

PID CONTROLLER DESIGN BY MODIFIED ZIEGLER-NICHOLS METHOD

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Abstract

The paper presents PID controller design using modification of the Åström and Hägglund's generalized Ziegler-Nichols rules. The design procedure consists in shifting the plant critical point into a specific point. Its position is defined by desired performance specification in term of achieving maximum overshoot and specified settling time of the closed-loop response or phase margin. The method is illustrated by the example.

1 Introduction

The frequency domain PID controllers tuning is a topic of great interest in the industries. Much of them are issued from [1]. The original Ziegler-Nichols method moves the critical point into a point with fixed coordinates according to the chosen type of controller. For a P-controller it is the point $[-0.5; -0j]$, for a PI controller the point $[-0.45; +0.0896j]$ and for a PID controller is it $[-0.6; -0.28j]$. This method gives relatively oscillating closed-loop performance; therefore various possibilities have been studied how to move the critical point. Up to now, more than 40 modifications were derived [2]. Using these methods, only small information is needed about the plant, however most of these methods can guarantee desired closed-loop performance.

In this paper the controller design have been presented by entered of system critical point and by moving calculating this point on the basis of performance specification. The method pursues from [3] and [4].

2 Design of PID controller

The critical gain and critical frequency can be obtained by several approaches, e.g. by increasing the gain of P controller to ultimate gain at which the output of the control loop oscillates with a constant amplitude.

Consider the following performance specifications:

- A) desired phase margin,
- B) maximum overshoot and settling time of the closed-loop step response.

At first we show how to find the aim point: where to move the plant critical point for A) specification.

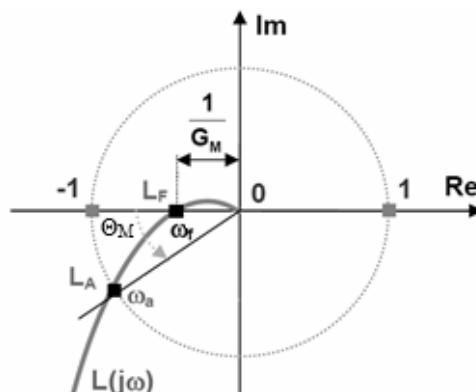


Figure 1: Graphical representation of points L_A and L_F on the Nyquist plot

Consider the Nyquist plot with points L_A and L_F which are shown in Figure 1. The point L_A intersects the Nyquist plot with unit circle. The corresponding frequency for L_A is ω_a with coordinates $[\cos\Theta_M, \sin\Theta_M]$, where Θ_M is the achieved desired phase margin. The point L_F has coordinates $[-1/G_M, 0]$, where G_M is gain margin. In Bode plot, point L_F is phase intersection and L_A is amplitude intersection.

Coordinates of point L_A are the aim of moving the plant critical point. This point is

$$[x, y] = [\cos\Theta_M, \sin\Theta_M]. \quad (1)$$

Now, we show determination of aim point: where to move the plant critical point for B) specification.

A perfect tracking of controlled variable means, that the magnitude of complementary sensitivity function

$$|T(j\omega)| = \left| \frac{G_P(j\omega)G_R(j\omega)}{1 + G_P(j\omega)G_R(j\omega)} \right| = \left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \quad (2)$$

will be equal to one on the full frequency range (i.e. the control variable tracks the reference signal without error). For this purpose it is necessary to choose only specific range of frequency $\langle 0, \omega_p \rangle$, where an increase of bound ω_p causes an increase of amplitude characteristic resonance peak $|T(j\omega)|$ and therefore also maximal magnitude

$$M_t = \sup_{\omega} |T(j\omega)| = \sup_{\omega} \left| \frac{L(j\omega)}{1 + L(j\omega)} \right| = |T(j\omega_r)| \quad (3)$$

In this way the tendency of closed loop vibration increases.

