Designed observer algorithms are verified by the measured data files, which contains values of electrical drive with FDC with two sensors, one on the motor shaft and other one on the load side.

1 Parts of the electrical drive

1.1 The control system of the electrical drive

The observer algorithms are designed for the electrical drive with a flexible coupling with a position sensor on the motor shaft. FDC used in the drive is described in [1], [3] and [4]. A speed control algorithm used in the simulation is defined:

$$i_{d\,dem} = 0 \tag{1}$$

$$i_{q\,dem} = \frac{2}{3p\Psi_{PM}}\left[\frac{J_R}{T_\omega}(\omega_{R\,dem} - \omega_R^*) + \Gamma_{Ls}^*\right] \tag{2}$$

Where $i_{d,q\,dem}$ are demanded stator currents, ω_R is angle speed of rotor shaft, p is number of pole pairs, J_R is rotor moment of inertia, Ψ_{PM} is magnetic flux of permanent magnets, T_ω is the settling time of speed FDC and Γ_{Ls} is motor torque. Position control algorithm is defined:

$$\dot{\theta}_{R\,dem} = T_\omega\left(\frac{J_L}{d^4K_s}\left(\theta_{L\,dem} - \hat{\theta}_L\right) - \left(\frac{4J_L}{d^3K_s} - \frac{4}{d}\right)\hat{\omega}_L - \left(\frac{4}{d} - \frac{1}{T_\omega}\right)\hat{\omega}_R - \left(\frac{K_s}{J_L} - \frac{6}{d^2}\right)\hat{\theta}_L - \right.$$

$$\left. - \left(\frac{6}{d^2} - \frac{K_s}{J_L}\right)\hat{\theta}_R + \frac{1}{K_s}\hat{\ddot{\Gamma}}_{Le} + \frac{4}{dK_s}\hat{\dot{\Gamma}}_{Le} - \left(\frac{1}{J_L} - \frac{6}{d^3K_s}\right)\hat{\Gamma}_{Le}\right) \tag{3}$$

Where θ_L is position of load shaft, θ_R is position of motor shaft, ω_L is angle speed of load, K_s is spring constant, J_L is load moment of inertia, Γ_{Le} is load torque and d is defined as $2T_\theta/15$, where the T_θ is the settling time of position FDC. Mathematical model in d, q frame for permanent magnet synchronous motor is used in the simulation [2]. The flexible coupling math model is used without damping and it is described in [3], [4], [5]. Model of PMSM with flexible coupling is defined:

$$\frac{di_d}{dt} = \frac{1}{L_d}\left(u_d - R_S i_d + p\omega_R L_q i_q\right) \tag{4}$$

$$\frac{di_q}{dt} = \frac{1}{L_q}\left(u_q - R_S i_q - p\omega_R\left(L_d i_d + \Psi_{PM}\right)\right) \tag{5}$$

$$\frac{d\omega_R}{dt} = \frac{1}{J_R}\left(\frac{3}{2}p\left(i_q\left(L_d i_d + \Psi_{PM}\right) - L_q i_q i_d\right) - \Gamma_{Ls}\right) \tag{6}$$

$$\frac{d\omega_L}{dt} = \frac{1}{J_L}\left(\Gamma_{Ls} - \Gamma_{Le}\right) \Rightarrow \frac{d\omega_L}{dt} = \frac{1}{J_L}\left(K_s\left(\theta_R - \theta_L\right) - \Gamma_{Le}\right) \tag{7}$$

$$\frac{d\theta_R}{dt} = \omega_R \tag{8}$$

$$\frac{d\theta_L}{dt} = \omega_L \tag{9}$$

u_d, u_q are stator voltages, i_d, i_q are stator currents, R_s is resistance of the stator windings, L_d, L_q are inductance in d, q axis, Ψ_d, Ψ_q are magnetic flux in d, g axis and Γ_{Le} is external load torque.

