# SPACE VECTOR PWM FOR TWO-PHASE INVERTER IN MATLAB-SIMULINK 

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#### Abstract

This paper deals with design of SVPWM for two-phase inverter in Matlab/Simulink. Nowadays is effort to reduce switching losses and increase efficiency of electrical devices. One of the many possibilities how to deal with this problem is to use inverter driving technique called space vector pulse width modulation (SVPWM).


## 1. SVPWM for two-phase inverter

## Two-phase two leg inverter

There are three possibilities how to create orthogonal two-phase voltage system. Every single one of these three options has advantages and disadvantages. First option is two-phase two leg inverter (Figure 1). The inverter topology consists of four switches, but is necessary to use two capacitors to create common point for common phase of two phase induction machine (TPIM).


Figure 1: Two-phase VSI with two legs.


Figure 2: Space vectors in two-phase VSI with two legs.

Figure 2 shows space vectors in two-phase VSI with two legs. Disadvantage for this topology is inability to turn on zero voltage vectors. Maximal trajectory of reference vector is $\mathrm{U}_{\mathrm{DC}} / 2$. Active voltage vectors are equally shifted by 90 degrees and they divide complex plane into the four operating area [1].

## Two-phase three leg inverter

Second option for creating two-phase orthogonal voltage system is two-phase three leg inverter (Figure 3). The advantage against two-phase two legs is in better utilization of the DC bus. Figure 4 shows space vectors in two-phase four leg inverter. Six active voltage vectors and two zero vectors can be turned on. Active voltage vectors divide complex plane into six operating area. Maximal trajectory of the reference vector is $\mathrm{U}_{\mathrm{DC}} / 1.414$.


Figure 3: Two-phase VSI with three legs.


Figure 4: Space vectors in two-phase VSI with three legs.

## Two-phase four leg inverter

The last option for creating two-phase orthogonal voltage system is VSI with four legs. Topology (Figure 5) consist of eight switches. High number of the switches is the biggest disadvantage of this topology. Thus, the twelve active voltage vectors and four zero vectors can be turned-on. Active voltages are equally shifted by $45^{\circ}$ and divide complex plane into eight operating area. Maximal trajectory of reference vector is $\mathrm{U}_{\mathrm{DC}}$ therefore four leg two-phase inverter has the best utilization of the DC bus [2.].


Figure 5: Two-phase four leg inverter.


Figure 6: Space vectors in two-phase VSI with three legs.

## 2. SVPWM for two-phase three leg inverter

For performing SVPWM it is necessary to determine length and angle of the reference vector. Length of the reference vector can be determined as vector sum of reference voltages [3.][4.].

$$
\begin{gather*}
\left|\bar{U}_{\operatorname{Re} f}\right|=\sqrt{U_{\alpha}^{2}+U_{\beta}^{2}}  \tag{1.}\\
\theta=\tan ^{-1} \frac{U_{\beta}}{U_{\alpha}}=\omega t=2 \pi f t \tag{2.}
\end{gather*}
$$

From the angle of the reference vector can be derived location of the reference vector. Figure 7 shows simple algorithm to determination of the sector where reference vector is located.


Figure 7: Sector determine algorithm.
Switching is performed in one sample period $\mathrm{T}_{\mathrm{S}}$. It means that it is necessary to spend this time to turn on adjacent active voltage vectors to the reference vector. The first adjacent vector $\bar{V}_{1}$ must be turned on in time $\mathrm{T}_{1}$. The second adjacent vector $\bar{V}_{2}$ must be turned on in time $\mathrm{T}_{2}$ [3.].

$$
\begin{equation*}
T_{S} \bar{U}_{\operatorname{Re} f}=T_{1} \bar{V}_{1}+T_{2} \bar{V}_{2} \tag{3.}
\end{equation*}
$$

Equation 3 is correct only if the length of the reference vector is equal to its maximal length. If the length of the reference vector is less than its maximal length, then the sum of times T 1 and T 2 is less than sample period Ts. In that case it is necessary to calculate difference between sample period Ts and sum of T1 and T2. Let's call this difference T0 [5.].