Let for transfer function of open loop holds

$$L(j\omega) = U(\omega) + jV(\omega) \quad (4)$$

Then the magnitude M_t square satisfies

$$M_t^2(\omega) = \frac{U^2(\omega) + V^2(\omega)}{[1 + U(\omega)]^2 + V^2(\omega)} \quad (5)$$

After substituting $\omega = \omega_r$ into (3) and after some modifications we obtain

$$\left[U(\omega_r) + \frac{M_t^2}{M_t^2 - 1} \right]^2 + [V(\omega_r)]^2 = \left[\frac{M_t}{|1 - M_t^2|} \right]^2 \quad (6)$$

The equation (6) gives circle with center in C_T and radius R_T

$$C_T = \left[-\frac{M_t^2}{M_t^2 - 1}, j0 \right] \quad (7)$$

$$R_T = \frac{M_t}{|1 - M_t^2|} \quad (8)$$

Therefore the points which lie in this circle satisfy also condition of maximum overshoot η_{\max} . The magnitude of M_t can be obtained from

$$\eta_{\max} \leq \frac{1.18M_t - T(0)}{T(0)} 100[\%] \quad (9)$$

where $T(0)=1$, if controller has integration component and η_{\max} is given for entered maximum overshoot.

Further condition is settling time. The position of L_A is:

$$L_A: 20 \log |L(j\omega_c)| = 20 \log \sqrt{x^2 + y^2} \quad (10)$$

This position relates with settling time and therefore with position of point $[x, y]$. The frequency of amplitude intersection ω_a can be estimated by relation

$$\omega_a = 10^{\left[\log \sqrt{x^2 + y^2} + \log \omega_c \right]} \quad (11)$$

and settling time can be computed from ω_a as follows

$$t_{reg} = \frac{\gamma}{\omega_a} \quad (12)$$

where γ is curve factor. For aperiodic closed-loop responses $\gamma = 3$ is chosen, for oscillatory output variable response $\gamma = \langle \pi, 4\pi \rangle$ is chosen (see [5]).

Let us denote

$$\frac{\omega_a}{\omega_c} = \sigma. \quad (13)$$

If σ is from interval $\langle 0.5, 0.95 \rangle$ then the closed-loop performance is satisfactory [6].

After substituting (13) into (12) and some manipulation with denominator we obtain

$$t_{reg} \omega_c = \frac{\gamma}{\sigma} = \tau_{reg} \quad (14)$$

where τ_{reg} is relative settling time, which expresses the real settling time weighted by the critical frequency ω_c of controlled plant. It is a dimensionless quantity which enables to express expected closed-loop dynamics for plants with various dynamics.

Let us make some manipulations with (11):

$$\begin{aligned} \omega_a &= \omega_c \sqrt{x^2 + y^2} \\ \frac{\omega_a}{\omega_c} &= \sqrt{x^2 + y^2} = \sigma = \frac{\gamma}{\tau_{reg}} \\ \sigma^2 &= x^2 + y^2 \end{aligned} \quad (15)$$

We have obtained another circle equation (15), this time for specification of settling time.

The aim point which has been found is the intersection of curve (6) and (15) and its coordinates were calculated in MATLAB symbolic toolbox.

The open-loop transfer function can be formulated as follows:

$$L(j\omega_c) = G_s(j\omega_c)G_R(j\omega_c) = x + jy \quad (16)$$

After substituting $[-1/K_c; j0]$ for critical point, it is possible to express the controller transfer function at critical frequency ω_c as complex number

$$G_R(j\omega_c) = \frac{L(j\omega_c)}{G_s(j\omega_c)} = -K_c(x + jy) \quad (17)$$

Consider frequency transfer functions of PI, PD and PID controllers:

$$\begin{aligned} G_R^{PI}(j\omega) &= K \left(1 - j \frac{1}{T_i \omega} \right) \\ G_R^{PD}(j\omega) &= K(1 + jT_d \omega) \\ G_R^{PID}(j\omega) &= K \left[1 + j \left(T_d \omega - \frac{1}{T_i \omega} \right) \right] \end{aligned} \quad (18)$$

then coefficients of individual controllers can be obtained by substitution (17) into (18). Namely, for PI controller:

$$\begin{aligned}
-K_c x &= K \\
-K_c y &= \frac{-K}{T_i \omega_c} \Rightarrow T_i = \frac{K}{K_c y \omega_c}
\end{aligned} \tag{19}$$

