TIME-DOMAIN ANALYSIS OF MICROWAVE STRUCTURES USING MATLAB

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Abstract

In this paper, the authors' experience with time-domain analysis of high-frequency electromagnetic field is presented. The essential algorithms of the FDTD method, their implementation in the form of MATLAB program and evaluation of output are described. Finally, functionality of the program is demonstrated on two types of microwave structures.

Introduction

Time-domain numerical analysis of high-frequency electromagnetic fields has recently expanded significantly, as the computer power increased and formerly time-consuming methods got available for wide audiences. Probably the most exploited method of time-domain analysis is the finite-difference time-domain method (FDTD), which emanates directly from Maxwell equations, discretized by central-differencing. This provides sufficient robustness for dealing with various types of practical problems.

The implementation of the method is straightforward, as it does not incorporate solution of large equation system, in contrast to many other numerical techniques. The algorithm consists of linear operations with large matrices only. A set of such operations is performed in cycle, representing the field evolution in time. This character of process designates the method for handling by MATLAB environment.

Theory

The fundamental of the FDTD method is system of Maxwell equations. The first two equations, Faraday's and Ampere's Laws, are central-differenced in space and time, while the remaining two Gauss's Laws are satisfied implicitly, when free charge is not present.

The finite-difference system for three-dimensional case can be written as [1]

$$H_x\big|_{i,j,k}^{n+1/2} = D_{a,H_x}\big|_{i,j,k} H_x\big|_{i,j,k}^{n-1/2} + D_{b,H_x}\big|_{i,j,k}\left(E_y\big|_{i,j,k+1/2}^{n} - E_y\big|_{i,j,k-1/2}^{n} + E_z\big|_{i,j-1/2,k}^{n} - E_z\big|_{i,j+1/2,k}^{n}\right) \quad (1a)$$

$$H_y\big|_{i,j,k}^{n+1/2} = D_{a,H_y}\big|_{i,j,k} H_y\big|_{i,j,k}^{n-1/2} + D_{b,H_y}\big|_{i,j,k}\left(E_z\big|_{i+1/2,j,k}^{n} - E_z\big|_{i-1/2,j,k}^{n} + E_x\big|_{i,j,k-1/2}^{n} - E_x\big|_{i,j,k+1/2}^{n}\right) \quad (1b)$$

$$H_z\big|_{i,j,k}^{n+1/2} = D_{a,H_z}\big|_{i,j,k} H_z\big|_{i,j,k}^{n-1/2} + D_{b,H_z}\big|_{i,j,k}\left(E_x\big|_{i,j+1/2,k}^{n} - E_x\big|_{i,j-1/2,k}^{n} + E_y\big|_{i-1/2,j,k}^{n} - E_y\big|_{i+1/2,j,k}^{n}\right) \quad (1c)$$

$$E_x\big|_{i,j,k}^{n+1} = C_{a,E_x}\big|_{i,j,k} E_x\big|_{i,j,k}^{n} + C_{b,E_x}\big|_{i,j,k}\left(H_z\big|_{i,j+1/2,k}^{n+1/2} - H_z\big|_{i,j-1/2,k}^{n+1/2} + H_y\big|_{i,j,k-1/2}^{n+1/2} - H_y\big|_{i,j,k+1/2}^{n+1/2}\right) \quad (2a)$$

$$E_y\big|_{i,j,k}^{n+1} = C_{a,E_y}\big|_{i,j,k} E_y\big|_{i,j,k}^{n} + C_{b,E_y}\big|_{i,j,k}\left(H_x\big|_{i,j,k+1/2}^{n+1/2} - H_x\big|_{i,j,k-1/2}^{n+1/2} + H_z\big|_{i-1/2,j,k}^{n+1/2} - H_z\big|_{i+1/2,j,k}^{n+1/2}\right) \quad (2b)$$

$$E_z\big|_{i,j,k}^{n+1} = C_{a,E_z}\big|_{i,j,k} E_z\big|_{i,j,k}^{n} + C_{b,E_z}\big|_{i,j,k}\left(H_y\big|_{i+1/2,j,k}^{n+1/2} - H_y\big|_{i-1/2,j,k}^{n+1/2} + H_x\big|_{i,j-1/2,k}^{n+1/2} - H_x\big|_{i,j+1/2,k}^{n+1/2}\right) \quad (2c)$$