$$
\begin{equation*}
T_{0}=T_{S}-\left(T_{1}+T_{2}\right) \tag{4.}
\end{equation*}
$$

Voltage vectors $\bar{V}_{1}, \bar{V}_{2}$ and $\bar{V}_{\operatorname{Re} f}$ can be written as follows.

$$
\begin{gather*}
\bar{V}_{1}=U_{1} e^{j \theta_{1}}=U_{1}\left(\cos \theta_{1}+j \sin \theta_{1}\right)  \tag{5.}\\
\bar{V}_{2}=U_{2} e^{j \theta_{2}}=U_{2}\left(\cos \theta_{2}+j \sin \theta_{2}\right)  \tag{6.}\\
\bar{V}_{\operatorname{Re} f}=U_{\operatorname{Re} f} e^{j \theta_{\operatorname{Re} f}}=U_{\operatorname{Re} f}\left(\cos \theta_{\operatorname{Re} f}+j \sin \theta_{\operatorname{Re} f}\right) \tag{7.}
\end{gather*}
$$

Zero voltage vectors have no role in creating of the reference voltage vector, thus:

$$
\begin{equation*}
T_{S} \bar{U}_{\operatorname{Re} f}=T_{1} \bar{V}_{1}+T_{2} \bar{V}_{2} \tag{8.}
\end{equation*}
$$

Substituting equations (5. - 7.) in equation (8.)

$$
\begin{align*}
& T_{S}\left(U_{\mathrm{Re} f} \cos \theta_{\mathrm{Re} f}+j U_{\mathrm{Re} f} \sin \theta_{\mathrm{Re} f}\right)=  \tag{9.}\\
& T_{1} U_{1} \cos \theta_{1}+j T_{1} U_{1} \sin \theta_{1}+T_{2} U_{2} \cos \theta_{2}+j T_{2} U_{2} \sin \theta_{2}
\end{align*}
$$

After separating real and imaginary parts and writing in matrix notation we get:

$$
T_{S}\left[\begin{array}{c}
U_{\operatorname{Re} f} \cos \theta_{\operatorname{Re} f}  \tag{10.}\\
U_{\operatorname{Re} f} \sin \theta_{\mathrm{Re} f}
\end{array}\right]=\left[\begin{array}{cc}
U_{1} \cos \theta_{1} & U_{2} \cos \theta_{2} \\
U_{1} \sin \theta_{1} & U_{2} \sin \theta_{2}
\end{array}\right]\left[\begin{array}{c}
T_{1} \\
T_{2}
\end{array}\right]
$$

From equation (10.) we can separate times $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$

$$
\left[\begin{array}{l}
T_{1}  \tag{11.}\\
T_{2}
\end{array}\right]=T_{S}\left[\begin{array}{ll}
U_{1} \cos \theta_{1} & U_{2} \cos \theta_{2} \\
U_{1} \sin \theta_{1} & U_{2} \sin \theta_{2}
\end{array}\right]^{-1}\left[\begin{array}{l}
U_{\mathrm{Re} f} \cos \theta_{\mathrm{Re} f} \\
U_{\mathrm{Re} f} \sin \theta_{\mathrm{Re} f}
\end{array}\right]
$$

Therefore:

$$
\begin{align*}
& T_{1}=T_{S} \frac{U_{\operatorname{Re} f}}{U_{1}} \frac{\sin \left(\theta_{2}-\theta_{\operatorname{Re} f}\right)}{\sin \left(\theta_{2}-\theta_{1}\right)}  \tag{12.}\\
& T_{2}=T_{S} \frac{U_{\operatorname{Re} f}}{U_{2}} \frac{\sin \left(\theta_{\operatorname{Re} f}-\theta_{1}\right)}{\sin \left(\theta_{2}-\theta_{1}\right)} \tag{13.}
\end{align*}
$$

