ROBUST DECENTRALIZED CONTROL
OF LARGE SCALE SYSTEMS

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Summary

In this paper, a new methodology is proposed for longitudial headway control design of platoons of automotive vehicles. It is based on the generalization of the Inclusion Principle to nominally linear dynamic systems with time-varying uncertain system parameters described in the state space. No statistical information about the uncertainty is required, only its bound is supposed to be known. Necessary and sufficient conditions for extensions and for contractibility of linear controllers are given as a necessary theoretical background. Expansions of input, state and output spaces are considered. Numerical simulatuion results show that the proposed methodology provides a reliable tool for systematic control design of longitudial vehicle controllers in general.

1. INTRODUCTION

A complex system is usually considered as a set of interconnected systems. The motivating reasons and the advantages offered by such approach when analyzing and designing controllers have been discussed by many authors. They are surveyed for instance in Bakule and Lunze (1988) and Šiljak (1991). The difficulties generated by particular features of complex systems such as high dimensionality, information structure constraint and uncertainty have motivated the development of new methodologies, such as decomposition, decentralization and robustness. These methodologies have been developed systematically since early seventies.

Overlapping decompositions are an useful mean of designing decentralized controllers, mainly in cases when disjoint decompositions fail. The controller design using the Inclusion Principle (Ikeda and Šiljak, 1980; Šiljak, 1991; Bakule et al., 2000a, 2000b) starts with expanding input, state and output spaces of the original system with overlapping subsystems into expanded larger spaces in which these subsystems appear disjoint. Decentralized control laws are designed in these larger spaces and then contracted into original spaces for implementation. It concerns both large and small scale systems. The Inclusion Principle has been adapted to stochastic systems, reduced-order design, dynamic output feedback, hereditary systems, time-varying systems, power and mechanical systems, longitudial headway control (Iftar, 1990; Šiljak, 1991; Bakule and Rodellar, 1995; Stanković et al., 1998).

This paper deals with a new extension of the Inclusion Principle to uncertain dynamic systems. The motivation for such generalization lies in the fact that, though the concept of extension has been introduced as an effective tool for constructing contractible controllers without any structural constraint when implementing expanded controllers to original systems (Iftar and Özgüner, 1990), it has not been specialized to uncertain systems up to now. Stabilization of uncertain systems via deterministic control derived from a constructive use of Lyapunov stability theory lies in one of the important methodology for robust control. Ikeda and Šiljak (1986) extended the expansion/contraction process to input, state and output inclusion in the state space. Iftar (1993) presented dynamic output feedback controller design for linear time-invariant systems.

The objective of this paper is to present a general methodology for decentralized stabilization using overlapping decompositions without any restrictions on constructing contractible linear controllers using extensions and including subsystems with overlapping input, state and output spaces for uncertain nominally linear systems, which provides a reliable tool for longitudial control design of platoons of automotive vehicles. As the author's knowledge, this problem has not been solved yet. The presented solution enables to construct block tridiagonal feedback controllers for uncertain systems. Numerical simulation results illustrate the advantages of this methodology.

2. THE PROBLEM

Consider the uncertain time-varying systems described by

$$\begin{aligned} S: \dot{x} &= (A + \sum_{i=1}^{i_r} A_i r_i(t))x + (B + \sum_{i=1}^{i_s} B_i s_i(t))u, \\ y &= (C + \sum_{i=1}^{i_v} C_i v_i(t))x, \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} S_e: \dot{x}_e &= (A_e + \sum_{i=1}^{i_{re}} A_{ei} r_{ei}(t))x_e + (B_e + \sum_{i=1}^{i_{se}} B_{ei} s_{ei}(t))u_e, \\ y_e &= (C_e + \sum_{i=1}^{i_{ve}} C_{ei} v_{ei}(t))x_e, \end{aligned} \tag{2.2}$$

where the n dimensional vector x represents the states, u is the m dimensional input vector and y is the p dimensional output vector of the system (2.1). A, A_i, B, B_i, C, C_i are constant matrices of appropriate dimensions. A, B, C and A_i, B_i, C_i correspond with nominal and uncertain part of the system S, respectively. Denote from now on $\Delta A(r(t)) = \sum_{i=1}^{i_r} A_i r_i(t)$, $\Delta B(s(t)) = \sum_{i=1}^{i_s} B_i s_i(t), \Delta C(v(t)) = \sum_{i=1}^{i_v} C_i v_i(t)$ time-varying perturbation matrices. $r = (r_1, ..., r_{i_r}), s = (s_1, ..., s_{i_s}), v = (v_1, ..., v_{i_v})$ are supposed to be fixed but unknown Lebesgue measurable functions $r: \mathbb{R} \to R_r \subset \mathbb{R}^{i_r}$, $s: \mathbb{R} \to R_s \subset \mathbb{R}^{i_s}$, $v: \mathbb{R} \to R_v \subset \mathbb{R}^{i_v}$, where R_r, R_s, R_v are compact bounding sets of the form

$$\begin{aligned} R_r &= \{r : r \in \mathbb{R}^{i_r}, |r_i| \le \overline{r}_i; i = 1, ..., i_r\} \\ R_s &= \{s : s \in \mathbb{R}^{i_s}, |s_i| \le \overline{s}_i; i = 1, ..., i_s\}, \\ R_v &= \{v : v \in \mathbb{R}^{i_v}, |v_i| \le \overline{v}_i; i = 1, ..., i_v\}, \end{aligned} \tag{2.3}$$