The complete block diagram of the electrical drive system is shown in Figure 1. Input for the position FDC block and the entire drive system is the demanded position of load shaft $\theta_{L\,dem}$. Output from position FDC block is generating demanded angle speed of motor shaft $\omega_{R\,dem}$ and it is the input for the speed FDC block. Demanded stator currents are developed in the speed FDC block. A comparison between them and stator currents from PMSM is the input for the inverter, which is supplying the motor. The inverter block is defined as a gain multiplication. Motor shaft is connected with the load by flexible coupling, for example a long shaft. The position sensor is used on the motor shaft. The position of motor shaft θ_R and stator current i_q are used observers. The observers are estimating variables for FDC control algorithms.

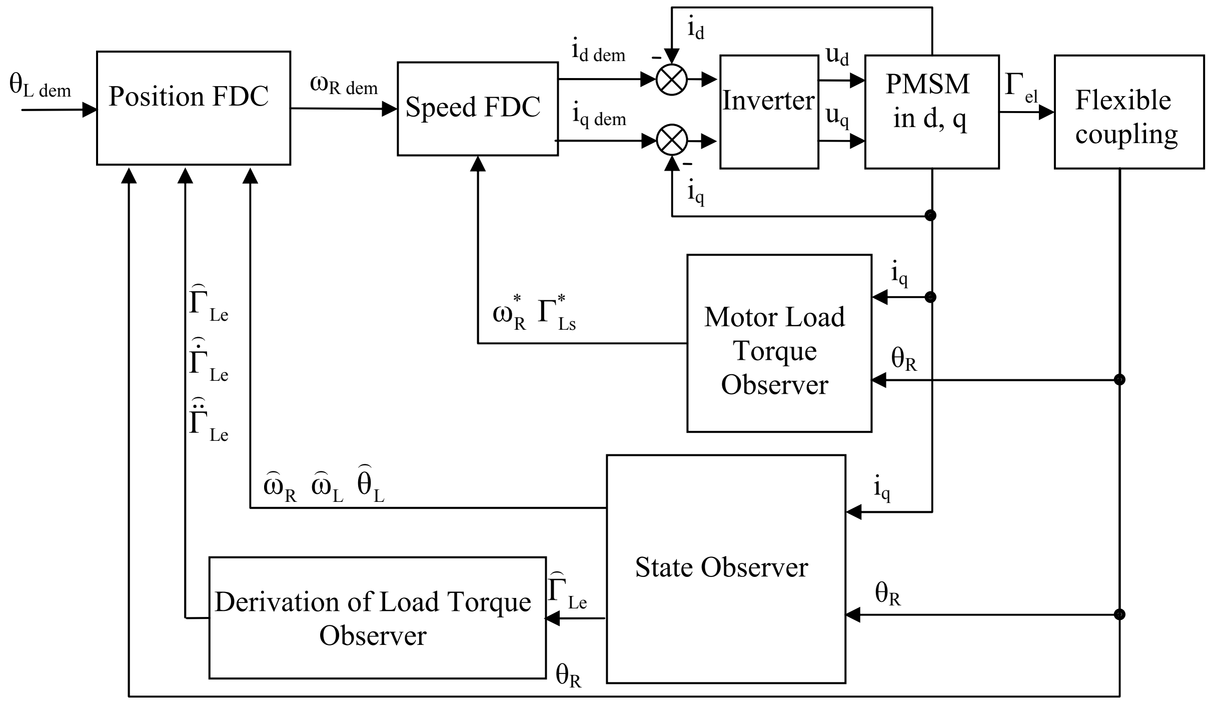


Figure 1: The complete block diagram of the electrical drive system

<u>1.2 Motor Load torque observer</u>

Measured position of rotor shaft θ_R and stator current i_q are used for the Motor Load Torque Observer. The load torque Γ_{Ls} and rotor angle speed ω_R are the goals of the estimation that are needed for FDC speed control block. The differential equations, which describe the observer, are based on the mechanical differential equations of permanent magnet synchronous motor (PMSM). The equation for the load torque is impossible to define exactly, therefore the load torque is considered to be a constant with regard to the settling time [3].

