METHODS FOR 2-D PHASE UNWRAPPING IN MATLAB

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Abstract: A short description of the phase unwrapping problem is presented. The simple method for phase unwrapping in MATLAB is mentioned. The unwrapping problems are described and possible solutions are shown. There is also described the principle of methods for unwrapping noisy phase maps, which are more valuable in practice than the MATLAB procedure.

1. Introduction

Many methods and applications in optical metrology use the interference principle to measure a wide range of physical quantities [1-3]. These methods produce a fringe pattern phase-modulated by the physical quantity being measured. These fringe patterns are mostly recorded by a CCD camera, digitized with a frame grabber and stored for further processing. An interferogram, i.e. the recorded intensity image, may be represented by the following expression:

$$I(x,y) = A(x,y) + B(x,y)\cos\varphi(x,y) + n(x,y), \tag{1}$$

where $A(x,y)$ is a slowly varying background intensity, $B(x,y)$ is the intensity modulation, $\varphi(x,y)$ is the phase related to the physical quantity being measured and $n(x,y)$ is an additive noise. The purpose of computer-aided fringe pattern analysis is to detect automatically the two-dimensional phase variation $\varphi(x,y)$ over the interferogram due to the spatial change of the measured physical quantity.

There are a number of techniques for measuring the phase $\varphi(x,y)$. One may analyze interferograms by using such well-known methods as the phase shifting technique, the Fourier transform method, the spatial phase-shifting, etc. Most of phase-retrieval techniques give the detected phase $\varphi_W(x,y)$ wrapped modulo 2π, i.e. the original phase $\varphi(x,y)$ is wrapped into the interval $[-\pi,+\pi]$. It is due to the non-linear wrapping function W involved in the phase-estimation process. The calculation of $\varphi(x,y)$ requires an integer multiple $k(x,y)$ of 2π to be added to the wrapped phase $\varphi_W(x,y)$. This operation is called *phase unwrapping* and can be expressed by the following equation

$$\varphi_W(x,y) = \varphi(x,y) + 2\pi k(x,y), \tag{2}$$

where $k(x,y)$ is an array of integers so that $-\pi < \varphi_W \le \pi$. The non-linear wrapping operator involved in phase-retrieval process is mostly arctangent function, so we can write

$$\varphi_W(x,y) = W[\varphi(x,y)] = \arctan\varphi(x,y). \tag{3}$$

The wrapped phase creates a sawtoothed pattern. Figure 1 shows a one-dimensional case of the phase unwrapping problem.

The unwrapping problem is trivial for phase maps calculated from good quality fringe data (*consistent phase maps*). Problems arise if the phase map is disturbed by the following error sources:

- low signal-to-noise ratio of the image caused by electronic noise, speckle noise or a low fringe modulation,
- the Nyquist sampling condition is violated (the signal is aliased),
- object discontinuities.

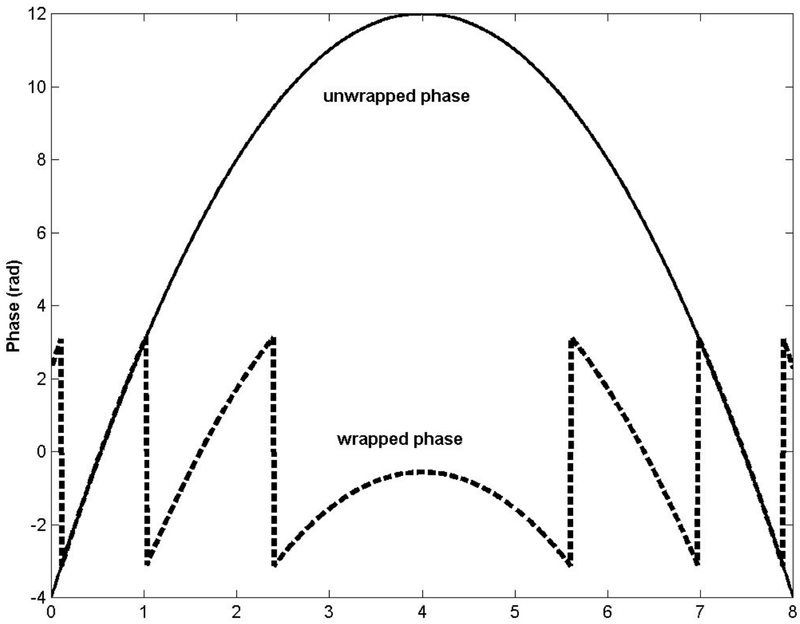

Fig.1.