For PD controller we have:

$$\begin{aligned}
-K_c x &= K \\
-K_c y &= K T_d \omega_c \Rightarrow T_d = \frac{-K_c y}{K \omega_c}
\end{aligned} \tag{20}$$

and for PID controllers:

$$\begin{aligned}
-K_c x &= K \\
-K_c y &= K T_d \omega_c - \frac{K}{T_i \omega_c}
\end{aligned} \tag{21}$$

If the derivative time constant is given or if $T_i = \beta T_d$ then

$$\begin{aligned}
\beta K \omega_c^2 T_d^2 + \beta K_c \omega_c y T_d - K &= 0 \\
T_d &= \frac{-\beta K_c \omega_c y \pm \sqrt{(\beta K_c \omega_c y)^2 + 4K^2 \beta \omega_c^2}}{2\beta K \omega_c^2} \\
T_i = \beta T_d &= \frac{-\beta K_c \omega_c y \pm \sqrt{(\beta K_c \omega_c y)^2 + 4K^2 \beta \omega_c^2}}{2K \omega_c^2}
\end{aligned} \tag{22}$$

The controller coefficients have to be nonnegative then the following constraints hold:

For PI controller it should be $x < 0$ and $y > 0$. For PD controller $x < 0$ and $y < 0$ have to hold. And for PID controller $x < 0$ holds and sign of y is determined so that $T_d > 0$. If $T_d > 0$, then $T_i = \beta T_d > 0$.

These conditions for position of specific point result from the natural effect of individual controller component. Their effect is depicted in Figure 2.

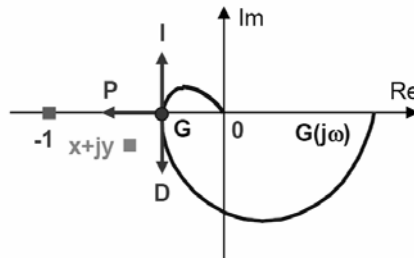


Figure 2: The effect of individual controller component for point moving

3 The program for PID controller design

For PID controller design obtained by modification of the Ziegler-Nichols method a program in MATLAB graphical user interface was made (see Figure 3).

GUI is divided into several parts:

- Input of the plant transfer function $G_s(s)$,
- Input of the desired performance specification (maximum overshoot, specified settling time of the closed-loop response or phase margin),
- window for new location of the points to be moved,
- window of controller coefficients,
- Nyquist plot in the complex plane,
- closed-loop step responses.