Analogously, the n_e dimensional vector x_e represents the states, u_e is the m_e dimensional input vector and y_e is the p_e dimensional output vector of the system (2.2). $A_e, A_{ei}, B_e, B_{ei}, C_e, C_{ei}$ are constant matrices of appropriate dimensions, where A_e, B_e, C_e and A_{ei}, B_{ei}, C_{ei} correspond with nominal and uncertain part of the system S_e, respectively. Further, denote $\Delta A_e(r_e(t)) = \sum_{i=1}^{i_{re}} A_{ei} r_{ei}(t), \Delta B_e(s_e(t)) = \sum_{i=1}^{i_{se}} B_{ei} s_{ei}(t)$, $\Delta C_e(v_e(t)) = \sum_{i=1}^{i_{ve}} C_{ei} v_{ei}(t)$ time-varying perturbation matrices. $r_e = (r_{e1}, ..., r_{ei_{re}})$,

$s_e = (s_{e1}, ..., s_{ei_{se}})$, $v_e = (v_{e1}, ..., v_{ei_{ve}})$ are supposed to be fixed but unknown Lebesgue measurable functions $r_e : \mathbb{R} \to R_{re} \subset \mathbb{R}^{i_{re}}$, $s_e : \mathbb{R} \to R_{se} \subset \mathbb{R}^{i_{se}}$, $v_e : \mathbb{R} \to R_{ve} \subset \mathbb{R}^{i_{ve}}$, where R_{re}, R_{se}, R_{ve} are compact bounding sets of the form

$$\begin{aligned} R_{re} &= \{r_e : r_e \in \mathbb{R}^{i_{re}}, |r_{ei}| \leq \overline{r}_{ei}; i = 1, ..., i_{re}\}, \\ R_{se} &= \{s_e : s_e \in \mathbb{R}^{i_{se}}, |s_{ei}| \leq \overline{s}_{ei}; i = 1, ..., i_{se}\}, \\ R_{ve} &= \{v_i : v_e \in \mathbb{R}^{i_{ve}}, |v_{ei}| \leq \overline{v}_{ei}; i = 1, ..., i_{ve}\}. \end{aligned} \tag{2.4}$$

The objective is to present a general theory of feedback control design for systems described by Eqs.(2.1) and (2.2) using overlapping decompositions.

Denote from now on the matrices $A_u(t) = A + \Delta A(r(t))$, $A_{ue}(t) = A_e + \Delta A_e(r_e(t))$, $B_u(t) = B + \Delta B(s(t))$, $B_{ue}(t) = B_e + \Delta B_e(s_e(t))$ and $C_u(t) = C + \Delta C(v(t))$, $C_{ue}(t) = C_e + \Delta C_e(v_e(t))$.

3. BACKGROUND - THEORY

The way of the solution proceeds as follows. First, the expansion process between systems S and S_e is presented in terms of extension and disextension. Second, conditions for contractible controllers with arbitrary feedback structure are given. Third, overlapping decompositions for systems of the form (2.1),(2.2) are presented for a considered prototype case and relations between rank 1 uncertainty matrices of systems S and S_e are given for this case.

3.1. EXTENSION/DISEXTENSION OF SYSTEMS

Consider the transformation matrices between the systems S and S_e satisfying the following assumption.

Assumption A1: $x_e = Tx, u_e = U^+u, y_e = Gy, x = T^+x_e, u = Uu_e, y = G^+y_e, T^+T = I_n, UU^+ = I_m, G^+G = I_p, rank(T) = n, rank(U) = m, rank(G) = p$. $I_{(.)}$ denotes the (.) dimensional identity matrix.

Definition 1: The system S_e is an *extension* of the system S, and S is a *disextension* of S_e, if there exist transformations T, U, G satisfying A1 such that for any initial state $x(0)$ and for any input $u_e(t) \in \mathbb{R}^{m_e}, t \geq 0$, the relations $x_e(0) = Tx(0), u(t) = Uu_e(t)$ imply that $x_e(t) = Tx(t), y_e(t) = Gy(t)$ for all $t \geq 0$.

Necessary and sufficient conditions for the extension of linear time-invariant system are given, for instance, in Iftar (1993). We extend these conditions for uncertain systems by the following theorem.

Theorem 1: The system S_e is an extension of the system S, or equivalently S is a disextension of S_e, if and only if there exist full-rank transformation matrices T, U, G satisfying (3.1) such that

$$\begin{aligned} (A_e + \Delta A_e(r_e(t)))T &= T(A + \Delta A(r(t))), \\ T(B + \Delta B(s(t)))U &= B_e + \Delta B_e(s_e(t)), \\ G(C + \Delta C(v(t))) &= (C_e + \Delta C_e(v_e(t)))T, \qquad \forall t \geq 0, \end{aligned} \tag{3.4}$$

for any uncertain functions r, r_e, s, s_e, v, v_e satisfying (2.3) and (2.4).

A logical consequence of Theorem 1 is a sufficient condition as follows.