These phase maps can be called *inconsistent.* The following text will focus on the unwrapping problem of consistent phase maps and its implementation in MATLAB. There will be also shortly mentioned some principles for unwrapping noisy phase maps.

2. Unwrapping consistent phase maps

The MATLAB procedure for unwrapping *unwrap* is not a very good one and fails even in some cases of absolutely consistent phase maps. A simple method for unwrapping of consistent or nearly consistent (small phase noise) phase maps will be described in this section. It can be shown [4] that the unwrapped phase can be obtained by integrating wrapped phase differences. The unwrapped result will equal the true phase provided the true phase differences $\Delta[\varphi(x,y)]$ are in the range $[-\pi,+\pi]$, i.e. that the phase signal is not aliased. Computing the differences of wrapped phases using Eq.(2) and Eq.(3) yields the equation

$$\Delta\{W[\varphi(x,y)]\} = \Delta\{\varphi(x,y)\} + 2\pi\Delta\{k_1(x,y)\}. \quad (4)$$

If we again apply the unwrapping operator W to the preceding equation, then we obtain

$$F(x,y) = W[\Delta\{W[\varphi(x,y)]\}] = \Delta\{W[\varphi_w(x,y)]\} = \Delta\{\varphi(x,y)\} + 2\pi[\Delta\{k_1(x,y)\} + k_2(x,y)], \quad (5)$$

where k_1, k_2 are integer arrays and $F(x,y)$ is the wrapped difference of wrapped phases. Due to the wrapping operator W the values of the function $F(x,y)$ must lie in the interval $[-\pi,+\pi]$. The requirement $-\pi < \Delta[\varphi(x,y)] \leq \pi$ implies that the term $2\pi[\Delta\{k_1(x,y)\} + k_2(x,y)]$ in Eq.(5) must vanish. Then we obtain

$$\Delta\{\varphi(x,y)\} = W\{\Delta[\varphi_w(x,y)]\}, \quad (6)$$

which may be simply rewritten into the following formula

$$\varphi(x_{i+1},y_j) = \varphi(x_i,y_j) + W\{\varphi_w(x_{i+1},y_j) - \varphi_w(x_i,y_j)\} \quad (7)$$

for $1 \le i \le N, 1 \le j \le M$, where *N*, resp.*M* is a number of pixels in the x-direction, resp. y-direction.We can see that we may unwrap the phase map (*N*x*M* pixels) by unwrapping the first row and then taking the last value of the row as the initial condition to unwrap along the following row in the backward direction (see Fig.2). In Eq.(7) we may use as the initial condition

$$\varphi(x, y) = \varphi_{in}, \tag{8}$$

where φ_{in} is an appropriately chosen value of the phase. This value can be arbitrary, because we are interested in the difference of phase values, not in the absolute values of the phase.