A value of $\mathrm{U} 1, \mathrm{U} 2, \theta 1$ and $\theta 2$ depends on operating area where reference vector is located.
Table 1: PARAMETERS DEFINITION FOR ACTIVE VECTORS

| Sektor | $\overline{\boldsymbol{V}}_{\mathbf{1}}$ | $\overline{\boldsymbol{V}}_{\mathbf{2}}$ | $\boldsymbol{U}_{\mathbf{1}}$ | $\boldsymbol{U}_{\mathbf{2}}$ | $\boldsymbol{\theta}_{\mathbf{1}}$ | $\boldsymbol{\theta}_{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bar{V}_{\mathbf{1}}(1,0,0)$ | $\bar{V}_{2}(1,0,1)$ | 1 | $\sqrt{2}$ | $0^{\circ}$ | $45^{\circ}$ |
| 2 | $\bar{V}_{3}(0,0,1)$ | $\bar{V}_{2}(1,0,1)$ | 1 | $\sqrt{2}$ | $90^{\circ}$ | $45^{\circ}$ |
| 3 | $\bar{V}_{3}(0,0,1)$ | $\bar{V}_{4}(0,1,1)$ | 1 | 1 | $90^{\circ}$ | $180^{\circ}$ |
| 4 | $\bar{V}_{5}(0,1,0)$ | $\bar{V}_{4}(0,1,1)$ | $\sqrt{2}$ | 1 | $225^{\circ}$ | $180^{\circ}$ |
| 5 | $\bar{V}_{5}(0,1,0)$ | $\bar{V}_{6}(1,1,0)$ | $\sqrt{2}$ | 1 | $225^{\circ}$ | $270^{\circ}$ |
| 6 | $\bar{V}_{1}(1,0,0)$ | $\bar{V}_{6}(1,1,0)$ | 1 | 1 | $0^{\circ}$ | $280^{\circ}$ |



Figure 8: Parameters definition for active vectors in simulink.

Figure 8 shows principle of the algorithm for parameters definition for active vectors in simulink. Implemented function in Fcn block for $\mathrm{V}_{1}$ can be for example:
$((u(1)==1) *(1)+(u(1)==2) * 1+(u(1)==3) * 1+(u(1)==6) * 1+(u(1)==4) * 1.414+(u(1)==5) * 1.414)$
The Figure 10 shows times $T_{1}, T_{2}$ and $T_{0}$ for active and zero vectors in one period.


Figure 9: Times $T_{1}, T_{2} T_{0}$ as function of time in one period.


Figure 10: Times calculator in simulink.
In Simulink can be times $T_{1}, T_{2}$ and $T_{0}$ calculated by function shown in Figure 10. Subsystem assign values $V_{1}, V_{2}, \theta_{1}$ and $\theta_{2}$ based on sector where the reference vector is located. Those values can be used as input values for equations 12,13 . and output values from the function are times $T_{1}, T_{2}$ and $T_{0}$.

If times $T_{1}, T_{2}$ and $T_{0}$ are known, active voltage vectors can be assigned to them and it is possible to generate switching signals. There are many ways how to generate switching signals. In this example it is symmetrical switching sequence. For first sector it looks like:

$$
\bar{V}_{0}(0,0,0)-\bar{V}_{1}(1,0,0)-\bar{V}_{2}(1,0,1)-\bar{V}_{7}(1,1,1)-\bar{V}_{2}(1,0,1)-\bar{V}_{1}(1,0,0)-\bar{V}_{0}(0,0,0)
$$

Table 2: SWITCHING SEQUENCE FOR THREE LEG TWO-PHASE INVERTER.

| Secto <br> r | $\frac{T_{0}}{2}$ | $T_{1}$ | $T_{2}$ | $T_{0}$ | $T_{2}$ | $T_{1}$ | $\frac{T_{0}}{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 2 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{2}=(1,0,1)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{0}=(0,0,0)$ |
| 3 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{3}=(0,0,1)$ | $\bar{V}_{0}=(0,0,0)$ |
| 4 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{4}=(0,1,1)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 5 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{5}=(0,1,0)$ | $\bar{V}_{0}=(0,0,0)$ |
| 6 | $\bar{V}_{0}=(0,0,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{7}=(1,1,1)$ | $\bar{V}_{6}=(1,1,0)$ | $\bar{V}_{1}=(1,0,0)$ | $\bar{V}_{0}=(0,0,0)$ |

Because of effort to reduction switching losses there is necessity to optimize switching sequence. Table 2 shows possible switching sequence for two-phase four leg inverter. As we can see, active vectors were chosen with consideration of switching states of switches. There is always only one switch which changes its switching state.