$$\frac{d\theta_R}{dt} = \omega_R \tag{10}$$

$$\frac{d\omega_R}{dt} = \frac{1}{J_R}\left(c\left(\Psi_d i_q - \Psi_q i_d\right) - \Gamma^*_{Ls}\right) \tag{11}$$

$$-\frac{d\Gamma_{Ls}}{dt} = 0 \tag{12}$$

The observer is based on the comparison between the measured θ_R and the estimated position of motor shaft θ_R^*, where the difference is defined as the measured error $\varepsilon_{\theta R} = \theta_R - \theta_R^*$. The Motor Load Torque Observer equations are created by adding this correction with the relevant gains k_θ, k_ω and k_Γ.

$$\frac{d\theta_R}{dt} = \omega_R + k_\theta\left(\theta_R - \theta_R^*\right) \tag{13}$$

$$\frac{d\omega_R}{dt} = \frac{1}{J_R}\left(c\left(\Psi_d i_q - \Psi_q i_d\right) - \Gamma_{Ls}^*\right) + k_\omega\left(\theta_R - \theta_R^*\right) \tag{14}$$

$$-\frac{d\Gamma_{Ls}}{dt} = k_\Gamma\left(\theta_R - \theta_R^*\right) \tag{15}$$

Dynamic error system is formed by subtraction of the observer equations from the observed system equations. The system is rewritten in a matrix form. The constant a is defined as $1/J_R$.

$$\begin{bmatrix} \dot\theta_R - \dot\theta_R^* \\ \dot\omega_R - \dot\omega_R^* \\ -\dot\Gamma_{Ls} + \dot\Gamma_{Ls}^* \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -a \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_R - \theta_R^* \\ \omega_R - \omega_R^* \\ -\Gamma_{Ls} + \Gamma_{Ls}^* \end{bmatrix} - \begin{bmatrix} k_\theta \\ k_\omega \\ k_\Gamma \end{bmatrix} \left(\theta_R - \theta_R^*\right) \tag{16}$$

Dynamic error system is changed by another adjustment and by substitution. The measured error ε is reinstated for the position, the angle speed and the torque differences and the matrix has a form $\dot{\boldsymbol{\varepsilon}}_\mathbf{i} = \mathbf{A} \cdot \boldsymbol{\varepsilon}_\mathbf{i}$, where the index i means θR, ωR, Γ:

$$\begin{bmatrix} \dot\varepsilon_{\theta R} \\ \dot\varepsilon_{\omega R} \\ \dot\varepsilon_\Gamma \end{bmatrix} = \begin{bmatrix} -k_\theta & 1 & 0 \\ -k_\omega & 0 & -a \\ k_\Gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{\theta R} \\ \varepsilon_{\omega R} \\ \varepsilon_T \end{bmatrix} \tag{17}$$

The observer behavior is intended so that the difference between the real and the estimated position variables is converging to the zero with increasing time of calculation. The eigenvalues of dynamic error system matrix **A** must have negative real parts. It is achieved by calculation of determinant.

$$\det\left[\mathbf{1}\cdot\boldsymbol{\lambda} - \mathbf{A}\right] = \det \begin{bmatrix} \lambda + k_\theta & -1 & 0 \\ k_\omega & \lambda & a \\ -k_\Gamma & 0 & \lambda \end{bmatrix} = \lambda^3 + \lambda^2 k_\theta + \lambda k_\omega + k_\Gamma a \tag{18}$$

The solution of this obtained equation has three multiple roots $\lambda_{1,2,3}=-\omega_0$ and prescribed settling time of T_{U1} can be achieved using Dodds formula [13], where for the third order polynomial concerns n = 3.

$$T_{U1} = 1,5\left(1+n\right)\frac{1}{\omega_0} \Rightarrow \omega_0 = \frac{6}{T_{U1}} \tag{19}$$

The characteristics third order equation is:

$$\left(\lambda + \omega_0\right)^3 = \left(\lambda + \frac{6}{T_{U1}}\right)^3 = \lambda^3 + \lambda^2\frac{18}{T_{U1}} + \lambda\frac{108}{T_{U1}{}^2} + \frac{216}{T_{U1}{}^3} \tag{20}$$

The gains are achieved by comparison between the matrix determinant and the characteristics equation. Parts λ with the same superscript are compared and the gains are

$$k_\theta = \frac{18}{T_{U1}},\ k_\omega = \frac{108}{T_{U1}{}^2},\ k_\Gamma = \frac{216}{T_{U1}{}^3} J_R \tag{21}$$

Block diagram of Motor Load Torque Observer shows Figure 2.