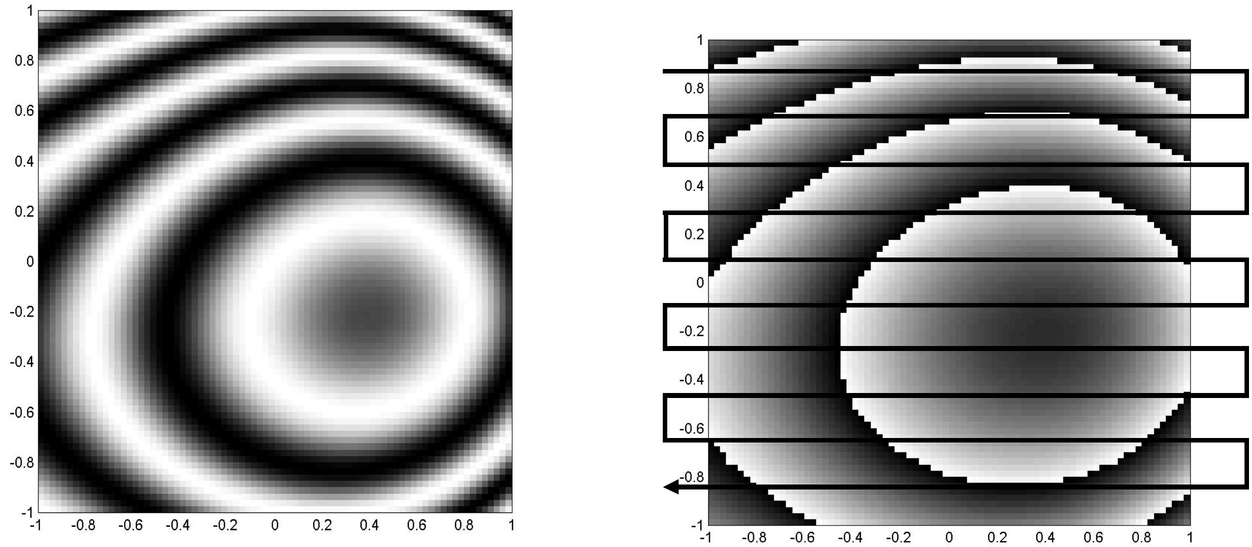

Fig.2.

Figure 2 shows the process of 2-D unwrapping (the original interferogram and the wrapped phase). The described procedure can be very easy programmed in MATLAB language in the following steps:

Step 1 > Reshape a matrix *N*x*M* to be a vector $\varphi_w(i)$, choose initial condition, e.g. $\varphi_{in}=0$

Step 2 > Compute the phase differences $D(i) = \varphi_w(i+1) - \varphi_w(i)$ for $i = 1,\ldots,NM-1$

Step 3 > Compute the wrapped phase differences:

$$\Delta(i) = \arctan\{\sin D(i) / \cos D(i)\}, \quad \text{for } i = 1,\ldots,NM-1$$

Step 4 > Unwrap the phase by summing the wrapped phase differences:

$$\varphi(i) = \varphi(i-1) + \Delta(i), \quad \text{for } i = 1,\ldots,NM-1$$

Step 5 > Reshape the vector $\varphi(i)$ to the matrix *N*x*M*

Figures 3 and 4 illustrate the phase unwrapping procedure on an example of phase map. There are shown the wrapped phase values (**Fig.3**) and the unwrapped phase data (**Fig.4**). The described unwrapping method can be also modified for unwrapping consistent phase maps bounded by a simply connected region.

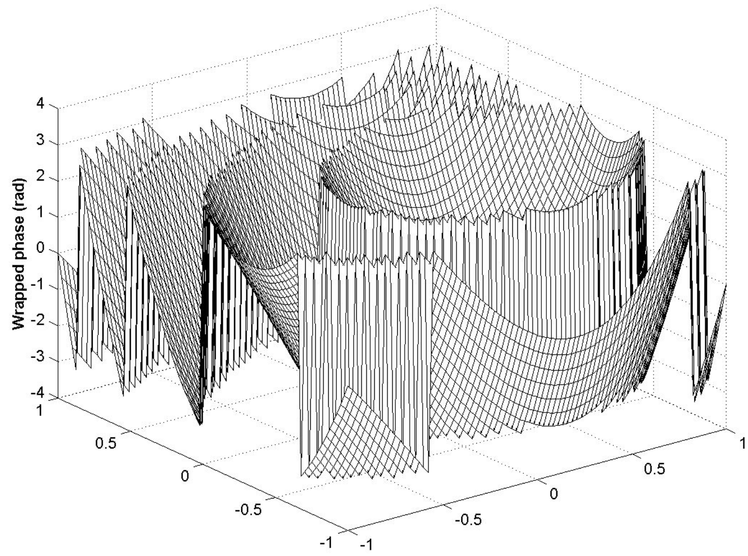

Fig.3.

