

SHORT PIPE SIMULATION

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Introduction

Dynamic systems simulation forms a basic tool for real technologic system analysis and control. So that it is necessary for a good process of education to show how to solve complex problems. The short pipe simulation was made up as a didactic task and is used in teaching.

1. Mathematic Model Definition

The term "short pipe" means the pipe of such a length that the difference of pressure, specific weight and velocity of the liquid at the start and at the end of the pipe are not significant enough. That means there is no necessity to use partial differential equations for solving the mathematic model. In the cause of laminar flow there is possible to apply the Poiseuill's equation:

$$P_0 - P_1 = 128lQ \frac{\mu}{\pi D^4} \quad (1)$$

where $P_0 [Pa]$ is the pressure at the pipe start, $P_1 [Pa]$ the pressure at the pipe end, $\mu [Pa.s]$ is the dynamic viscosity, $l [m]$ is the pipe length, $D [m]$ is the pipe diameter and $Q [m^3.s^{-1}]$ the flow of the liquid. Comparing this equation with the Ohm's Law:

$$U = RI \quad (2)$$

where $U [V]$ is the electromotive force, $I [A]$ the electric current and $R [\Omega]$ stands for the electric resistance the mathematic isomorphism is seen. There are also the analogies between the electromotive force $U [V]$ and the pressure difference in the pipe $P_0 - P_1 [N.m^{-2}]$, the electric current $I [A]$ and the pipe flow $Q [m^3.s^{-1}]$, and the electric resistance $R [\Omega]$ and the coefficient $128 \mu l D^{-4} / \pi$ which is called hydrodynamic resistance $R_h [N.s.m^{-5}]$.

In case of the turbulent flow the system is described by the equation:

$$P_0 - P_1 = 32lQ^2 f \frac{\rho}{\pi^2 D^5} \quad (3)$$

where $\rho [kg.m^{-3}]$ is the liquid density, f the Fanning's friction coefficient and the value $32 l f \rho \pi^2 D^{-5}$ is called turbulent hydrodynamic resistance $R_{hT} [N.s^2.m^{-8}]$.

Considering the linear time-invariant system, the equation for the turbulent flow must be linearized resulting in the expression:

$$R_{hT}(lin) = 2R_{hT}Q_0 \quad (4)$$

where Q_0 is the flow in the stable state.

The fact of laminar or turbulent flow is given by the value of Reynold's number Re . Its critic area is not given exactly but usually it is about $Re=6000$ for the smooth pipes.

Let the initial Navier-Stokes equation is given in the form:

$$\iiint_V \rho \frac{D\bar{v}}{Dt} dV = \iiint_V \mu \nabla^2 \bar{v} dV - \iiint_V \nabla P dV \quad (5)$$

Considering the constant pressure drift in the pipe it is possible to express:

$$\frac{\partial P}{\partial z} = -\frac{P_0 - P_1}{l} \quad (6)$$
$$\iiint_V dV = \pi \frac{D^2}{4} l$$

Than the equation is reduced into the following expression:

$$\pi \frac{D^2}{4} l \rho \frac{dv}{dt} = L_h \rho \frac{dQ}{dT} \quad (7)$$

as the convection components of substantial derivation are equal to zero. Using Eq. 1 for the laminar flow or Eqs. 3 and 4 for the turbulent one, the Navier-Stoke's equation is modified with the Eqs. 6 and 7 to form expression [1]:

$$L_h \frac{dQ}{dT} + R_{hT} Q = P_0 - P_1 \quad (8)$$

where $L_h [kg.m^{-4}]$ is the hydraulic inertion defined as:

$$L_h = \frac{4l\rho}{\pi D^2} \quad (9)$$

This component is analogous to the coefficient of the electric induction.

Using the Laplace operator Eq. 8 can be transformed into the form:

$$(L_h p + R_h) Q(p) = \Delta P(p) \quad (10)$$

2. Problem Formulation

To present principles of simulation in the environment MATALAB - SIMULINK the short pipe is selected.