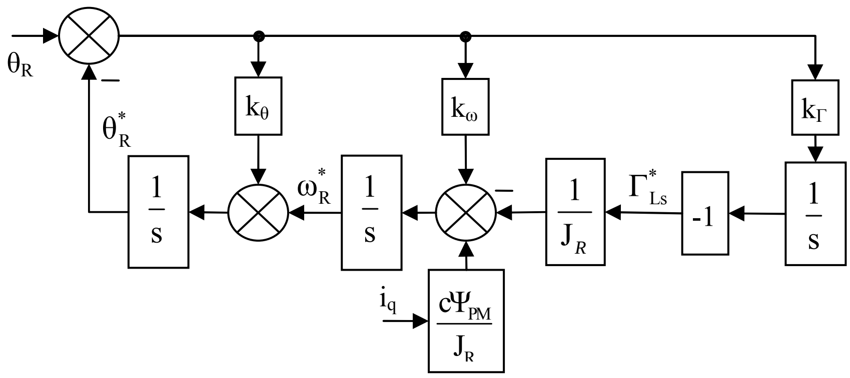


Figure 2: Motor Load Torque Observer block diagram

1.3 State Observer

State Observer is used for the behavior estimation of mechanical part of PMSM and flexible coupling. The observer inputs are the position of rotor shaft θ_R and the stator current i_q. Position of load θ_L, angle speed of load ω_L, angle speed of motor shaft ω_R and external load torque Γ_{Le} are estimated in this observer and are needed for positional FDC algorithm. Description of equations is based on the flexible coupling math model and with one added equation for external load. It is assumed that the external load torque is equal to the constant with regard to the settling time T_{U2} [3]. Block diagram of State Observer is shown in Figure 3. State Observer equations are:

$$\frac{d\hat{\theta}_L}{dt} = \omega_L + k_{\theta L}\left(\hat{\theta}_R - \theta_R\right) \tag{22}$$

$$\frac{d\hat{\theta}_R}{dt} = \omega_R + k_{\theta R}\left(\hat{\theta}_R - \theta_R\right) \tag{23}$$

$$\frac{d\hat{\omega}_L}{dt} = \frac{1}{J_L}\left[K_s\left(\theta_R - \theta_L\right) - \Gamma_{Le}\right] + k_{\omega L}\left(\hat{\theta}_R - \theta_R\right) \tag{24}$$

$$\frac{d\hat{\omega}_R}{dt} = \frac{1}{J_R}\left[\Gamma_{el} - K_s\left(\theta_R - \theta_L\right)\right] + k_{\omega R}\left(\hat{\theta}_R - \theta_R\right) \tag{25}$$

$$-\frac{d\hat{\Gamma}_{Le}}{dt} = k_T\left(\hat{\theta}_R - \theta_R\right) \tag{26}$$

Calculated gains for State Observer are:

$$k_{\theta L} = \frac{J_L}{K_S}\left(\frac{7290J_L - 45K_ST_{U2}}{J_LT_{U2}^3}\right); k_{\theta R} = \frac{45}{T_{U2}}$$

$$k_{\omega L} = \frac{J_L}{K_S}\left(\frac{32805J_L^2J_R - 810K_SJ_LJ_RT_{U2}^2 + K_S^2J_LT_{U2}^4 + K_S^2J_LT_{U2}^4}{J_L^2J_RT_{U2}^4}\right) \tag{27}$$

$$k_{\omega R} = \frac{810}{T_{U2}^2} - \frac{K_S}{J_L} - \frac{K_S}{J_R}; k_T = \frac{J_LJ_R59049}{K_ST_{U2}^5}$$