Theorem 2: The system S_e is an extension of the system S, or equivalently S is a disextension of S_e, if and onlu if there exist full-rank transformation matrices T, U, G satisfying (3.1) such that

$$
\begin{aligned}
A_e T = TA, \qquad TBU = B_e, \qquad GC = C_e T, \\
\Delta A_e(r_e(t))T = T\Delta A(r(t)), \qquad \Delta B_e(s_e(t))T = T\Delta B(s(t)), \\
G\Delta C(v(t)) = \Delta C_e(v_e(t))T, \quad \forall t \geq 0,
\end{aligned} \tag{3.5}
$$

for any uncertain functions r, r_e, s, s_e, v, v_e satisfying (2.3) and (2.4).

Suppose that the matrices of systems S and S_e can be related as follows:

$$
A_{ue}(t) = TA_u(t)T^+ + M_u(t), B_{ue}(t) = TB_u(t)U + N_u(t), C_{ue}(t) = GC_{ue}(t)T^+ + P_u(t), \tag{3.7}
$$

where $M_u(t), N_u(t), P_u(t)$ are complementary matrices. For S_e to be an extension of S, these complementary matrices must satisfy certain conditions given by the following theorem.

Theorem 3: Suppose that the matrices in the systems S, S_e and the transformation matrices defined by Eq.(3.1) are related as given by Eq.(3.7). Then the system S_e is an extension of the system S if and only if $M_u(t)T = 0, N_u(t) = 0, P_u(t)T = 0$ for all $t \geq 0$ and for uncertain functions r, r_e, s, s_e, v, v_e satisfying (2.3) and (2.4).

Consider now the matrix $M_u(t)$ decomposed in the form $M_u(t) = M + M_d(t)$, where M corresponds with the nominal part of the system S and $M_d(t)$ with its uncertain part. Similarly, consider $P_u(t) = P + P_d(t)$. We can formulate the following condition.

Theorem 4: Suppose that matrices in the systems S, S_e and the transformation matrices defined by Eq.(3.1) are related as given by Eq.(3.7). Then the system S_e is an extension of the system S if and only if $MT = 0, PT = 0, M_d(t)T = 0, N_u(t) = 0, P_d(t)T = 0$ for all $t \geq 0$ and for uncertain functions r, r_e, s, s_e, v, v_e satisfying (2.3) and (2.4).

3.2. CONTRACTIBILITY OF CONTROLLERS

Consider feedback control now. Since we deal with a stabilization problem, we consider static control laws in the form $u = Fy$ and $u_e = F_e y_e$ for the systems S and S_e, respectively.

We are interested in designing control laws for the extended system that can be implementable into the original system. The concept of contractibility of controller F_e into F is defined now as a concept contributing to this objective.

Definition 2: The control law $u_e = F_e y_e$ is *contractible* into the control law $u = Fy$ if there exist full rank transformations T, U satisfying A1 such that, for any initial states $x(0), x_e(0)$ and controls u, u_e verifying $x_e(0) = Tx(0), u(t) = Uu_e(t)$, the relation $Fy(t; x(0), u) = UF_e y_e(t; x_e(0), u_e)$ holds.

Note here that the use of any inclusion concept other than extension can result in non contractible controller (Iftar and Özgüner, 1990; Ikeda and Šiljak, 1986). Sufficient conditions for the contractibility of the control laws using the extension defined in the previous section are given by the following theorem.

Theorem 5: Suppose that the system S_e in (2.2) is an extension of the system S in (2.1). The control law $u_e = F_e y_e$ is contractible to the control law $u = Fy$ in the system S if $FC = UF_e GC, F\Delta C(v(t)) = UF_e G\Delta C(v(t))$ for all $t \geq 0$ and for uncertain function v satisfying (2.3).

Stability in the context of extension of the systems S and S_e in (2.1) is determined by the following theorem.

Theorem 6: If S_e is an extension of S and S_e is a stable (respectively asymptotically stable) system, then S is a stable (respectively asymptotically stable) system.

Thus the (asymptotic) stability of the system S_e guarantees the (asymptotic) stability of the original uncertain system. Though the relation is presented for open–loop systems, it also holds for closed–loop systems provided that both systems are in extension/disextension relation.
Theorem 7: If S_e in (2.2) is an extension of S in (2.1) and control law $u_e = F_e y_e$ is contractible to $u = Fy$, then the closed–loop system

$$\begin{aligned} S_{ec} : \dot{x}_e &= (A_e + \Delta A_e(r_e(t)) + (B_e + \Delta B_e(s_e(t)))F_e(C_e + \Delta C_e(v_e(t))))x_e \\ &= (A_{ec} + \sum_{i=1}^{i_{ecr}} A_{eci} r_{eci}(t))x_e \end{aligned} \tag{3.14}$$

is an extension of the closed–loop system

$$\begin{aligned} S_c : \dot{x} &= (A + \Delta A(r(t)) + (B + \Delta B(s(t)))F(C + \Delta C(v(t))))x \\ &= (A_c + \sum_{i=1}^{i_{cr}} A_{ci} r_{ci}(t))x. \end{aligned} \tag{3.15}$$