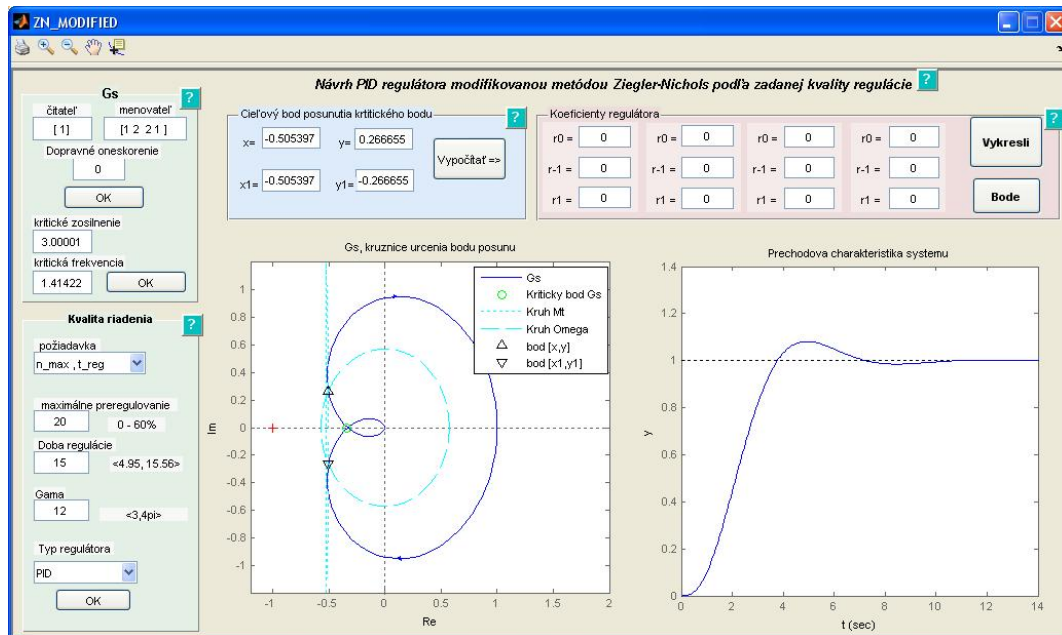


Figure 3: The main window GUI for modified Ziegler-Nichols method

The program use steps:

1) Input of the plant transfer function $G_s(s)$:

at this step, numerator and denominator are verified whether numerical values were inserted. GUI offers the possibility to input transport delay. After pressing „OK“, the critical frequency is computed as well as ultimate gain of the plant. These can be modified. Step response and Nyquist plot in the complex plane are depicted in corresponding windows.

2) Input of desired performance specification

There are two possibilities:

A) phase margin – window with a request on phase margin is shown, with possibility to choose the input value of phase margin,

B) maximum overshoot and specified settling time of the closed-loop response – there are three parameters: desired maximum overshoot, settling time and gamma curve factor.

For all entered values in the part for performance specification, intervals of allowed values are displayed.

After inserting all necessary parameters it is necessary to choose one of PID structures, it is possible to choose PI, PD or PID controller. Button “OK” starts computation, the results are coordinates of aim point (to which the critical point should be moved). On the Nyquist plot and step response, particular curves are shown and aim points are depicted.

3) Window for new location of the moved points:

There can be two points in this window, where the result of the phase margin request is only one point $[x, y]$ and by combination of maximum overshoot and settling time, two points $[x, y]$, $[x1, y1]$ are received.

For study aim it is possible to edit these points.

After pressing button „Vypočítat=>“, all possible controller coefficients are computed, where for PID controller there are at least two. In MATLAB command window all controller transfer functions are written on the background.

By pressing button „Vykresli“ open and closed-loop transfer functions are formed, but only if is the conditions of nonnegative controller coefficients are satisfied. Then Nyquist plot and step responses are depicted, but only for control loop, which are stable. This constraint has practical reason, because nonstable system would require improper scaling.

For verification of the phase margin there is a possibility to depict Bode frequency logarithmic characteristic for built up systems with designed controllers.

4 Example

Consider the plant $G_s(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$

The part of main window GUI for the input of the plant transfer function is depicted in Figure 4 together with its critical gain and critical frequency.

The screenshot shows a GUI window titled "Gs". It contains several input fields and buttons:

- čitateľ** (numerator): 1
- menovateľ** (denominator): 1 2 2 1
- Dopravné oneskorenie** (transport delay): 0
- OK** button
- kritické zosilnenie** (critical gain): 3.00001
- kritická frekvencia** (critical frequency): 1.41422
- OK** button

Figure 4: The window for input of plant transfer function Gs

Let us choose performance specification in the form of phase margin equal to 30° and PID structure of controller (see Figure 5).

The screenshot shows a GUI window titled "Kvalita riadenia". It contains several input fields and buttons:

- požiadavka** (requirement): fázová rezerva
- požadovaná fázová rezerva** (required phase margin): 30
- 0 - 90°** (range)
- Typ regulátora** (controller type): PID
- OK** button

Figure 5: The window for choosing performance specification in the form of phase margin

Computed coordinates of aim point are in the Figure 6:

The screenshot shows a GUI window titled "Cieľový bod posunutia kritického bodu". It contains the following information:

