BOOTSTRAP AND 2D IMAGE DENOISING

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Abstract: Digital signal processing of the 2D images plays role in optical monitoring of technological process. Noise suppression is a basic aim of image filtering. A wide class of 2D filters is based on pixel neighborhood analysis using statistical estimates. Every statistical sample can be re-sampled and re-estimated. This non-parametric technique is called Bootstrap. Combining neighborhood data, original filter as estimate and re-sampling technique we obtain Bootstrap estimate of central pixel intensity and its variance. Both linear and nonlinear filters are re-sampled and compared using SNR criteria. Bootstrap variances enable to construct compromise estimates with adaptive behavior.

Keywords: Bootstrap, re–sampling, pixel neighbourhood, adaptive filtering, 2D de–noising.

1 INTRODUCTION

The gray image is supposed to be a matrix of pixels. The gray level of every pixel $x_{ij} \in [0;1] \subset \mathbf{R}$. The pixel neighborhood as a sub–matrix of size $(2r+1)^2$ plays the main role in 2D image processing. The pixel neighborhood processing is a process producing single value from the neighborhood and it is represented by a function of $(2r+1)^2$ variables. This function can be presented as a statistical estimate on the neighborhood sample.

2 TRADITIONAL DE-NOISING

The main aim of de–noising can be realized for $r = 1$ on 3x3 matrix neighborhood. The traditional linear approach is based on FIR filters with various weight schemes. The non–linear approach can be presented by weighted medians, quasi medians, Hodges–Lehmann medians, BES or any other rank based estimates.

3 SIGNAL TO NOISE RATIO

The signal to noise ratio criterion (SNR) agrees with digital filtering standard. It is defined as

$$SNR = 10 \cdot \log \frac{Var(X)}{Var(Y-X)}$$

where $Var(A) = \mathbf{E}\{(A - \mathbf{E}A)^2\}$ and $\mathbf{E}$ is symbol of average value. The prototype image matrix X consists of $x_{ij} \in [0;1]$ while the intensities y_{ij} form the matrix Y of noised or de–noised images. Both noised and de–noised images consist noised pixels. But noised level should be less after de–noising.

4 BOOTSTRAP TECHNIQUE

This technique of re–sampling was originally developed [1] as no more than an useful heuristic estimate. Lately Efron [1] proved some efficiency properties and made the Bootstrap reputable. Let $n \in \mathbf{N}$ be sample size. Let $\vec{x} = (x_1, \ldots, x_n)$ be statistical sample from given distribution. Let $\mathcal{T}(\vec{x})$ be a statistic estimate of distribution parameter. Let $B \in \mathbf{N}$ be Bootstrap repetition factor. Then the **Bootstrap estimate** is defined as

$$\mathcal{B}(\vec{x}) = \frac{1}{B} \cdot \sum_{k=1}^{B} \mathcal{T}(\vec{r}_k)$$

where

$$r_{kj} = x_{RND_{kj}}$$

and $RND_{kj} \in \{1, \ldots, n\}$ is a discrete random variable with constraint density function. The **variance of the Bootstrap estimate** is also defined as

$$\text{var}\ \mathcal{B} = \frac{1}{B-1} \cdot \sum_{k=1}^{B} \left(\mathcal{T}(\vec{r}_k) - \mathcal{B}(\vec{x})\right)^2 .$$

5 BOOTSTRAP DE–NOISING OF 2D IMAGE

There are various possibilities how to use Bootstrap technique in 2D image de–noising.

5.1 Single Bootstrap

Using single estimate $\mathcal{T}(\vec{x})$ and the Bootstrap estimate $\mathcal{B}(\vec{x})$ for given B we can compare which of them gives the higher value of SNR. The results of biomedical MRI 2D image de–noising are collected for $B = 30$ and $B = 100$ in the Tab. 1. It is easy to recognize that the single Bootstrap estimates are better than the point estimates according to SNR values. When the original point estimate has good quality then the effect of Bootstrap is symbolic only. Anyway the Bootstrap technique grow up the quality of the worse filters.

5.2 Winner takes all Bootstrap

Using a set of estimates $\mathcal{T}_1(\vec{x}), \ldots, \mathcal{T}_M(\vec{x})$ for given B we obtain a set of Bootstrap estimates

$$\mathcal{B}_1(\vec{x}), \ldots, \mathcal{B}_M(\vec{x})$$

and their variances

$$\text{var}\ \mathcal{B}_1(\vec{x}), \ldots, \text{var}\ \mathcal{B}_M(\vec{x})$$

Then the **winner Bootstrap estimate** is defined as

$$W(\vec{x}) = \mathcal{B}_W(\vec{x})$$

where

$$W = \arg \min_{k=1,\ldots,M}\ \text{var}\ \mathcal{B}_k(\vec{x}).$$

The winner estimate is a model of hard fixing to the best possible result.