4. HEADWAY CONTROL OF A PLATOON OF VEHICLES - SIMULATION EXPERIENCE

We consider the error regulation problem of a string of moving vehicles by Ikeda and Šiljak (1986). They consider a string of four vehicles as a representative situation for strings of any length. The motion of each vehicle is described by two states: position and velocity. The equation of motion of the string can be written in terms of the deviations from a given nominal distance between adjacent vehicles and the deviations from a nominal string velocity. The model is given in normalized (dimensionless) distances, velocities, input forces and nominal model parameters. All states are supposed to be available as outputs. In contrast to that work, we consider here a string of moving vehicles with uncertain model parameters and design a static output feedback controller.
Formulation: Consider the system (4.1) described in accordance with (2.1),(2.3) with $i_r = i_s = 4$ and $i_v = 1$. Uncertainty function r_i denotes the deviation from the normalized reciprocal value of the mass of the i-th vehicle. This mass enters equally into the input matrix (Bakule and Lunze, 1988). Thus $r_i(t) = s_i(t)$. v_i is the sensor uncertainty in the distance between the 1st and the 2nd vehicles. All uncertainties are supposed to be maximally 13% of the parameter deviations from their nominal values. Thus, their bounds on uncertainty of the original system are selected as follows: $\overline{r}_1 = \overline{r}_2 = \overline{r}_3 = \overline{r}_4 = \overline{v}_1 = 0.13$.

The normalized states are $x = (w_1, d_{12}, w_2, d_{23}, w_3, d_{34}, w_4)^T$. w_i denotes the velocity deviation of the i-th vehicle, $d_{i,i+1}$ is the distance between the i-th and $(i+1)$-th vehicles. The normalized control inputs are $u = (u_1, u_2, u_3, u_4)^T$, where u_i is the force applied to the i-th vehicle. The normalized outputs are $y = (d_{12}, w_2, d_{23}, w_3, d_{34})^T$.

The objective is to construct decentralized static output feedback controller.
Solution: The solution proceeds as follows: First, an overlapping decomposition is defined by matrices satisfying A1. Second, local output static controllers are designed for the extended system so that its nominal closed-loop system is stable. Third, uncertainty bounds are computed for such stable system. The last step is to contract the controller for the implementation on the original system. To illustrate the procedure, simulation verification is supplied.

$$
\begin{gathered}
A=\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, B=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
C=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, A_1=\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
A_2=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, A_3=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
A_4=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, B_1=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
B_2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B_3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B_4=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
C_1=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
\end{gathered}
\tag{4.1}
$$

Overlapping decomposition is considered for the string of four vehicles into three nonoverlapping systems in extension in accordance with Ikeda and Šiljak (1986). Using the matrix $A = (a_{ij})$ in (4.1), overlapping appears in its elements a_{33} and a_{55} as indicated by the broken lines in (4.1). In our case, the original system is considered as including two different overlapped parts. Therefore, the matrix A is considered as composed of submatrices as follows: $A = (A_{i,j}),\quad i,j = 1,...,5$, where $dimA_{1,1} = dimA_{5,5} = 2 \times 2, dimA_{2,2} = dimA_{3,3} = dimA_{4,4} = 1$. Note that using this partitioning of A, the overlapped parts are the submatrices $A_{2,2} = a_{33}$ and $A_{4,4} = a_{55}$. The same reasoning as for matrix A holds for all uncertainty matrices A_i given by (4.1). In matrix $B = (b_{ij})$ in (4.1) the overlapping appears in elements b_{32} and b_{53}. No partitioning is necessary because the complementary matrix for B is zero. Denoting $C = (c_{ij})$, overlapping appears in elements c_{23} and c_{45}. Consider its partitioning into submatrices as follows: $C = (C_{i,j}),\quad i = 1, ..., 7; j = 1, ..., 5$, where $dimC_{1,1} = dimC_{5,5} = 1 \times 2, dimC_{2,2} = dimC_{3,3} = dimC_{4,4} = 1$. Consider C_1 in (4.1) partitioned in the same way as C. Denote the complementary matrices $M_d(t) = \sum_{i=1}^{i_r} A_i r_i(t)$ and $P_d(t) = P_1 v_1(t)$ corresponding to the structure of $\Delta A(t)$ and $\Delta C(t)$ in (2.1).With these partitioning, the transformation matrices T, U, G and the complementary matrices can be described as follows:

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, [.] = \begin{pmatrix} 0 & 0.5[.]_{12} & -0.5[.]_{12} & 0 & 0 & 0 & 0 \\ 0 & 0.5[.]_{22} & -0.5[.]_{22} & 0 & [.]_{24} & -[.]_{24} & 0 \\ 0 & -0.5[.]_{22} & 0.5[.]_{22} & 0 & [.]_{34} & -[.]_{34} & 0 \\ 0 & -0.5[.]_{32} & 0.5[.]_{32} & 0 & [.]_{44} & -[.]_{44} & 0 \\ 0 & -0.5[.]_{42} & 0.5[.]_{42} & 0 & -[.]_{44} & [.]_{44} & 0 \\ 0 & 0 & 0 & 0 & -0.5[.]_{54} & [.]_{54} & 0 \end{pmatrix} \tag{4.2}$$

with $[.]$ denoting the complementary matrices $M, M_i, P,\ P_1$, where the submatrix $[.]_{ij}$ means the $(i, j) - th$ submatrix of the corresponding matrices $A,\ A_i,\ C, C_1$, respectively.