- x =** -0.866024
- y =** -0.500003
- Vypočítať =>** button

Figure 6: Computed coordinates of aim points

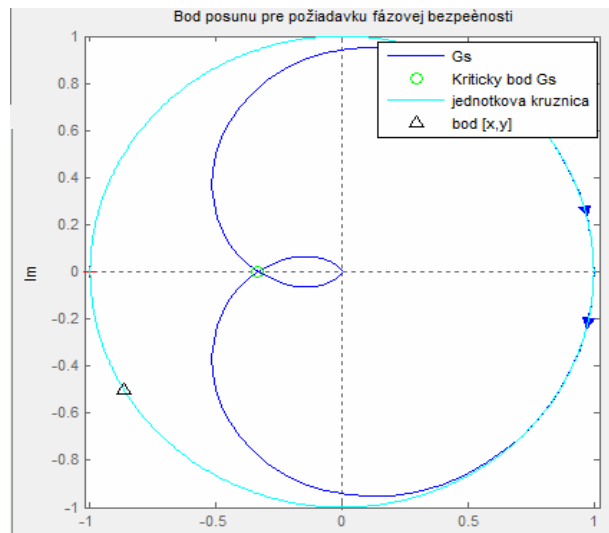


Figure 7: Complex plane, Nyquist plot of G_s , critical point, unit circle and aim point (x,y)

All necessary values and curves are depicted in complex plane in Figure 7.

By pressing the button „Vypočítat“ individual coefficients of controller are depicted. Because there is PID structure, the solutions are two (see Figure 8).

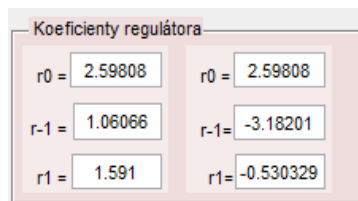


Figure 8: Computed coefficients of PID controller

By pressing the button „Vykresli“ the controller with negative coefficients is red highlighted, what means, that this controller is not suitable for close loop control.

Open loop Nyquist plot is depicted (see Figure 9) in the complex plane and in the second window, close loop step response is shown (see Figure 10). Similarly, it is possible to depict also open loop Bode plot (see Figure 11).

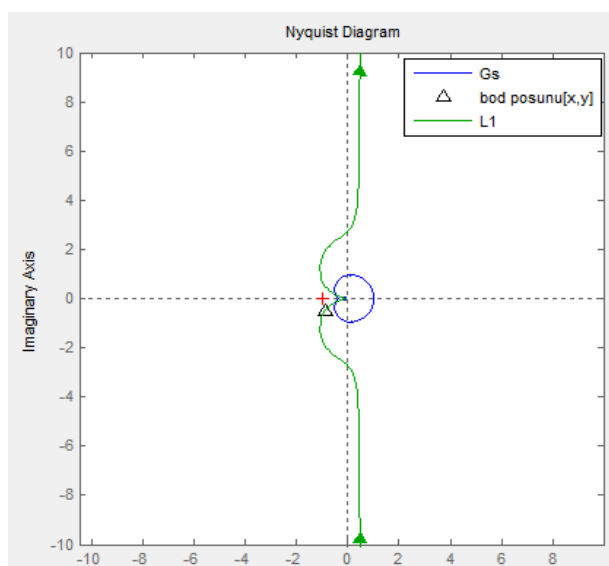


Figure 9: Complex plane with depicted aim point, Nyquist plot of the plant and of the open-loop

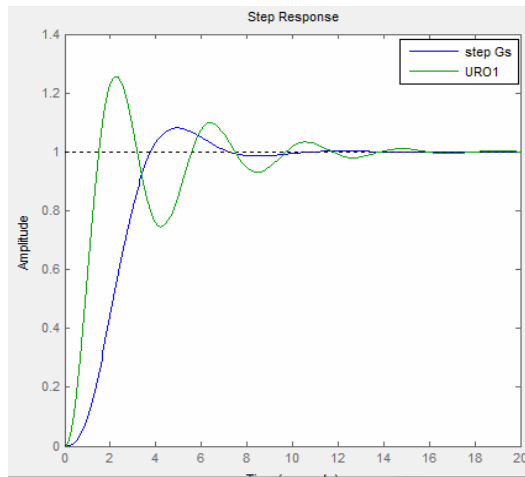


Figure 10: Closed loop step response with PID designed by phase margin and step response of plant $G_s(s)$