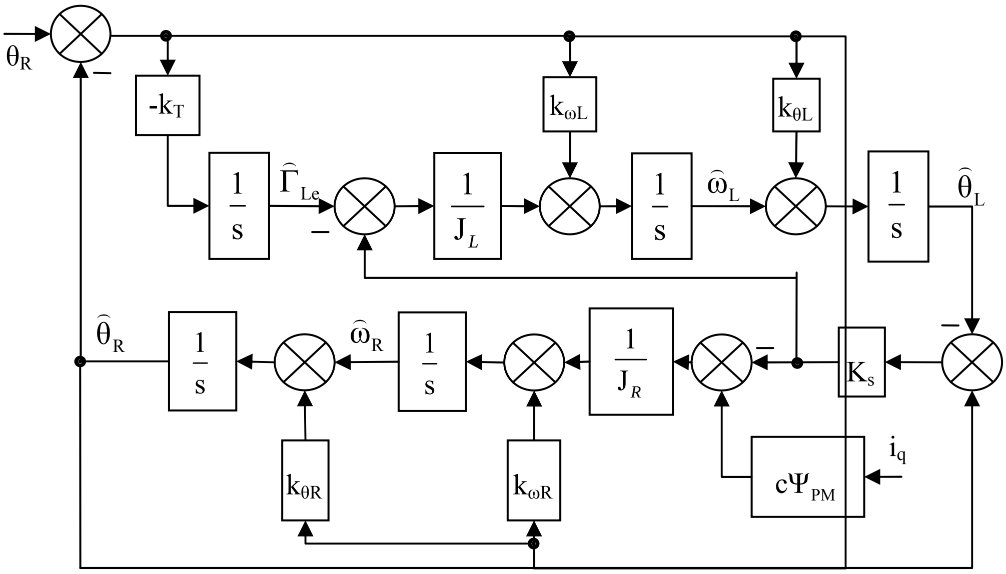


Figure 3: State Observer block diagram

1.4 Derivation of Load Torque Observer

Estimated external load torque Γ_{Le} from State observer is the input for Derivation of Load Torque Observer. Its goal is to estimate first and second derivation of the external load torque for the position FDC block. The observer equations contain only relevant derivation of external load torque with a correction. The equation for the third derivation is supposed to be constant with regard to the settling time T_{U3} [3]. In this case, the observer equations are:

$$\frac{d\hat{\Gamma}_{Le}}{dt} = \hat{\dot{\Gamma}}_{Le} + k_1\left(\widehat{\Gamma}_{Le} - \hat{\Gamma}_{Le}\right) \tag{28}$$

$$\frac{d\hat{\dot{\Gamma}}_{Le}}{dt} = \hat{\ddot{\Gamma}}_{Le} + k_2\left(\widehat{\Gamma}_{Le} - \hat{\Gamma}_{Le}\right) \tag{29}$$

$$\frac{d\hat{\ddot{\Gamma}}_{Le}}{dt} = k_3\left(\widehat{\Gamma}_{Le} - \hat{\Gamma}_{Le}\right) \tag{30}$$

The relevant gain values are:

$$k_1 = \frac{18}{T_{U3}},\ k_2 = \frac{108}{T_{U3}^{\ 2}},\ k_3 = \frac{216}{T_{U3}^{\ 3}} \tag{31}$$

Block diagram of Derivation of Load Torque Observer is shown in Figure 4.

