PREDICTIVE CONTROL OF AMBIENT AIR TEMPERATURE
BY AUXILIARY CONTROL VARIABLE IN LARGE
OCCUPIED SPACE

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Abstract

In the article is presented a control method of indoor air temperature of larger
occupied space. The desired temperature is set on a standard PI-controller handling
an input signal of CO₂-concentration parameter, which serves as auxiliary control
quantity. The space in which may gather many persons in relatively short time,
represents controlled object with thermal conditions for which a predictive controller
response is preferred. The controller handles the auxiliary parameter as time-
depending variable on pre-programmed basis and has a hygienic priority while
setting controller’s output.

1 Work Place Safety And Indoor Air Contamination

1.1. Carbon Dioxide As Air Pollution Contaminant

The goal of any indoor environment control includes the keeping of air pollution contaminants far
from hazardous concentration levels. These levels are known from medical studies and ventilation
rates should push carbon dioxide (CO₂) concentrations below 1000 ppm acceptable to most individuals
[1], [2]. Negative health effects caused by carbon dioxide concentration level by longer time
occupancy within enclosure spreads from tiredness to asphyxiation as it replaces oxygen in the blood.
It is preferred tight or near hermetically closed conditions for mechanically ventilated space: the more
tightness means more accurate indoor CO₂ measurement, more effective control as well as energy
savings.

There is minimal hygienic limit of 27 m³/h.person under exactly defined conditions [2] for an
unadapted person, while for adapted persons it is decreased to 9 m³/h.person. In good agreement are
other recommendations ([4]) that should essentially set control boundary limits. In terms of an air
change (the number of times that the air in a room is fully changed within one hour), ACH value is
often prescribed, and the outdoor air rate can be obtained if the air change is multiplied by the room
volume. Because there is also a certain correlation among the CO₂ amount produced by the gathered
occupant(s) and its/their characteristics: adaptation degree (time period of presence), physical activity,
age and clothing, the air rate estimation was further detailed.

1.2. CO₂-Concentration Monitoring

The carbon dioxide (CO₂) amount, generated by occupants in the controlled enclosed space,
represents one of the three controllable quantities: input air rate \( \dot{m} \) (kg.s⁻¹), the operative temperature
t₀ (°C) and CO₂ concentration (ppm). Under assumption that people gathered in the room are the main
source of air pollution (human metabolism) compared to:

- other internal sources of carbon dioxide (technology devices, animals),
- odor agents, released from building materials,

the amount of both, the water vapor and CO₂ amount could be considered as reference indoor air
parameters. If measured in indoor environment, they indirectly indicate approximately how much
outdoor air is entering an enclosed space in relation to the number of occupants under assumption of
knowing these quantities for the entering air. However, the quality of controlled quantities depends
also on their measurement accuracy, i.e. others appropriate sensors, their placement, control method
and sampling time period; it is always a compromise between the control accuracy demands and
rationally chosen appropriate control system.
The CO\textsubscript{2}-sensor placement was one of decisive tasks in order to get sufficiently reliable and representative measurement values. There are several rules regarding the temperature as well as CO\textsubscript{2} sensors placement with respect to the vertical and horizontal temperature profiles. The CO\textsubscript{2} concentration map basically copies the similar one of water vapor small deviations throughout the enclosure. Stemming from individual sources – occupants, it quickly spreads around (Fig.1), thus the concept of microclimate near the source was useless for positioning the CO\textsubscript{2}-sensor there, not to mention the mobility near the sensor. Rather, upon the indoor air velocity profile estimation were proposed places, in which lesser fluctuations of CO\textsubscript{2} concentration levels were expected, furthermore minimal influences of inlets/outlets including openings (doors, windows), thus minimizing possible CO\textsubscript{2}-measurement of outdoor air. As the low air velocities one would expect in central part of the enclosure, on the other site, the basic requirements dictate to avoid the sensor placement in any boundary layer zone, regardless of character flows (laminar, transient or turbulent one). Finally, on the margins of the occupied zone the sensors were placed, thus avoiding false signals with expectation of obtaining values without any unnecessary delay.