Here, $E_{x,y,z}$ and $H_{x,y,z}$ are the components of electric and magnetic field vectors, C_a, C_b, D_a, D_b are the updating coefficients, which are related to the field components and dependent on the environment and lattice properties, *i*, *j*, *k* are spatial coordinates and *n* is a time coordinate. These relations express the basic principle of the method – the electric and magnetic field components in the whole area of interest are updated in time, depending on the neighboring field values.

For two-dimensional problems, a useful simplification can be made. When zero field variation in the z-axis direction is assumed, the original set of equations (1) and (2) splits into two mutually independent cases – the TE and TM mode.

The TM mode incorporates the H_x, H_y and E_z field components:

$$H_x\Big|_{i,j}^{n+1/2} = D_{a,H_x}\Big|_{i,j} H_x\Big|_{i,j}^{n-1/2} + D_{b,H_x}\Big|_{i,j}\left(E_z\Big|_{i,j-1/2}^{n} - E_z\Big|_{i,j+1/2}^{n}\right) \quad (3a)$$

$$H_y\Big|_{i,j}^{n+1/2} = D_{a,H_y}\Big|_{i,j} H_y\Big|_{i,j}^{n-1/2} + D_{b,H_y}\Big|_{i,j}\left(E_z\Big|_{i+1/2,j}^{n} - E_z\Big|_{i-1/2,j}^{n}\right) \quad (3b)$$

$$E_z\Big|_{i,j}^{n+1} = C_{a,E_z}\Big|_{i,j} E_z\Big|_{i,j}^{n} + C_{b,E_z}\Big|_{i,j}\left(H_y\Big|_{i+1/2,j}^{n+1/2} - H_y\Big|_{i-1/2,j}^{n+1/2} + H_x\Big|_{i,j-1/2}^{n+1/2} - H_x\Big|_{i,j+1/2}^{n+1/2}\right) \quad (3c)$$

The remaining components are employed by the TE mode:

$$E_x\Big|_{i,j}^{n+1} = C_{a,E_x}\Big|_{i,j} E_x\Big|_{i,j}^{n} + C_{b,E_x}\Big|_{i,j}\left(H_z\Big|_{i,j+1/2}^{n+1/2} - H_z\Big|_{i,j-1/2}^{n+1/2}\right) \quad (4a)$$

$$E_y\Big|_{i,j}^{n+1} = C_{a,E_y}\Big|_{i,j} E_y\Big|_{i,j}^{n} + C_{b,E_y}\Big|_{i,j}\left(H_z\Big|_{i-1/2,j}^{n+1/2} - H_z\Big|_{i+1/2,j}^{n+1/2}\right) \quad (4b)$$

$$H_z\Big|_{i,j}^{n+1/2} = D_{a,H_z}\Big|_{i,j} H_z\Big|_{i,j}^{n-1/2} + D_{b,H_z}\Big|_{i,j}\left(E_x\Big|_{i,j+1/2}^{n} - E_x\Big|_{i,j-1/2}^{n} + E_y\Big|_{i-1/2,j}^{n} - E_y\Big|_{i+1/2,j}^{n}\right) \quad (4c)$$

These equations are useful when specific analysis of long and homogenous structures, such as transmission lines, is needed. Then, a significant reduction of computer burden is achieved.

3D Application

The described algorithm was implemented in the form of MATLAB function. In the program kernel, the matrix formulation was used to the maximum, because every amount of machine time spent in the cycle body multiplies with high number of time steps.