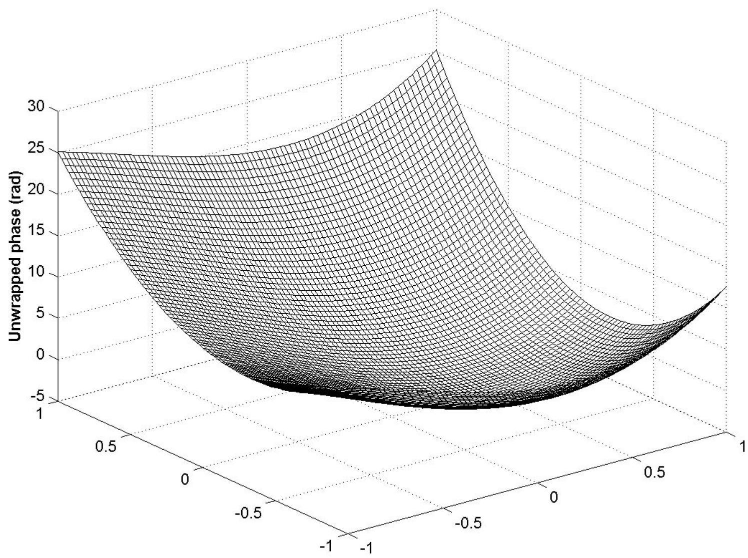

Fig.4.

3. Unwrapping inconsistent phase maps

It was noticed that the phase unwrapping process becomes difficult or even impossible if the phase data are noisy. In reality, a noise is always present in certain amount in experimental data, e.g. electronic noise, speckle noise, etc. The need for a more or less noise immune algorithms has led to the exploration of many unwrapping procedures recently [4-5]. There are two main groups of algorithms that are based on a different mathematical principle. The first group may be called *the path-following algorithms*. These techniques are based on the principle of integration of the phase gradient

$$\varphi = \int_C \nabla\varphi \, \mathrm{d}\boldsymbol{x} + \varphi(\boldsymbol{x}_0)\,, \tag{9}$$

where C is any path connecting the points $\boldsymbol{r_0}$ a $\boldsymbol{r}$. In the presence of noise the integral become dependent on the integration path and the path-following unwrapping algorithms are concerned with selecting the appropriate integration path for integrating phase gradients over a good quality data. The well-known are the branch-cut method and quality-guided path following algorithm [4].

The second group of algorithms may be called the minimizing techniques for phase unwrapping. These methods are based on the principle of minimization of a merit function J (functional) [4] that describes the error between the true phase differences and the wrapped differences of the wrapped phase, e.g.

$$J = \sum_{i=2}^{N}\sum_{j=2}^{M} U(x_i, y_j)\left[\varphi(x_i, y_j) - \varphi(x_{i-1}, y_j) - \Delta^x \varphi_w(x_i, y_j)\right]^2 + \sum_{i=2}^{N}\sum_{j=2}^{M} V(x_i, y_j)\left[\varphi(x_i, y_j) - \varphi(x_i, y_{j-1}) - \Delta^y \varphi_w(x_i, y_j)\right]^2 \tag{10}$$

where $\Delta^x \varphi_w(x_i, y_j) = W\{\varphi_w(x_i, y_j) - \varphi_w(x_{i-1}, y_j)\}$,

$\Delta^y \varphi_w(x_i, y_j) = W\{\varphi_w(x_i, y_j) - \varphi_w(x_i, y_{j-1})\}$,

and $U(x,y)$ and $V(x,y)$ are the weights that are selected to avoid integrating through pixels with low quality phase data. Described techniques are more time consuming than the path-following algorithms, but in the case of very noisy phase maps they offer the only possible solution.

4. Conclusion

The methods for two-dimensional phase unwrapping were presented. It was shown that the phase unwrapping for the phase maps of a very good quality is an easy process. The simple method for unwrapping consistent phase maps in MATLAB was described with examples of unwrapping process. This method can be simply implemented into MATLAB. It was noted that in the presence of noise, discontinuities or aliasing the unwrapping process is very difficult or even impossible. There was also described the principle of methods for unwrapping noisy phase maps, which are more valuable in practice. The problem of phase unwrapping is very important in many non-contact measurement techniques, e.g. holographic interferometry, electronic speckle pattern interferometry, synthetic aperture radar applications, etc.

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