Applying the transforms by (3.7), (4.2) on the system (2.1),(4.1), we get the matrices $A_e, B_e, C_e, A_{e1}, A_{e2}, A_{e3}, A_{e4}, B_{e1}, B_{e2}, B_{e3}, B_{e4}, C_{e1}$

Note that the expanded system is composed of three non-overlapped subsystems. The uncertainty functions for the extended system are as follows: $r_{e1}(t) = s_{e1}(t) = r_1(t)$, $r_{e2}(t) = s_{e2}(t) = r_2(t)$, $r_{e3}(t) = s_{e3}(t) = r_3(t)$, $r_{e4}(t) = s_{e4}(t) = r_4(t)$, $v_{e1}(t) = v_1(t)$ with the corresponding bounds.

Consider now the decentralized controller design for the nominal extended system.

This system has two uncontrollable modes, both in -1, and is observable. The original system is controllable. It means that the extended system is stabilizable. Choose the local feedback $F_e = diag(F_{e1}, F_{e2}, F_{e3})$, where $F_{e1} = \begin{pmatrix} -0.1557 & 0.0702 \\ 0.1442 & -0.1480 \end{pmatrix}, F_{e2} = \begin{pmatrix} 0.9214 & 0.8217 & -0.9155 \\ -0.8849 & -0.7789 & 0.8786 \end{pmatrix}, F_{e3} = \begin{pmatrix} -0.15 & -0.1248 \\ -0.0007 & 0.1544 \end{pmatrix}$. The nominal closed-loop for the extended system has the form $\dot{x}_e = A_{ec}x_e = (A_e + B_e F_e C_e)x_e$,
where the matrix A_{ec} has the eigenvalues: -0.1673, -0.5213, -0.5444, -0.8± 0.138, -1.0813, -1, -1, -2.1403. Considering this pole placement as satisfactory, no dynamic compensator is neccessary to stabilize the extended system using the decentralized feedback F_e.

Now, the closed-loop system has the form (3.14) with $i_{ecr} = 7$, where the matrices A_{ec5}, A_{ec6} occur from the relation $\Delta B_e F_e \Delta C_e$. Their values depend on F_e. The uncertain variables are $r_{ec1}(t) = r_1(t), r_{ec2}(t) = r_2(t), r_{ec3}(t) = r_3(t), r_{ec4}(t) = r_4(t)$, $r_{ec5}(t) = r_1(t)v_1(t), r_{ec6}(t) = r_2(t)v_1(t), r_{ec7}(t) = v_1(t)$ with the corresponding bounds.

