COMPLETE RESPONSE OF CIRCUITS CONTAINING OPERATIONAL AMPLIFIERS IN MATLAB

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Abstract

Finding the complete response of circuits having operational amplifiers is time-consuming activity also in case when the operational amplifiers are analyzed as black boxes. It means that only terminal behavior of operational amplifiers is taken into account and it is not attended to the internal characteristic of the operational amplifiers. Such problem could be done by transforming the circuit directly into the complex frequency domain using the Laplace transform and then writing and solving node equations. The better way is to apply the Laplace transform together with the sparse tableau analysis and symbolic computation of MATLAB, because formulation of the circuit equations is done in systematic and automatic way for such type of circuits and there is no difficulty in writing equations as well as solving circuits with operational amplifiers.

1 Principles of Finding Complete Response of Electric Circuits Containing Operational Amplifiers

Finding complete response of electric circuits containing operational amplifiers implies problem of solving dynamic circuits. Analysis of these circuits is sometimes time-consuming activity, because the equations for such type of circuits take the form of integrodifferential equations.

One of the tools for the analysis of these circuits is to transform the circuit directly into the complex frequency domain (s-domain) using the Laplace transform. The Laplace transform for a function $x(t)$ of a real variable t (meaning the time) is defined as [1], [2]

$$\hat{x}(s) = \pounds\left\{x(t)\right\} = \int_{0}^{\infty} x(t)e^{-st}\mathrm{d}t\,, \tag{1}$$

where s is a complex variable (the complex frequency $s = \sigma + \mathrm{j}\omega$, where σ, ω are real, $\mathrm{j}=\sqrt{-1}$ is an imaginary unit), and $x(t)$ is zero for all $t < 0$.

After transforming a given circuit with its initial conditions into its the s-domain equivalent, it can be dealt with it as if it consisted of sources and resistors only, because the passive elements have impedances, which can be regarded as generalized resistances.

In order to find all branch currents and voltages it is necessary to obtain a complete description of a general circuit having arbitrary elements in the s-domain by writing a set of simultaneous equations. The set of these equations consists of the equations that depend on the topology of the circuit and the equations that depend on the type of the circuit elements. The equations, depending on the topology of the circuit, represent how the circuit elements are connected to one another in the circuit. These equations are called the connection equations. The equations, which depend on the type of the circuit element, describe the voltage-current relationship and they are called the element equations or branch equations. If the voltage-current characteristics for two-terminal elements and two-port elements in the circuit are linear, and time-invariant, then the set of the equations describing the circuit in s-domain, is also linear.

Sparse tableau analysis is a very useful method for systematic and automatic formulation of the circuit equations for every circuit as well as for circuits containing operational amplifiers. Sparse tableau analysis is most general formulation of the equations describing the circuit because the solution provides the currents through all elements, the voltages across all elements and all nodal voltages simultaneously.

For s-domain equivalent circuit the connection equations are obtained by applying Kirchhoff's laws to the circuit, which leads to the two sets of linear algebraic equations in terms of the branch currents and the branch voltages. The set of connection equations must be linearly independent.

The first set of the connection equations can be expressed [1], [3]:

$$\mathbf{A}\hat{\mathbf{i}} = \mathbf{0}\,, \tag{2}$$

where $\hat{\mathbf{i}}$ is a matrix of s-domain branch currents, $\mathbf{A}$ being the node versus branch reduced-incidence matrix.

The second set of the connection equations can be expressed in terms of the s-domain branch voltages using the fundamental loop versus branch incidence matrix. The better way is to convey this set of the equations in terms of the s-domain branch voltages and the s-domain node voltages as [3]:

$$\hat{\mathbf{u}} = \mathbf{A}^{\mathrm{T}}\hat{\mathbf{v}}\,, \tag{3}$$

where $\mathbf{A}^{\mathrm{T}}$ is the transposed matrix $\mathbf{A}$, $\hat{\mathbf{u}}$ being the s-domain branch voltage vector, $\hat{\mathbf{v}}$ being the s-domain node voltage vector.

The element equations are related according to the voltage-current characteristics of the elements. For linear circuits, containing the ideal operational amplifiers too, the element equations can be expressed [3]:

$$\hat{\mathbf{K}}_{\mathrm{u}}\hat{\mathbf{u}} + \hat{\mathbf{K}}_{\mathrm{i}}\hat{\mathbf{i}} = \hat{\mathbf{z}}\,, \tag{4}$$

where $\hat{\mathbf{K}}_{\mathrm{u}}, \hat{\mathbf{K}}_{\mathrm{i}}$ are the matrices containing the coefficients that define the s-domain linear voltage-current relationships for the circuit elements uniquely, $\hat{\mathbf{z}}$ being the vector containing the s-domain parameters of the independent voltage and current sources.

Assembling the equations (2), (3), and (4), the sparse tableau equations are constituted. It is convenient to rewrite the sparse tableau equations as a single matrix equation [3]:

$$\hat{\mathbf{T}}\hat{\mathbf{x}} = \hat{\mathbf{w}}\,, \tag{5}$$

where $\hat{\mathbf{T}}$ is the square tableau matrix, $\hat{\mathbf{x}}$ being the vector of unknown variables, $\hat{\mathbf{w}}$ being the vector containing zero vectors of appropriate dimensions and the vector $\hat{\mathbf{z}}$.