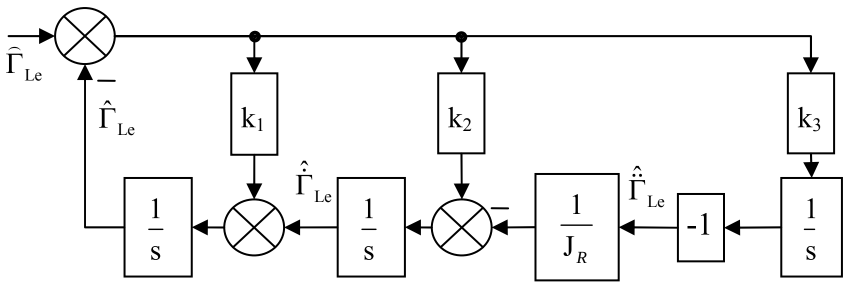


Figure 4: Block diagram of load torque derivation observer

2 Verification by Simulation

Matlab/simulink was used for the simulation. The simulation comparison was done between the system without and with the data files. The data files were obtained by measurement of a real electrical drive with flexible coupling and position sensors on both shaft sides and controlled using FDC. Demanded load position $\theta_{L\ dem}$ was 2π rad with the prescribed settling time T_θ = 0,2s. Speed FDC algorithm settling time T_ω was 0,017s. The drive was loaded during the measurement with an external load Γ_{Le} = 3Nm in the time 1s [4]. Settling times for Motor Torque Observer is T_{U1}=0,04s, for State Observer is T_{U2}=0,08s and for Derivation of Load Torque Observer is T_{U3}=0,08s. Simulation block diagram used in the simulation with no data files is shown in Figure 1 and with the data files in Figure 5. The position of the motor shaft θ_R and the current i_q is taken from the files.

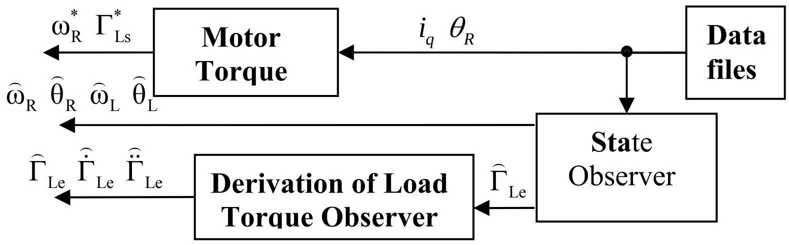
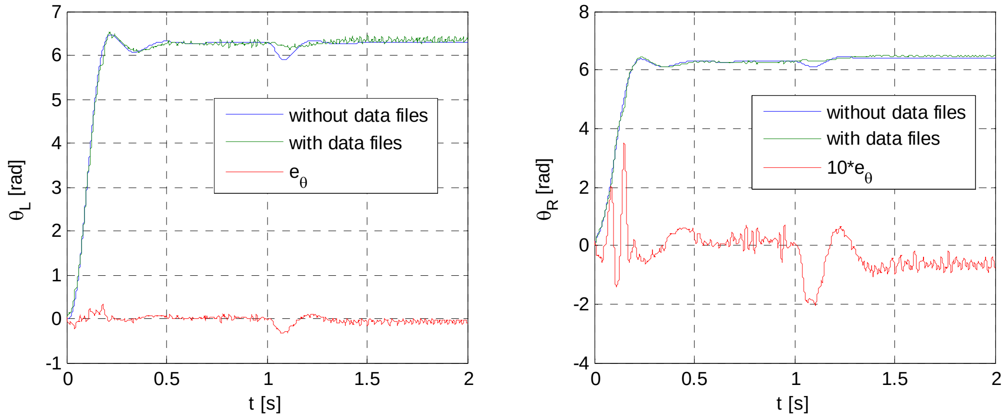


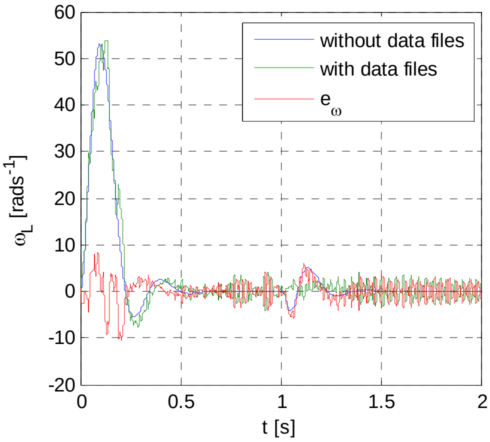
Figure 5: Block diagram of simulation with data files