### 1.3. TVOC-Monitoring and Removal Efficiency

Where other odor agents (released from building material, furniture and equipment, etc.) are decisive for outdoor air rate, its minimum value should be used:

\[
R_{TVOC} = \frac{G_{TVOC}}{3.6(\rho_{i,TVOC} - \rho_{e,TVOC})}
\]

In Eq. (1) symbols denote:
- \(R_{TVOC}\) – minimum outdoor air rate per unit surface area (\(\text{dm}^3 \cdot \text{s}^{-1} \cdot \text{m}^{-2}\)),
- \(G_{TVOC}\) – TVOC rate production within the interior per unit surface area (\(\mu\text{g} \cdot \text{h}^{-1} \cdot \text{m}^{-2}\)),
- \(\rho_{i,TVOC}\) – TVOC interior concentration limit (\(\mu\text{g} \cdot \text{m}^{-3}\)),
- \(\rho_{e,TVOC}\) – TVOC concentration in outdoor air (\(\mu\text{g} \cdot \text{m}^{-3}\)),
- TVOC – total volatile organic compounds (ppm).

Then, the required outdoor air rate is the sum of the two air rates (if they occur), i.e. based on CO\textsubscript{2} estimation in previous chapter and calculated from TVOC level. In addition, if air recirculation is used, the outdoor air rate must not be lower than 10 % of supplied air into the room.

**Odor substances material removal efficiency**

Basically, there are three ways of deodorization: chemical, mechanical and biological (introducing plants). There are several known organic as well as anorganic groups of material capable of odor substances removal. The most subtitle group is based on carbon variations: from activated carbon, charcoal to various synthetic materials. The material produced in shapes of blocks, roles, etc., is one of filter active components in air ventilation system.

### 2 Controlling with Auxiliary Control Variable

#### 2.1. PS-Controller (SISO) Adjustment Takahashi Settings

In the following chapter is introduced the conventional PS-controller designed to maintain required operative temperature \(t_o\) in the middle-sized library room. The SISO (single-input-single-output) controller’s temperature sensor is placed at the internal wall out of reach of direct sunshine or possible mobile heat source (computer or any other heat generating device). It provides basic signal of indoor air temperature \(t_o\), while the three other temperature sensors stick to the external wall, opposite and side wall of the room. Their values are summarized and represent the average radiant temperature of surrounding walls \(t_{mrt}\) - the mean radiant temperature as an area mean parameter. Together with indoor air temperature is calculated operative temperature (EN ISO 7730):

\[
t_o = A \cdot t_o + (1 - A) \cdot t_{mrt}
\]
Where \( t_o \) is the operative temperature [\(^\circ\)C], \( t_a \) is the air temperature [\(^\circ\)C], \( t_{med} \) is the mean radiant temperature [\(^\circ\)C], \( A \) is a factor accordance to the relative air velocity (\( A = 0.5 \) for \( var = 0.2 \) m/s, \( A = 0.6 \) for \( var = 0.2-0.6 \) m/s, \( A = 0.7 \) for \( var = 0.6-1.0 \) m/s).

The aim of the control task was to maintain the desired operative temperature 293 K, resp. 294 K (20 or 21 °C) within the occupied zone [3], while the infiltration was assumed to be 0.9 h\(^{-1}\), still above the limit for a non-smoking space as is stated in most national hygienic standards and normatives [2], [5]. Takahashi control algorithm [6] with respect to proportional-sum processing of control deviation \( e \) should minimize the integral functional \( J \) formed from its squares differences:

\[
J(\tau) = \int_{\tau_0}^{\tau} [e(\tau)^T S e(\tau)]
\]

where \( S \) – symmetrical positive-definitive matrix 3 x 3,
\( e \) – deviation vector, \( e(\tau) = w(\tau) - y(\tau). \)

There is an input vector \( e(\tau) \) into controller of deviations of operative temperature and airflow rate and on the controller output \( u(\tau) \) single actuator that serves to change the fan heater output, outdoor airflow rate and flap for recirculating air remain constant, Fig. 1:

\[
\text{Figure 1: The reference space controlled by (a) operative temperature sensor and (b) operative temperature sensor with CO}_2\text{-sensor. 1 – operative temperature sensor, 2 – CO}_2\text{-sensor, 3 – heater unit, 4 – flap of air recirculation unit, 5 – air rate sensor, 6 – central control unit, MIMO-controller (multi-input-multi-output).}
\]

For the control algorithm is utilized time-discretisation form with proportional and sum part (PS), i.e. its continuous formula follows from:

\[
\text{(4)}
\]

\[
\text{where integration is substituted through sum of control deviation } e:
\]

\[
\text{(5)}
\]

\[
\text{with } \Delta \tau \text{ - sampling period (s). Recommended values of both parameters of PS-controller – proportional constant and time constant [6]:}
\]

\[
\text{(6)}
\]

\[
\eta, \tau, \Delta \tau, r_0, K - \text{ variables}
\]
Substituting \( R = K/\tau \), \( K_p = r_0 \), and \( K_i = r_0 \cdot \Delta \tau/\tau \) follows:

\[
K_p = \frac{0.9}{R(\tau + \Delta \tau/2)} \cdot \frac{1}{2} K_i \quad K_i = \frac{0.27 \Delta \tau}{R(\tau + \Delta \tau/2)}
\]  

(7)

The settings with PS-controller were used while simulating heating up the space during daylight period, and adequate change of the average temperature of the external wall, Fig. 2. These transient responses were modeled in MATLAB software with its Simulink toolbox [7].