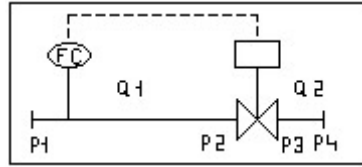


Fig. 1: The short pipe system and its variables

Its length is 200 m and diameter 10 mm . The Fanning's coefficient is 0.015 . The liquid has the density of 1000 kg.m^{-3} and the dynamic viscosity $1.005 \cdot 10^{-3}\text{ kg.m}^{-1}.s^{-1}$. There is the sensor with transducer on the pipe. The sensor has the extent $1.5\text{ m}^3.h^{-1}$ and the signal is transformed into the pneumatic one with the extent 0.1 MPa . The sensor time constant is 0.2 s . The input and output of the pneumatic controller are pneumatic signals with the extent 0.1 MPa , too. There is the valve on the pipe. The pressure difference on it $P_r = P_2 - P_3$ is 0.25 MPa and the other coefficients are: $K_v 0.5\text{ m}^3.h^{-1}$, $\Phi 0.02$ and maximal valve lift $z_R 9\text{ mm}$. The valve is actually open for $50\% z_R$. The valve time constant is 5 s . The problem is considering the disturbance of signal ΔP_1 and simulation the behavior of this loop control.

```
% Short pipe simulation
clear all
% Data:
% Liquid
l=200; ro=1000; eta=1.005e-3; d=10;
% Valve
Kv=0.5; fi=0.02; f=0.015; xr=9; x=xr/2; Tv=5; Tc=0.2; deltaP=2.5*1e5;
% gains
rr=1e5/1e5; % controller
rc=1e5*3600/1.5; % sensor {Pa.s/m3}
rv=10/1000/1e5;% valve (m/Pa)
```

Fig. 2: Input data in MATLAB

The valve is one of the most important parts of this controlled system. While valve types may vary in the design, their function can be usually described by the one equation:

$$Q = 0.1K_v \left[\Phi + (1 - \Phi) \frac{z}{z_R} \right] \sqrt{\frac{\Delta P}{\rho}} \quad (11)$$

where $K_v [m^3 \cdot h^{-1}]$ is the nominal flow which is given in the special table of valve parameters, $\rho [kg \cdot m^{-3}]$ is the liquid density, $\Delta P [Pa]$ the pressure difference of the valve, Φ is the relative flow coefficient, $z [mm]$ is actual and $z_R [mm]$ maximal valve lift.

At first there is necessary to find the value of Reynold's number Re . To know this value must be the value of the flow enumerated using the Eq. 11 for $\Delta P = P_r$ and velocity $v [m \cdot s^{-1}]$ using relations:

$$v = \frac{4Q}{3600\pi D^2}$$

$$Re = \frac{vD\rho}{\mu}$$
(12)

The solution for given input data is $Re = 14,189$. There is the turbulent flow under such condition there and in order to evaluate R_h the Eqs. 3 and 4 must be used. The dynamic pipe qualities could be described using Eq. 10.

$$[L_{1h}p + R_{1h}]Q_1(p) = P_1 - P_2$$

$$[L_{2h}p + R_{2h}]Q_2(p) = P_3 - P_4$$
(13)

The flow is 3 variables function: $Q=Q(z, P_2, P_3)$. Considering the incompressible liquid implying that $Q=Q_1=Q_2$ the deviation from the steady state can be formed:

$$\Delta Q = \frac{\partial Q}{\partial z} \Delta z + \frac{\partial Q}{\partial P_2} \Delta P_2 + \frac{\partial Q}{\partial P_3} \Delta P_3$$
(14)

The total hydrodynamic pipe resistance and the total hydrodynamic pipe inertia can be expressed in the following forms:

$$R_h = R_{1h} + R_{2h}$$

$$L_h p = L_{1h}p + L_{2h}p$$
(15)

Assume that

$$\frac{\partial Q}{\partial P_2} \Delta P_2 = -\frac{\partial Q}{\partial P_3} \Delta P_3$$
(16)

Using Eq. 14 and applying Eqs. 11, 13 and then 15 and 16 the final relation form could be found:

$$\Delta Q = \frac{\frac{\partial Q}{\partial z} \Delta z + \frac{\partial Q}{\partial P_2} \Delta P_2 (\Delta P_1 - \Delta P_4)}{1 + \frac{\partial Q}{\partial P_2} (R_h + L_h p)}$$
(17)

The controlled system gain and time constant are given by relations:

$$k_a = \frac{1}{1 + \frac{\partial Q}{\partial P_2} R_h}$$

$$\tau_a = k_a L_h \frac{\partial Q}{\partial P_2}$$
(18)

The following equations for the coefficients $\partial Q/\partial z$ and $\partial Q/\partial P_2$ are valid:

$$\frac{\partial Q}{\partial z} = 0.1K_v \frac{1-\Phi}{3.6z_R} \sqrt{\frac{P_r}{\rho}}$$

$$\frac{\partial Q}{\partial P_2} = \frac{Q}{2P_r \cdot 3600}$$
(19)

There is the resulting block scheme of the controlled system presented in Fig. 3.