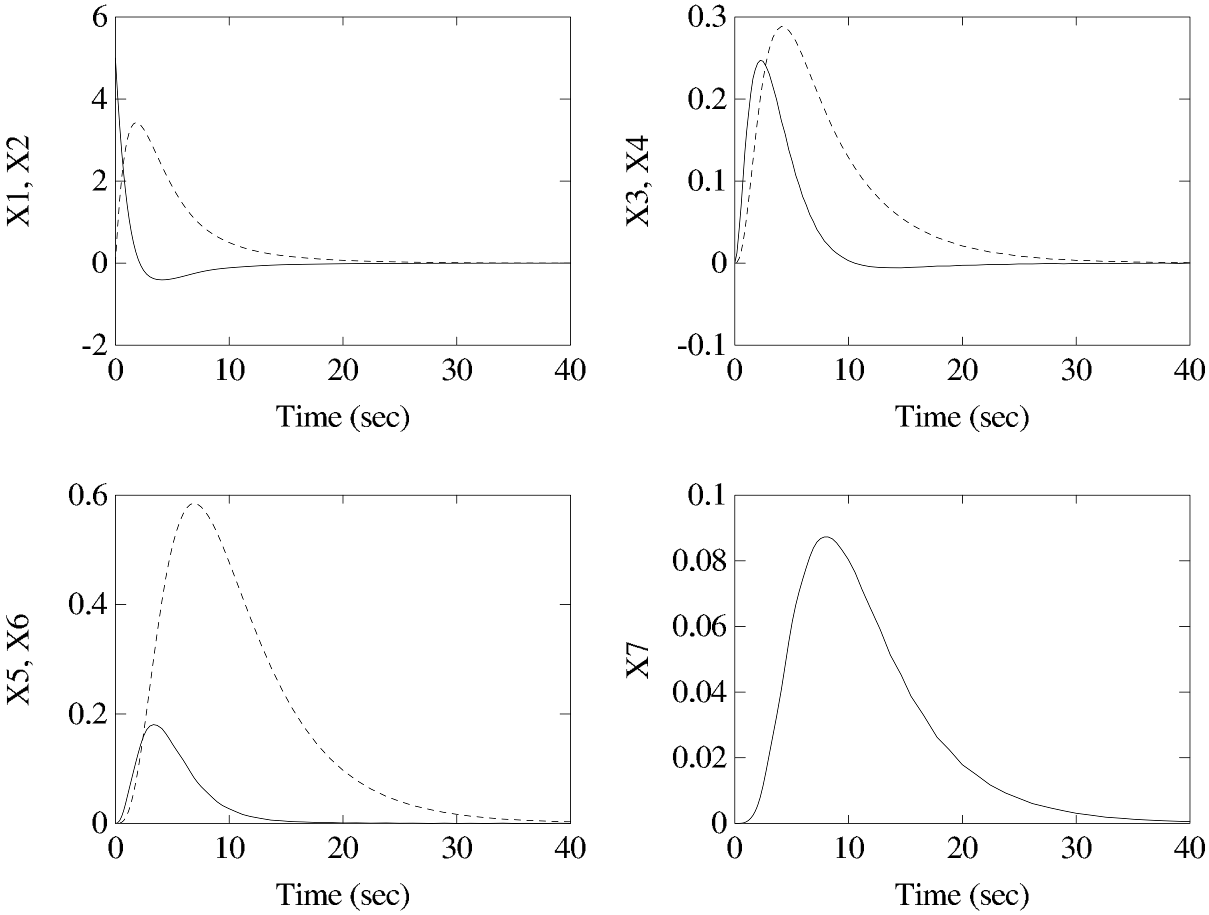

Fig. 1. String of moving vehicles - case 1: (—) velocities; (- - -) distances.

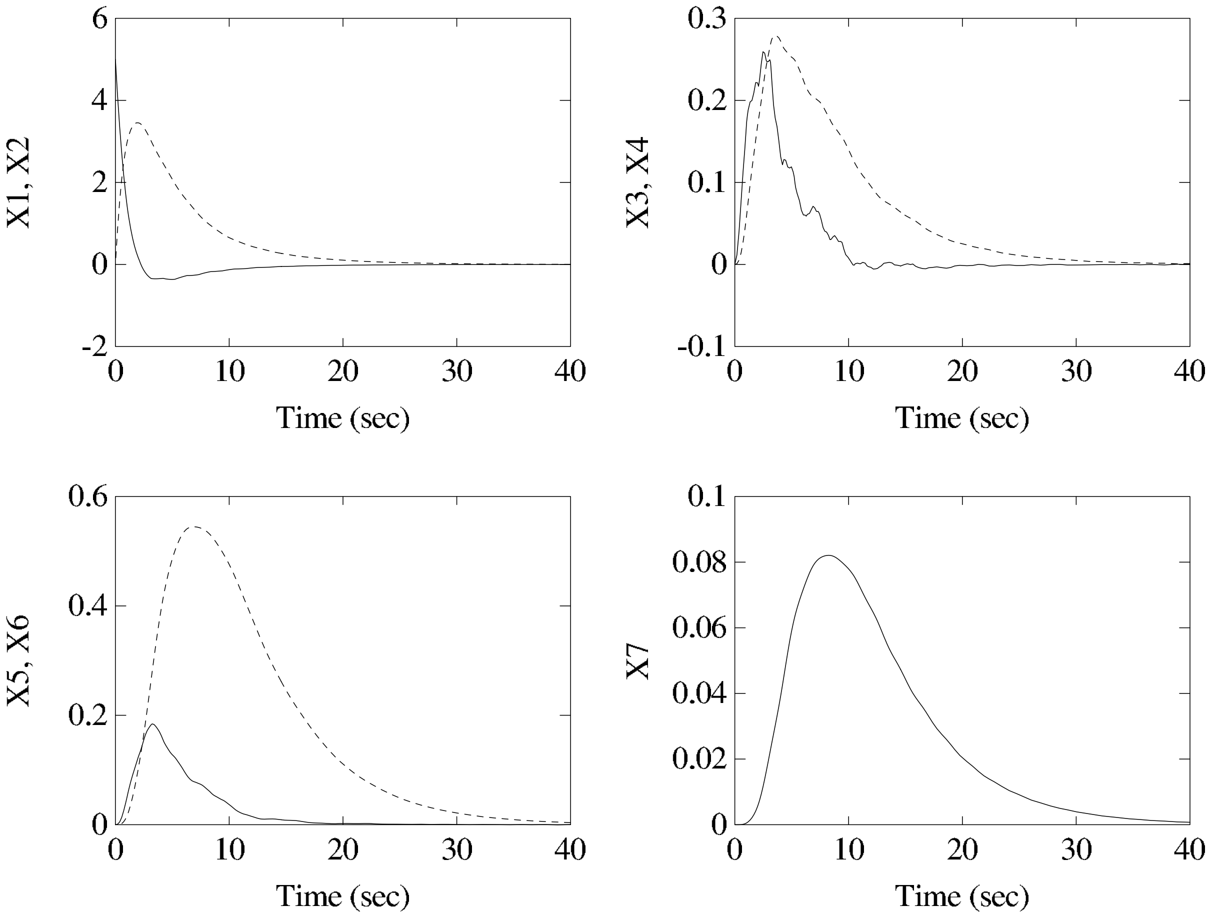

Fig. 2. String of moving vehicles - case 2: (—) velocities; (- - -) distances.