![Figure 2: Average surface temperature \( t_w \) transient response for heating up the outdoor wall of reference space (unit-step heat load). \( P \) – point of inflexion.](image)

![Figure 3: Mean radiant temperature \( t_{mrt} \) transient response for heating up the reference space (unit-step heat load). \( P \) – point of inflexion.](image)

From the transient responses yields differences between the operative air temperature and the mean radiant temperature in the tested room much smaller than 4 °C. With assumption of low velocities of the indoor air within the occupied zone (< 0.2 m.s\(^{-1}\)), the simplified design operative temperature \( t_o \):

\[
t_o = \frac{1}{2}(t_i + t_{mrt})
\]  

(8)

was used instead of Eq. (2) as the main input onto MIMO-controller (Chap. 2.2).

The simulation test of the tested room response, including periodic (sinus) daytime temperature swing caused by exposing the external wall to the external heat load, allows to determine its thermodynamic characteristics – crucial to the investigated space [3]. Provided that the air in the space
is perfectly stirred (as is usually assumed in models of that kind), the simulation model order $n$ enables to estimate the system time constant together with the dead time (lag):

$$\tau_s = \frac{\tau_{inf} - \tau_u}{n-1} \quad (9)$$

$\tau_s$ – system time constant, $\tau_u$ – time lag and $n$ – system order. The comparison of temperature amplitudes on exterior- and interior-faced outdoor wall, under outdoor climate-driven thermal disturbances, is useful for evaluating gains, if thermal equilibrium would not be sufficiently reached. For the external wall, the overall damping coefficient was:

$$\nu_{uo} = \frac{\Delta t_{uo, e}}{\Delta t_{uo, i}} = 0.19(-) \quad (10)$$

$\Delta t_{uo, e}$ - two amplitudes of temperature harmonic swing on outdoor-faced external wall (K), $\Delta t_{uo, i}$ - two amplitudes of temperature harmonic swing at room-faced inner surface of the external wall (K).

### 2.2. MIMO-Controller with Auxiliary Control Variable

With the CO$_2$-control concept is decisive the CO$_2$ concentration of outdoor air rate while heater maintains the desired operative temperature $t_o$, (eventually indoor air temperature $t_d$). To the rise of CO$_2$ concentration level must be adequate increase of outdoor air rate. It could be done by increasing fan revolving or often by flap position control (Fig. 1), recommended ventilation rates are given for inst. in EN 15251:2007, Annex B.

For this the input vector $\mathbf{e}(\tau)$ into MIMO (multiple-input-multiple-output) controller contains deviations of operative temperature $t_o$, airflow rate $m$ and CO$_2$ concentration $C$, and on the controller output similar vector of actuators $\mathbf{u}(\tau)$ that serve heater $Q$, full fresh-air (outdoor) airflow rate $m$ and flap position $l_a$ for recirculating air (Fig. 1, Chap.2.1). The basic CO$_2$-profile in the tested room was estimated as the space is occupied during daylight hours by number of occupants with patterns of gathering larger number of occupants in certain time periods as is seen in Table 1:

<table>
<thead>
<tr>
<th>Average no of occupants</th>
<th>7-9 A.M.</th>
<th>9-11 A.M.</th>
<th>11 A.M. - 1 P.M.</th>
<th>1-3 P.M.</th>
<th>3-5 P.M.</th>
<th>5-7 P.M.</th>
<th>7-9 P.M.</th>
<th>9 P.M. - 7 A.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>30</td>
<td>50</td>
<td>20</td>
<td>50</td>
<td>40</td>
<td>10</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The apriori information about CO$_2$-concentration level together with the internal heat sources (metabolism heat rate) was employed to minimize afterwards controller’s actions through simultaneous comparison with actual CO$_2$-value. Their difference is set as the auxiliary variable forming the term of deviation vector $\mathbf{e}(\tau)$. The primary controller input represents the operative temperature of the tested space with approximated transfer function of the 3rd-order and different time constants:

$$ F_3(p) = \frac{K_s}{(1 + p\tau_1)(1 + p\tau_2)(1 + p\tau_3)} e^{-p\tau_d} \quad (9)$$

where $K_s$ is system gain and $\tau_1$, $\tau_2$, $\tau_3$ are time constants of the reference room. Employing system order reduction rules [7], the 4th-order equation would substitute the same model with the 4-time multiple time constant $\tau_s$ and with dead time $\tau_{d4}$:

$$ F_4(p) = \frac{K}{(1 + p\tau_s)} e^{-p\tau_d} \quad (10)$$

The reference space transfer function $F_d(p)$ with the transfer function of outdoor airflow rate, resp. CO$_2$-concentration level fill deviation vector $\mathbf{e}(\tau)$. 
3 Closed Loop Control with Auxiliary Variable