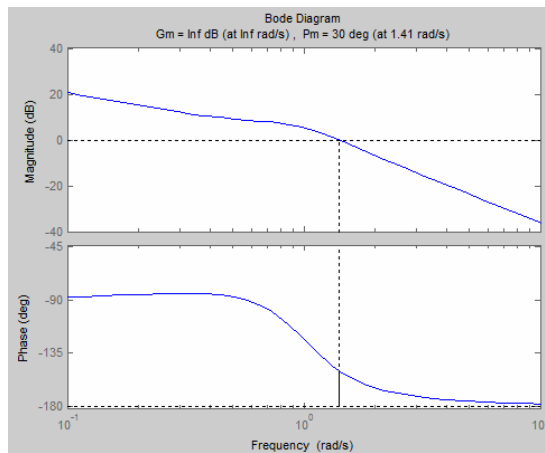


Figure 11: Open loop Bode plot for a designed PID controller with desired phase margin

Similar procedure is when we consider the second possibility of performance specification:

Maximum overshoot 20%, settling time 15s and curve factor 12. The results are shown in the Figure 12 and Figure 13.

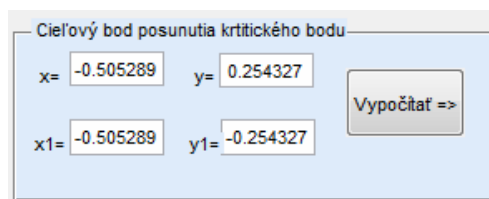


Figure 12: Coordinates of aim point

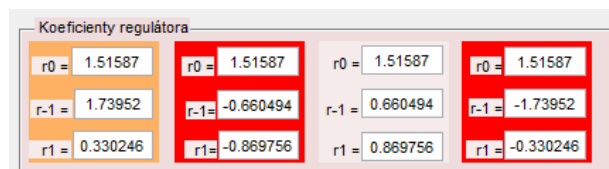


Figure 13: Computed controller coefficients

Two controllers (mark by red colour) have negative coefficients, and therefore they cannot be used for control. The controller marked by orange colour gives non stable closed loop characteristic polynomial. Therefore, only third controller is suitable for control. The closed loop step response only for third controller is shown in the Figure 14.

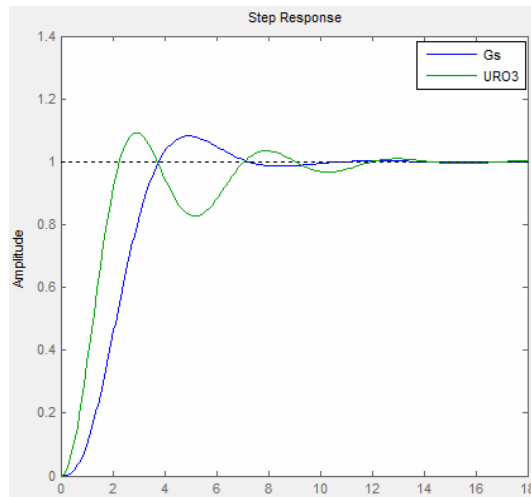


Figure 14: Step response of plant $G_s(s)$ and step response of closed loop for desired maximum overshoot, settling time and gama curve factor.

Designed controllers in both cases satisfy performance specifications. Some of systems are not suitable for the presented method, because they cannot be driven to the stability margin. It is necessary to note that we know only critical attributes, which for some systems do not guarantee appropriate controller design, because it is necessary to know more information.

5 Conclusion

The proposed PID controller design method provides two variants of modification of Ziegler-Nichols method for the improvement of performance specification. The method allows to obtain a desired phase margin or maximum overshoot with settling time. The procedure is designed in MATLAB graphical user interface for arbitrary system. For systems of the first or second orders the method cannot be used because the plant cannot be get at the stability boundary.

References

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