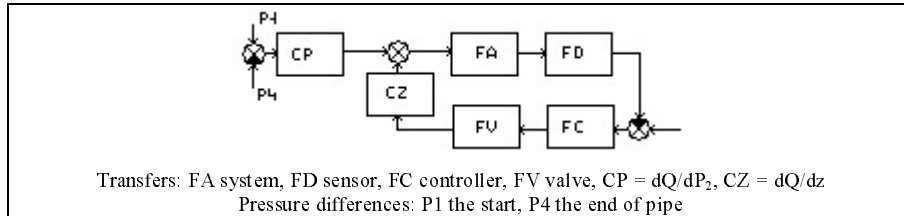


Fig. 3: Block scheme of the controlled loop

The program in MATLAB to solve the problem was created and the following parameters were enumerated: the flow Q , the velocity v (both in steady state), Reynold's number Re , the hydrodynamic resistance $R_{HT}(lin)$, the hydraulic inertia L_h and the coefficients $\partial Q/\partial z$ and $\partial Q/\partial P_2$. The same program can also evaluate the gains and time constants for the parts of the control loop: k_a , τ_a for the system, k_d for the sensor and k_v for the valve which are ratios of output extents and input signals.

```

% flow in the stable state
Q=0.1*Kv*(fi+(1-fi)*x/xr)*sqrt(deltaP/ro); % m3/h
% velocity
d=d/1000; S=pi*d^2/4; v=Q/S/3600; % m/s
% Reynold's number
Re=v*d*ro/eta;
% turbulent resistance and inertia
Rt=2*Q*32*fi*ro/(pi^2)/(d^5)/3600; L=1*ro*4/pi/(d^2);
% partial derivations
xr=xr/1000; Qx=0.1*Kv*sqrt(deltaP/ro)*(1-fi)/xr; Qx=Qx/3600;% s
QP2=Q/2/deltaP/3600;
Ta=QP2*L/(1+Rt*QP2); ra=1/(1+Rt*QP2);

```

Fig. 4: The computation of parameters

Then the scheme in SIMULINK could be created using the values of the MATLAB program. The final SIMULINK scheme is presented in Fig. 5. The results of the simulation can be displayed by the mean of Scope. Some of the results are seen in Fig. 6 in the condition of using P controller. There is small permanent control deviation but in fact it is inconsiderable.

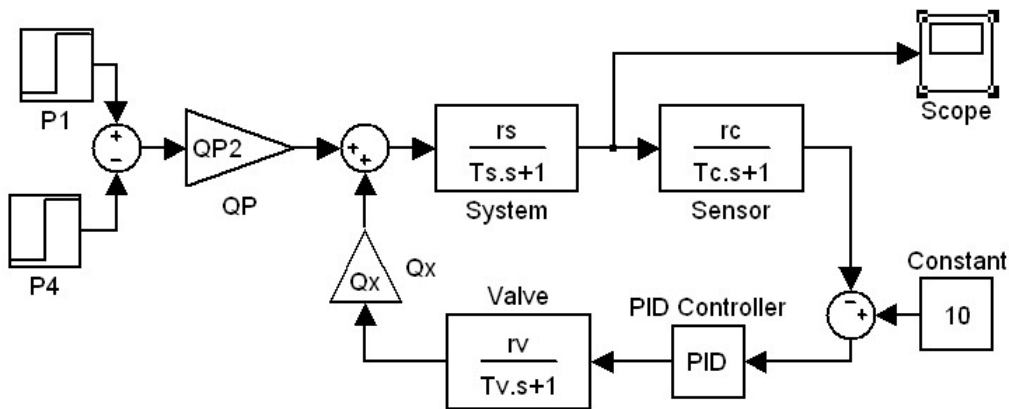


Fig. 5: SIMULINK scheme

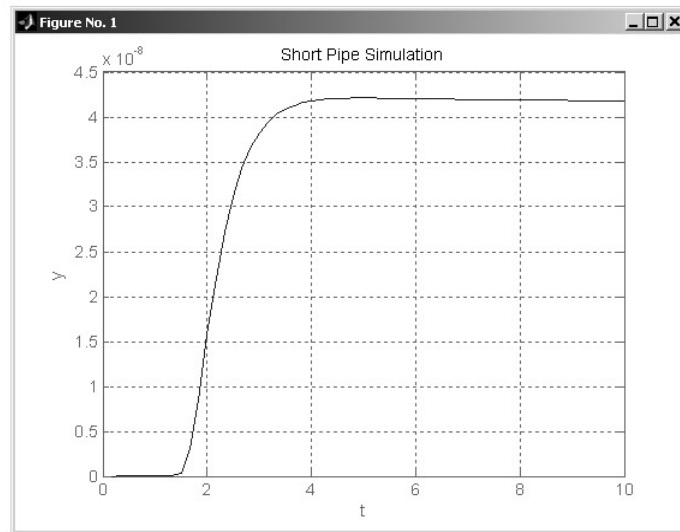


Fig. 6.: The simulation results

3. Conclusions

The model presented in this paper is very simple but it can form the first step that shows the students how to access to the complex solution of more complicated problem, e.g. signal processing, the system identification, the statistic system analysis, the multiparametric control and neural network applications.

Literature

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- [4] Ziegler, B.P.: Simulation Methodology and Model Manipulation. In: System and Control Encyclopedia, Pergamon Press, Vol. 6, 4311-4313, 1987

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