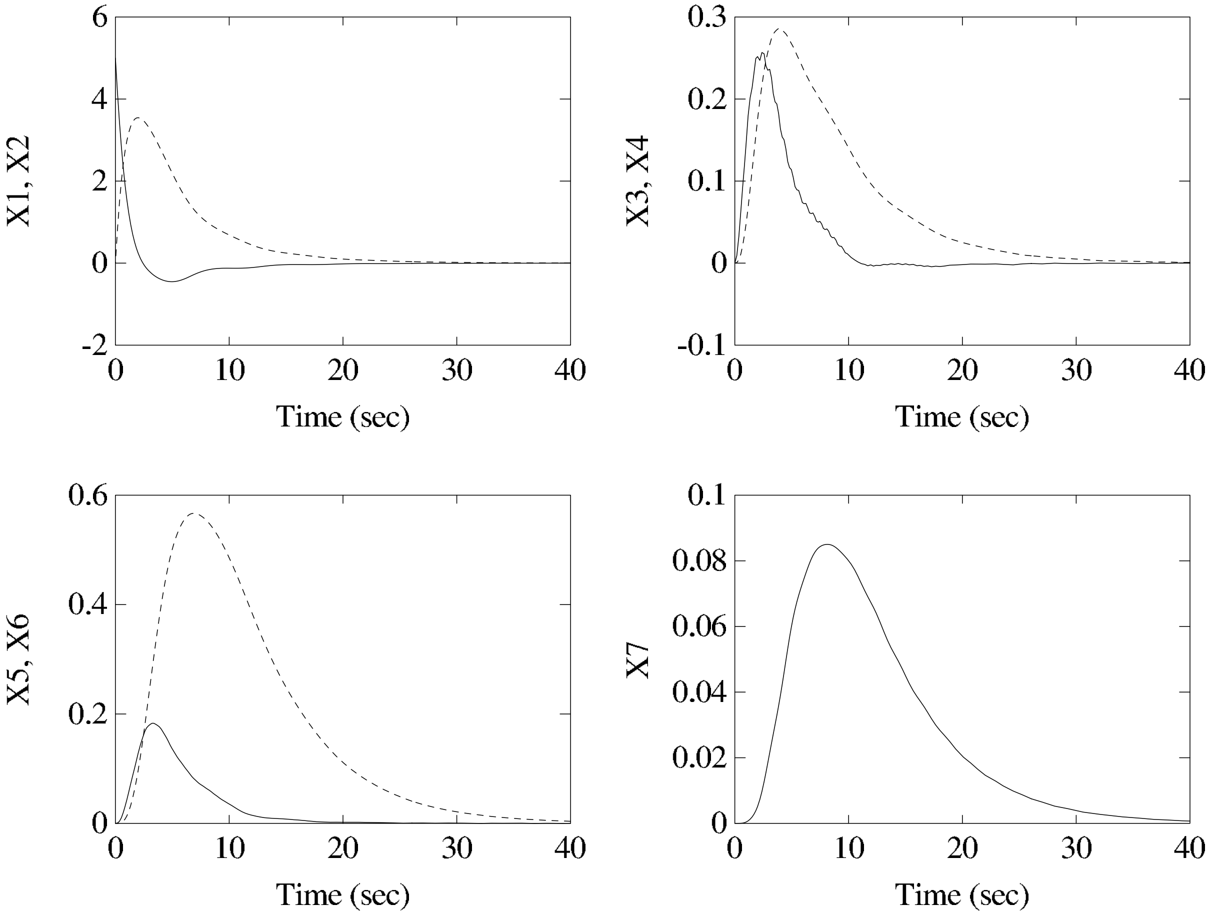

Fig. 3. String of moving vehicles - case 3: (—) velocities; (- - -) distances.