3 Results

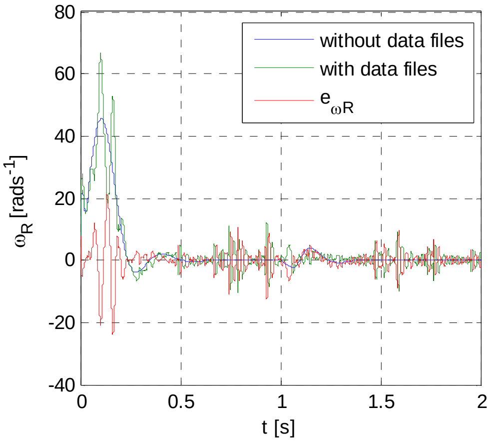
Figure 6 shows the comparison between variables estimated in State Observer. Figure 6a is showing estimated positions of loads θ_L, one for the ideal simulation and the other one for simulation with the data files. In Figure 6b are the compared positions of motor shaft θ_R. Figure 4c and 4d are showing the estimated angle speeds, in 6c speed of load shaft ω_L and in 6d speed of motor shaft ω_R. Estimated external load torque Γ_{Le} is in 6e. The again estimated external load torque Γ_{Le} by Derivation of Load Torque Observer is shown in the Figure 7a and its derivation in 7b, 7c. The motor load torque Γ_{Ls} and the angle speed of motor shaft ω_R estimated by Motor Load Torque Observer are shown in Figure 8a and 8b. All estimated variables are imaged with an error e between them, only derivations of load torque not.