The program kernel for three-dimensional cavity resonator is written as follows:

```
for t=1:Nt,                        % time-marching cycle

  Hx = Hx + Db.*( Ey(:,:,indz2) - Ey - Ez(:,indy2,:) + Ez );
  Hy = Hy + Db.*( Ez(indx2,:,:) - Ez - Ex(:,:,indz2) + Ex );
  Hz = Hz + Db.*( Ex(:,indy2,:) - Ex - Ey(indx2,:,:) + Ey );

  Ez(20,20,1) = Ez(20,20,1) .* pom2;

  Ex = Ex + Cb.*( Hz - Hz(:,indy1,:) - Hy + Hy(:,:,indz1) );
  Ey = Ey + Cb.*( Hx - Hx(:,:,indz1) - Hz + Hz(indx1,:,:) );
  Ez = Ez + Cb.*( Hy - Hy(indx1,:,:) - Hx + Hx(:,indy1,:) );

  % resistive source
  Ez(20,20,1) = Ez(20,20,1) ./ pom3 - V(t);

  Ez(20,20,2:5) = 0;               % PEC current probe

  % E field display
  figure(1); h1=imagesc(squeeze(Ez(:,20,:)).',[-1 1]);
  axis image; title(['t=' int2str(t)]); drawnow;

end;
```

The cavity is fed by a current probe located on the position (20,20,1) of the lattice coordinate system. The source is realized with finite internal resistance and the probe itself is considered to be constructed of perfect electric conductor. These objectives are attained by the three additional lines for E_z. The remaining stuff serves for visualization of the evolving field component E_z. The `ind` subscripts are prepared in advance, with respect to the overall speed.

The algorithm does not comprise any output probe, since this part of program is for demonstration only. However, the output can be integrated easily, as it will appear next.

In Figs. 1, 2 the evolution of electric field component is presented. The first snapshot is taken at time step 59, when the wave surface had not been spread enough. The second "photograph"

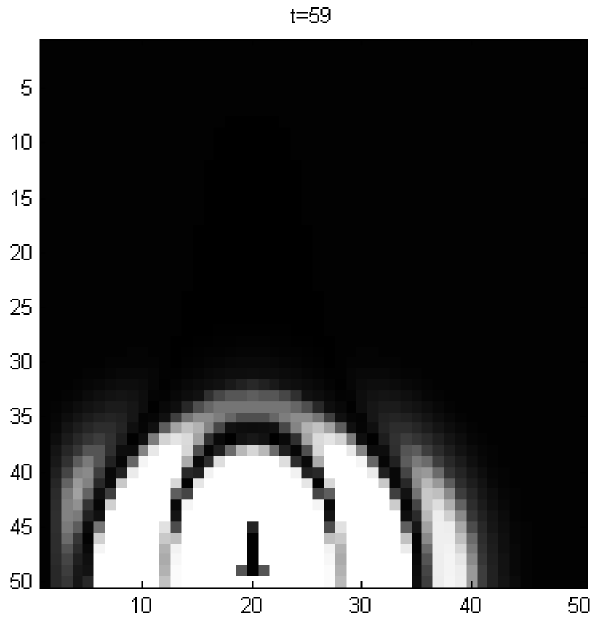

Fig. 1: The E_z field component in cavity resonator after 59 time steps

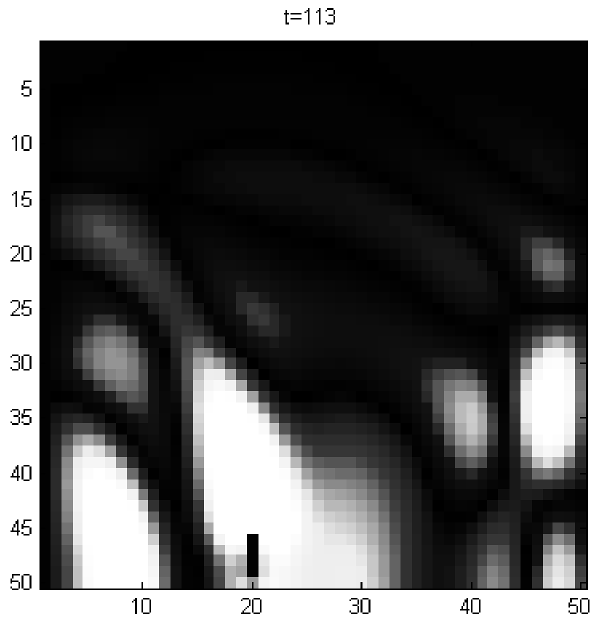

Fig. 2: The E_z field component in cavity resonator after 113 time steps

characterizes the field after some reflections, at time step 113. The feeding metal probe can be recognized as a dark vertical strip on the bottom of the cavity. The field continues to fill the whole volume of the cavity, where it will remain oscillating, unless the environment is lossy.