It is possible to derive switching pattern from Table 2. It is necessary to derive it only for upper switches, because lower switches switch complementary to upper.

Table 3: SWITCHING PATTERN FOR TWO-PHASE THREE LEG INVERTER.

|  |  | Sector 1 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 2 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 3 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 4 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 5 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |


|  |  | Sector 6 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Ts |  |  |  | Ts |  |  |  |
|  |  | t0/2 | t1 | t2 | t0/2 | t0/2 | t2 | t1 | t0/2 |
| Upper switches | S1 |  |  |  |  |  |  |  |  |
|  | S3 |  |  |  |  |  |  |  |  |
|  | S5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| Lower switches | S2 |  |  |  |  |  |  |  |  |
|  | S4 |  |  |  |  |  |  |  |  |
|  | S6 |  |  |  |  |  |  |  |  |

From Table 3 it is possible to derive duty cycles for upper switches of two phase three leg inverter. Lower switches switch complementary to upper, thus it is not necessary to determine duty cycles to lower switches of two-phase three leg inverter.


Figure 11: Duty cycles for upper switches for one period.
Figure 11 shows duty cycles for switches in one period. Blue curve shows duty cycle for switch S1. Red curve shows duty cycle for switch S5 and green curve shows duty cycle for switch S3. Switch S3 switches middle leg of inverter. Common point of TPIM is connected to this leg.

Simulation was performed with the simulation step of $1 \mathrm{e}^{-6}$ and sample time $\mathrm{T}_{\mathrm{S}}=0,0002 \mathrm{~s}$. Figure 12 shows stator current at start-up and in steady state. As load of the inverter, asymmetrical two phase induction motor (TPIM) was used.

Table 4: PARAMETERS OF TWO-PHASE INDUCTION MOTOR.

| $\mathrm{R}_{\mathrm{sd}}=61.3 \Omega$ | $\mathrm{R}_{\mathrm{sq}}=68.8 \Omega$ | $\mathrm{R}_{\mathrm{rd}}=87.25 \Omega$ | $\mathrm{R}_{\mathrm{rq}}=109.95 \Omega$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{~L}_{\mathrm{sd}}=1.154 \mathrm{H}$ | $\mathrm{L}_{\mathrm{sq}}=1.645 \mathrm{H}$ | $\mathrm{L}_{\mathrm{rd}}=1.174 \mathrm{H}$ | $\mathrm{L}_{\mathrm{rq}}=1.665 \mathrm{H}$ |
| $\mathrm{M}_{\mathrm{srd}}=1.12 \mathrm{H}$ | $\mathrm{M}_{\mathrm{srq}}=1.6 \mathrm{H}$ | $\mathrm{T}_{\mathrm{N}}=0.15 \mathrm{Nm}$ | $\mathrm{n}_{\mathrm{p}}=1$ |
| $\mathrm{U}_{\mathrm{N}}=230 \mathrm{~V}$ | $\mathrm{f}=50 \mathrm{~Hz}$ | $\mathrm{n}_{\mathrm{N}}=2850 \mathrm{rpm}$ | $\mathrm{J}=25.10^{-5} \mathrm{Kg} \cdot \mathrm{m}^{2}$ |



Figure 12: Stator current at start-up and in steady state.
The simulink tool powergui offers option of Fast Fourier Transformation (FFT). With this tool it is possible to make FFT analysis of stator currents and it can also compute THD of currents. For example stator current in phase $\beta$ has THD equal to value $6,33 \%$ (Figure 13).


Figure 13: FFT analysis of stator current.

## 3. Conclusion

The two-phase space vector PWM for three inverter topologies is proposed in this paper. Principles of the SVPWM for three leg inverter with respect to switching reduction are also described. Switching reduction increase efficiency and this could be the main reason for using of the SPWM.

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