3.1. Auxiliary Variable Performance as Function of Transfer Functions

The control concept with auxiliary quantity of CO$_2$ concentration level shows Fig. 4:

![Block scheme of closed control loop with auxiliary control quantity $y_{SH}$ of CO$_2$ concentration. $F_S$ - transfer function of controlled reference space, $F_{SH}$ - transfer function of controlled auxiliary reference space (CO$_2$, TVOC), $F_R$ - PS-controller transfer function, $F_{RH}$ - transfer function of auxiliary controller.](image)

Transfer functions for all four outputs, i.e. [$y_S(t)$, $y_{SH}(t)$] for the tested room space, resp. [$x_S(t)$, $y_{SH}(t)$] for the MIMO-controller were set in order to compose a simulation model of the tested space with the controller. Takahashi rules for the MIMO-controller parameters adjustment remain the same as is described in Chap. 2.1. Disturbance vector $z(t)$ acts together with controller output $x_S$ and control quantity (program) $w$ with controlled quantity $y_S$. Transfer function of auxiliary controller $F_{RH}(p)$ yields from known transfer function of PI-controller:

$$F_{RH}(p) = K_c \left( 1 + \frac{1}{T_i \cdot p} \right) \quad (11)$$

where $K_c$ is controller gain and $T_i$ time integration constant. The transfer function of the reference space of the 3rd order (Eq. (9) with the CO$_2$-concentration transfer function $F_{SH}(p)$ is:

$$F_S(p) = \frac{F_{SH}(p)}{y_R(p)} y_S(p) \quad (12)$$

Thus, the auxiliary control quantity yields:

$$y_{SH}(p) = \frac{F_{SH}(p)}{F_S(p)} y_S(p) \quad (13)$$

Transfer functions of other units in block scheme of closed control loop in Fig. 4 are obtained through disconnecting the circuit in three different circuit points (at $y_S$, $y_{SH}$ and $x_S$). By their substitution back into the condition of closed control loop $z - x_R = x_S$ would be assessed influence of disturbance variable:

$$y_S(p) = \frac{F_S(p)}{1 + F_S(p) F_{SH}(p) F_{RH}(p)} \quad (14)$$

The Eq. (14) finally indicates the influence of auxiliary control variable on the quality of the control process: the denominator became bigger with the term $F_{SH}(p) F_{RH}(p)$, so the control deviation unit becomes smaller in the thermal equilibrium state.

3.2. Results and Conclusion

There was presented a control method with a partially time-predictive variable of occupant(s) production of carbon dioxide as main source pollution. From the hygienic directives (for inst. [2]) the limits of CO$_2$ content (in the modeled case of 1800 mg.m$^{-3}$ (1000 ppm(v))) could be served through fixed airflow rate as well as flap position in air recirculation unit. More efficient approach would control both, the fresh-air flow rate and the recirculated air. To the controlled operative temperature was suitably associated the variable upon which are estimated internal heat sources, odorous particles
and primarily, the carbon dioxide concentration stemming from occupant(s) human metabolism. The measurement of CO\textsubscript{2}-quantity is carried out with regular sampling time to on-line modify default values (Table 1), thus it is providing actual values upon which is based multilevel control: standard temperature control of operative (alternatively indoor air) temperature and simultaneously keeping the limits of CO\textsubscript{2} concentration level. The later parameter – the auxiliary variable – is part of a control process depicted in Fig. 5, green dashed line:

![Figure 5: Indoor air temperature unit-step response for load change of the disturbance variable – heating up of the modeled space.](image)

It shows the transient response of the modeled space to unit-step input of the heating load of the air heater with auxiliary control variable. The difference is shown in the plot in comparison to the transient response on the unit-step of the same disturbance variable (heat load) without auxiliary control function. It represents nearly 2 %-improvement of the deviation from desired indoor air temperature in the equilibrium state and it is accordingly percentage fraction of the saved energy.

References

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