The introduced FDTD code can be supplied by some kind of absorbing layer to simulate the free space. Then, various types of antennas can be simulated with advantage.

2D Application

In two dimensions, only the TM mode formulation was chosen for detail description, as the TE mode is analogical to a great extent. The listing of the program kernel follows:

```
for t=1:Nt,                     % time-marching cycle

 Ez(ind5)=Ez(ind5)+V(t);        % the soft source - input

 out(t)=Ez(ind6);               % reading of the field - output

 Hx=Hx+Db.*(Ez-Ez(:,ind3));
 Hy=Hy+Db.*(Ez(ind4,:)-Ez);

 Ez=Ez+Cb.*(Hy-Hy(ind1,:)-Hx+Hx(:,ind2));

 if t==t2,                      % time indication
     t2=t2+1000;
     disp(t);
 end;

end;
```

Here, the E_z field component is excited by a predefined soft source and, in the following line, read directly from the field. The `if` structure displays every thousandth time step number as an indication of running process.

The constructed 2D algorithm was used to analyze the wavemode frequencies of microwave transmission lines, particularly the square waveguide and a shielded air-strip line [2, 3]. Cross sections of the structures are displayed in Fig. 3.

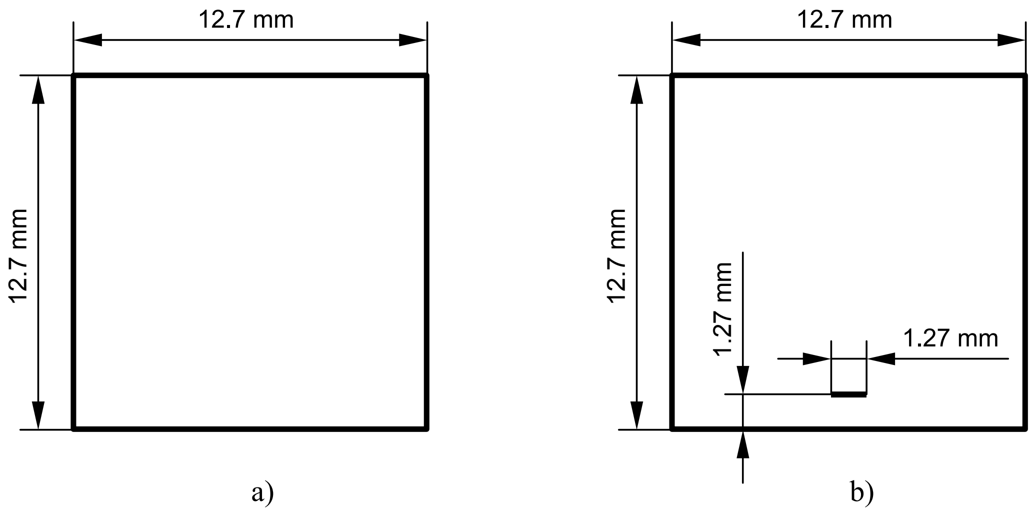

Fig. 3: Cross sections of analyzed types of transmission lines: a) square waveguide, b) shielded air-strip line

The only difference between these two sections is the strip, which perturbs the field inside and causes frequency shifts and splits of wavemode frequencies. The corresponding magnitude spectra obtained simply by

```
spec = abs(fft(out));
```

are shown in Figs. 4 and 5.

Obviously, the presence of the strip causes acquisition of the additional wavemode frequencies in the vicinity of the original modes. Replacing the input and output probes affected the magnitude of the spectrum in the second case.