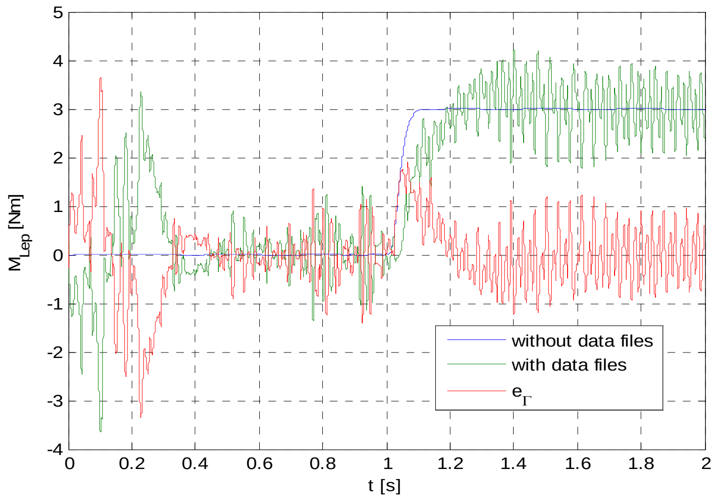
a, b,



c,

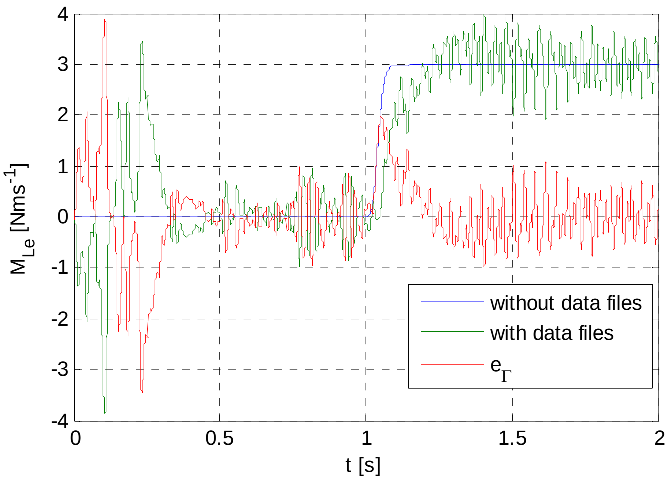


d,

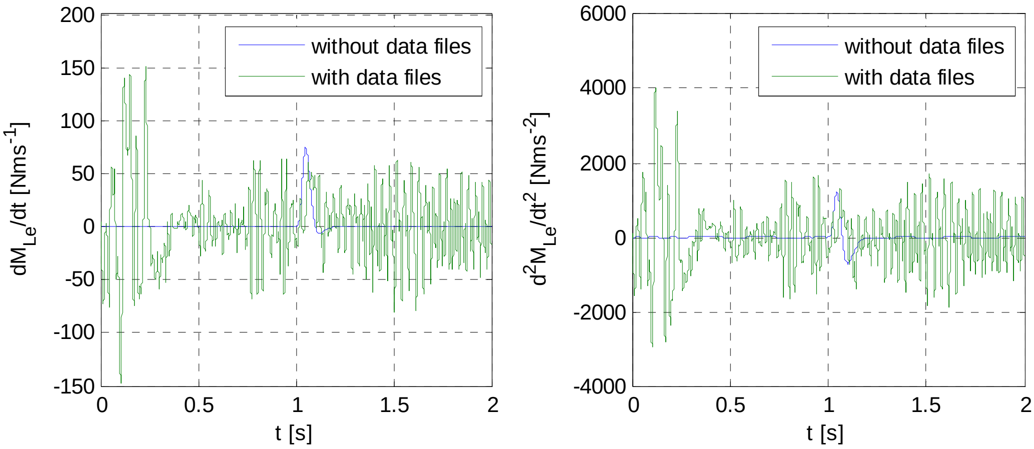


e,

Figure 6: Comparison of estimated variables by State Observer

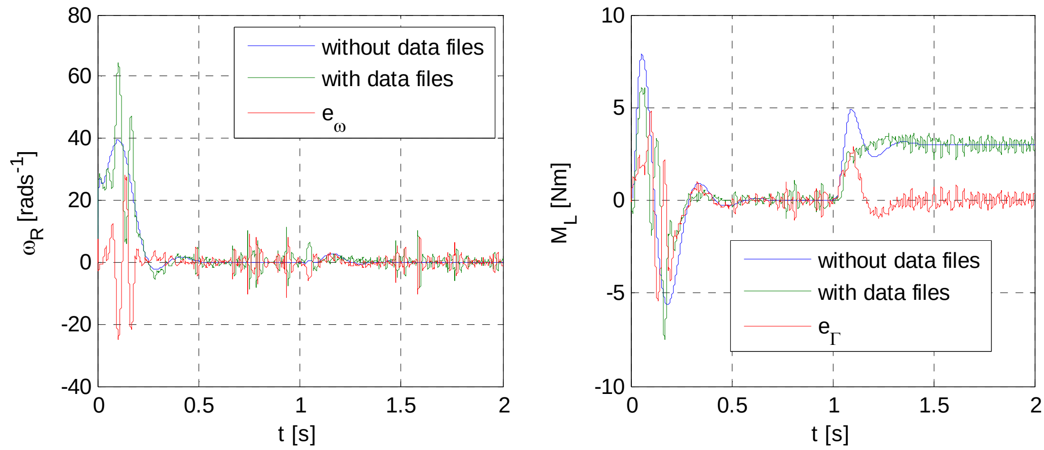


a,



b, c,

Figure 7: Estimated variables by Derivation of Load Torque Observer



a, b,

Figure 8: Estimated variables by Motor Load Torque Observer

4 Conclusion

The proposed observer algorithms were verified. Estimated positions, angle speeds and torques are following the demanded courses. Observer algorithms in the simulation with the data files are working well. The differences between simulation with data files and without are the result of sensor sensitivity and used math model. Derivations of load torque are very noisy and that could be a problem in a real application. The simulation has proved that the measured data files can be used for the verification of proposed algorithm parts.

Acknowledgment

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Appendix

$R_s = 1{,}3\ \Omega$, $L_d = 14{,}4$ mH, $L_q = 16{,}3$ mH, $\Psi_{PM} = 0{,}13$ Wb, p = 5, $J_R = 0{,}0037$ kgm^2, $J_L = 0{,}01$ kgm^2, $K_s = 23$ Nm rad^{-1}

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