The operational amplifier is an electronic circuit element that is designed for using with other circuit elements to perform a specified signal-processing function. In order to solve the circuits containing operational amplifiers it is necessary to create a model of operational amplifier. Several models of operational amplifiers, of varying accuracy and complexity, are available for operational amplifiers [1]. However, when the black-box approach is chosen, the simplest model of the operational amplifier can be used. This model is called as the ideal operational amplifier and it is shown in Figure 1.

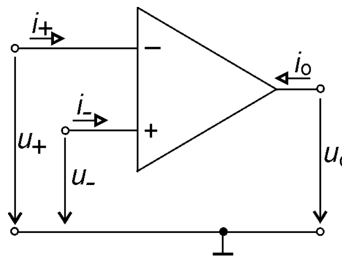

Figure 1: The ideal operational amplifier

For the ideal operational amplifier the following two conditions are valid.

The first condition implies that the input resistance between the two input terminals of the ideal operational amplifier is assumed zero. In this case, the input terminals can be considered as being open in terms of current in the sense that no current flows into or out of them [1]:

$$i_{+} = 0 \text{ and } i_{-} = 0\,, \tag{6}$$

where i_{+}, i_{-} are the currents flowing into input positive, negative terminal, respectively.

The second condition implies that the two input terminals of the ideal OP amp with a negative feedback path can be considered as being short in terms of voltages in the sense that the voltage levels at the two input terminals are almost equal [1]:

$$u_+ \cong u_- , \tag{7}$$

where u_+, u_- are the voltages at input positive, negative terminal, respectively.

After solving the set of the sparse tableau equations, the s-domain expression of the branch currents, the branch voltages, and the node voltages is obtained. To get the t-domain solution, which represents the complete response of the given circuit, it is necessary to take the inverse Laplace transform of the s-domain solution. The inverse Laplace transform for a function $\hat{x}(s)$ of a complex variable s is defined as [1], [2]

$$x(t) = \pounds^{-1}\{\hat{x}(s)\} = \frac{1}{2\pi \mathrm{j}} \int_{\sigma - \mathrm{j}\infty}^{\sigma + \mathrm{j}\infty} \hat{x}(s) e^{st} \mathrm{d}s , \tag{8}$$

where t is a real variable, and $x(t)$ exists for $t \geq 0$.

All the above given steps which must be done to solve the circuits containing operational amplifiers can be easily executed using MATLAB, especially the symbolic computation.

2 Results

The proposed procedure for finding complete response of circuit containing operational amplifiers was applied to the circuit shown in Figure 2. In this circuit, all the initial conditions are assumed zero and the values of the input voltage, the resistors, and the capacitors are $u_{\text{in}} = 1$ V, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $C_1 = 0.25$ mF, $C_2 = 16$ µF [1]. Since all initial conditions are assumed zero, the complete response equals to the transient response of the circuit to unity step function applied in $t = 0$.

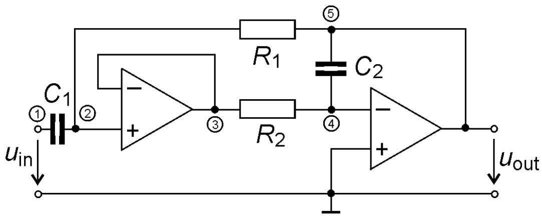

Figure 2: The circuit for finding the complete response

All the computations for analysis of the circuit (with given parameters) depicted in Figure 2 were done by running MATLAB program. The time domain representation of the voltage u_{out} of the operational amplifier for the time $t \geq 0$ was determined in symbolic form (Figure 3 left) and in graphical form (Figure 3 right).

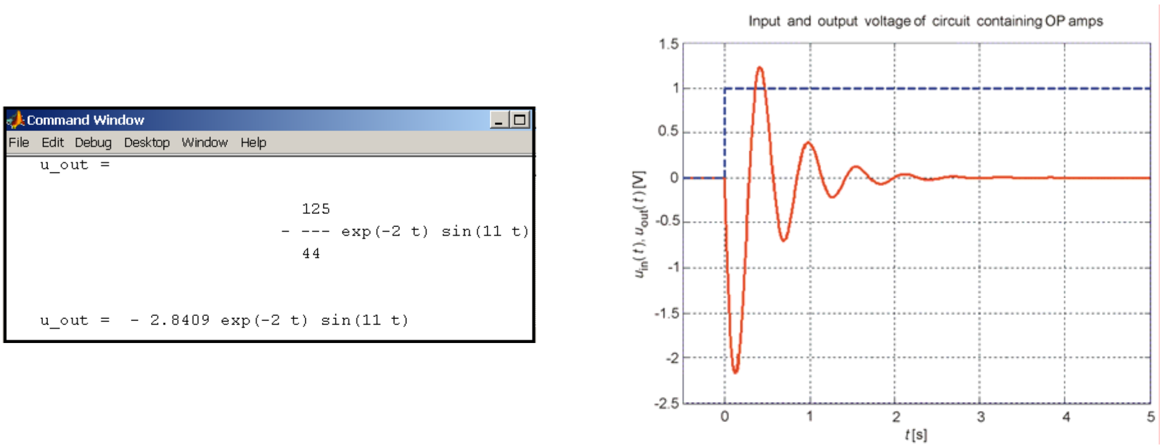

Figure 3: The output voltage u_{out} of the operational amplifier expressed in symbolic form (left) and in graphical form (right)

Using MATLAB saves a lot of time and effort thus everybody who uses it can concentrate on gaining insights, and making conclusions.

References

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