The area of transverse sections was discretized with 20 cells per side. The problem was solved in frequency range from 0 to 50 GHz, with 2 MHz resolution.

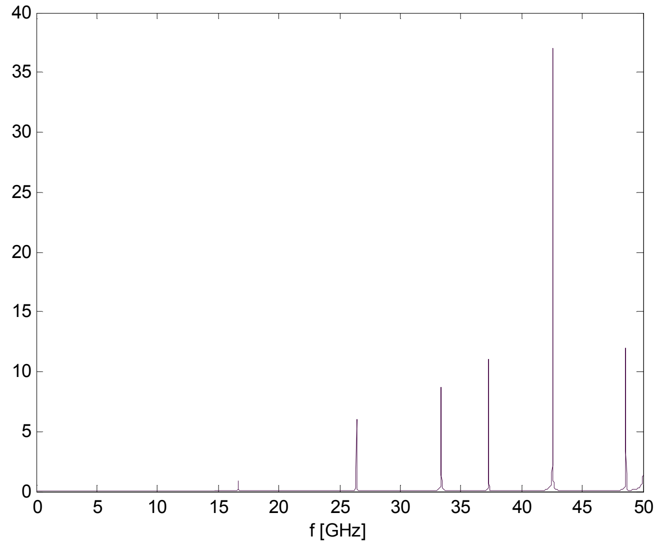

Fig. 4: Frequency spectrum for square waveguide, 20 cells per side

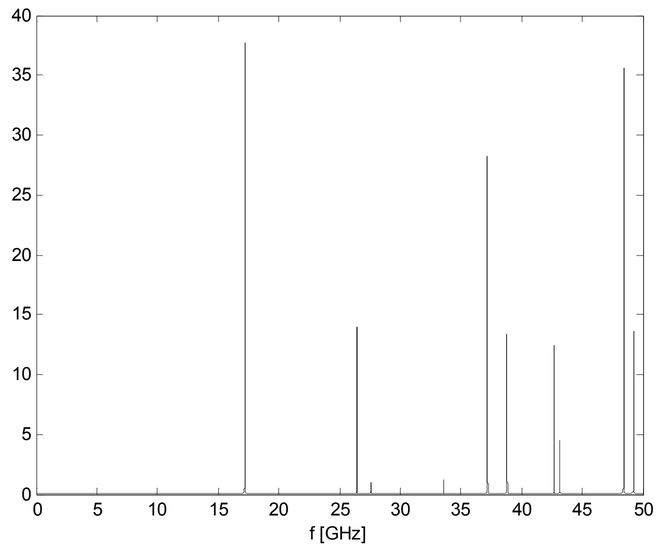

Fig. 5: Frequency spectrum for shielded air-strip line, 20 cells per side

The results and machine demands of the engaged method were also confronted with other time-domain methods [2, 3]. The time consumption was minimal in comparison with the finite element complex frequency hopping [2] and the time-domain finite element method [3], while the accuracy remained good.

Conclusion

Time-domain analysis using FDTD method implemented in MATLAB has many advantages. The time-evolving field component arrays perfectly fit to the matrix oriented MATLAB and the program execution is very fast. As the method works in time domain, Fourier transforming (provided by built-in `FFT`) is often necessary for spectral-domain evaluation. Finally, easy visualization helps to understand the processes taking place in the dynamic electromagnetic field.

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Reference

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[2] MOTL, M., FRANEK, O., RAIDA, Z. “Comparison of Finite Element Complex Frequency Hopping (FE/CFH) and Finite-Difference Time-Domain (FDTD) Methods,” *Proceedings of the 12th International Czech-Slovak Scientific Conference RADIOELEKTRONIKA 2002*.

[3] MOTL, M., FRANEK, O., RAIDA, Z. “Comparison of Time-Domain Finite Element (TD-FE) and Finite-Difference Time-Domain (FDTD) Methods,” *International Symposium on Antennas JINA 2002*. Submitted.

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