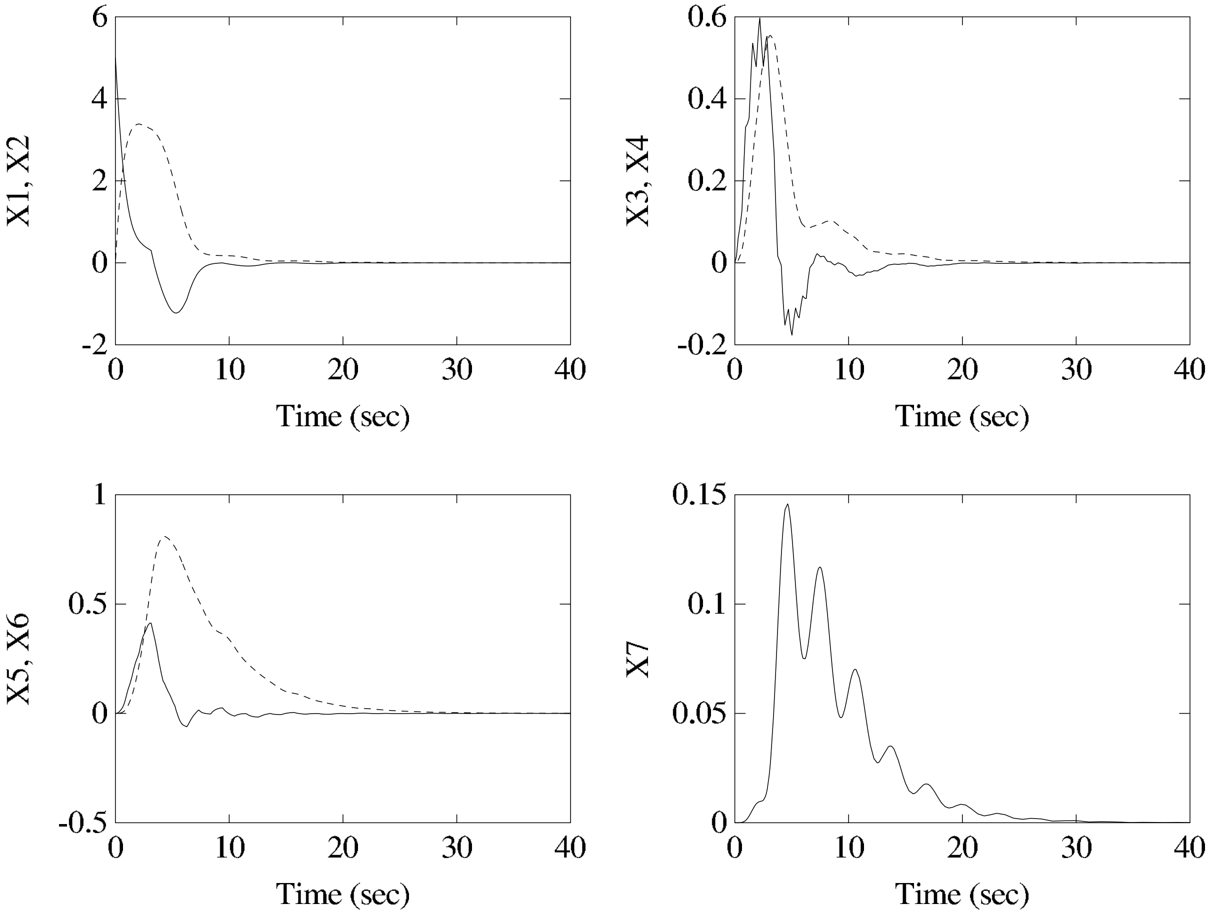

Fig. 4. String of moving vehicles - case 4: (—) velocities; (- - -) distances.

Applying now the condition (iii) of the Theorem by Zhou and Khargonekar (1987) on the system (3.14) given by (4.1), (4.2) to find allowable bounds on uncertainty parameters r_{eci}, we get $|r_{eci}| < 0.1373$ for $i = 1, ..., 7$.
Therefore, since the bounds for the extended system 0.13, it is stable with the feeback (4.5), (4.6). By using Theorem 5, the contracted feedback for the original system has the form

$$F = \begin{pmatrix} -0.1557 & 0.0702 & 0 & 0 & 0 \\ 0.1442 & -1.0694 & -0.8217 & 0.9155 & 0 \\ 0 & 0.8849 & 0.7789 & -1.0296 & -0.1248 \\ 0 & 0 & 0 & 0.0007 & 0.1544 \end{pmatrix}. \tag{4.3}$$

The nominal closed-loop system has the same eigenvalues as its extension but without uncontrollable modes. They are: -0.1673, -0.5213, -0.5444, -0.8± 0.138, -1.0813, -2.1403. The closed-loop system (3.15) is now defined by the corresponding matrices with $i_{cr} = 7$.

To illustrate this result by simulation consider four cases. The original system is controlled by the feedback F as selected above for the same initial condition $x(0) = (5, 0, 0, 0, 0, 0, 0)^T$. These cases consider uncertainty functions as follows:

1) $r_1(t) = 0.13sin(t), r_2(t) = 0.13sin(10t), r_3(t) = 0.13sin(3t), r_4(t) = 0.13sin(3t)$, $v_1(t) = 0.13$.

2) $r_1(t) = 0.13sign(sin(3t)), r_2(t) = 0.13sin(10t), r_3(t) = 0.13sin(3t)$, $r_4(t) = 0.13sin(3t)$, $v_1(t) = 0.13sign(sin(3t))$.

3) $r_1(t) = 0.13sign(sin(3t)), r_2(t) = 0.13sign(sin(10t)), r_3(t) = 0.13sin(3t), r_4(t) = 0.13sin(3t), v_1(t) = 0.13sign(sin(3t))$.

4) $r_1(t) = 0.13sign(sin(3t)), r_2(t) = 0.13sign(sin(10t)), r_3(t) = 0.13sign(sin(3t))$, $r_4(t) = 0.13sign(sin(3t)), v_1(t) = 0.13sign(sin(3t))$.

Fig.1, Fig.2, Fig.3 and Fig.4 show the responses for the cases 1, 2, 3 and 4,respectively. The states in the Figures are denoted as $x = (w_1, d_{12}, w_2, d_{23}, w_3, d_{34}, w_4)^T = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)^T$. The responses are smoother in the first case, particularly in the velocities of the second and the third vehicles. It corresponds with sinusoidal change of the mass of the first vehicle, while this change is stepwise in the second case. The fourth vehicle has sufficiently smooth velocity response. The case 4 is the worst possible case with respect to uncertainties. The responses are asymptotically stable in all considered cases.

5. CONCLUSIONS

A methodology for decentralized stabilizing controller design of uncertain nominally linear dynamic systems using overlapping decompositions has been presented. Neccesary and sufficient conditions for extensions and for contractibility of controllers are given as a necessary theoretical background.

With a design perspective, the methodology presented may be used through the following steps: 1) Transformation of the original uncertain system to the expanded uncertain system; 2) Design of a robust decentralized controller for the expanded system; 3) Contraction of this controller for its implementation on the original system.

This new methodology has been proposed for longitudial headway control design of platoons of automotive vehicles. First, the control for a nominal linear system is designed in the expanded space, and then the bounds of uncertanties guaranteeing the closed loop system stability of uncertain system are computed and evaluated when comparing them with the bounds given for the original system. Numerical simulation results show that the proposed methodology provides a reliable tool for systematic design of longitudial vehicle controllers in general.

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