5.3 Mixture of independent Bootstrap estimates

Supposing the statistical independence of Bootstrap estimates $\mathcal{B}_1(\vec{x}), \ldots, \mathcal{B}_M(\vec{x})$ we can mix them in inverse proportions to their variances. The **independent Bootstrap estimate** is defined as

$$\mathcal{I}(\vec{x}) = \sum_{k=1}^{M} w_k\, \mathcal{B}_k(\vec{x})$$

where

$$\sum_{k=1}^{M} w_k = 1,$$

$$w_k^{-1} \sim\ \text{var}\ \mathcal{B}_k(\vec{x}).$$

5.4 Micture of dependent Bootstrap estimate

In case of dependent estimates we can use the covariance matrix **C** as a tool for weight calculations. Let **C** be the covariance matrix estimate for the vector $(\mathcal{T}_1(\vec{x}), \ldots, \mathcal{T}_M(\vec{x}))$. The **dependent Bootstrap estimate** is defined as

$$\mathcal{D}(\vec{x}) = \sum_{k=1}^{M} v_k \, \mathcal{B}_k(\vec{x})$$

where

$$\sum_{k=1}^{M} v_k = 1,$$

$$v_k \geq 0,$$

$$\vec{v}\,' \mathbf{C} \, \vec{v} = \min_{\vec{v}} .$$

6 EXPERIMENTAL RESULTS

The previous techniques were also used to MRI 2D image de–noising (Fig. 1. - 4.) and the results of winner takes all, independent and dependent Bootstrap are in the Tab. 1. It is clear that the principle of winner takes all is not effective while the dependent Bootstrap is the best one mixed principle. But the dependent Bootstrap is not better than the best individual Bootstraps. It is caused by mixing of various quality estimates. Studying this effect will be the subject of future research.

No.	Filter name	SNR [dB]		
		Point	$\mathrm{Bootstrap}_3 0$	$\mathrm{Bootstrap}_1 00$
	NO	-2.2653		
3	full Hodges	3.3695	3.4734	3.5055
4	mean	3.6407	3.5466	3.6144
5	center	2.6582	3.2232	3.2745
6	median	2.7719	3.1504	3.2146
7	Hodges–Lehmann	3.3626	3.4604	3.5119
8	light Hodges	3.3838	3.4482	3.5145
11	pivot	3.0706	3.4916	3.5773
12	BES	3.3745	3.4633	3.5364
13	quasi median	3.0818	3.3574	3.4430
14	Hodges BES	3.5032	3.4609	3.5337
	winner takes all		2.7803	2.9076
	independent		3.5024	3.5481
	dependent		3.4919	3.5726

Table 1: Individual and combine Bootstraps

References

[1] EFRON, B., TIBSHIRANI, R.J. *An Introduction to the Bootstrap.* Chapman & Hall, 1993.

[2] MITRA, S.K., KAISER, J.F. *Handbook for Digital Signal Processing.* John Wiley & Sons, New York, 1993.

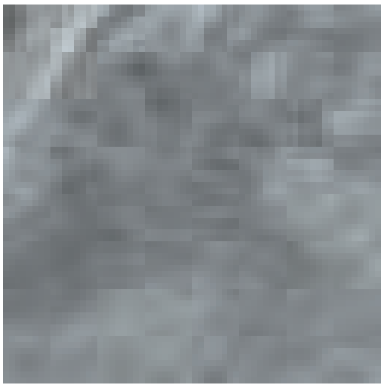

Figure 1: Original image
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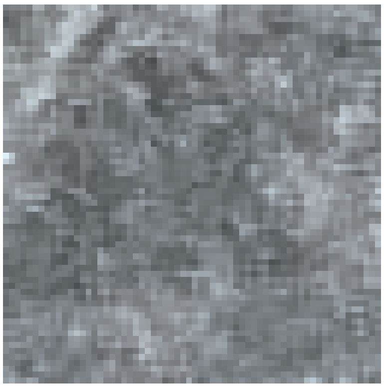

Figure 2: Winner takes all Bootstrap

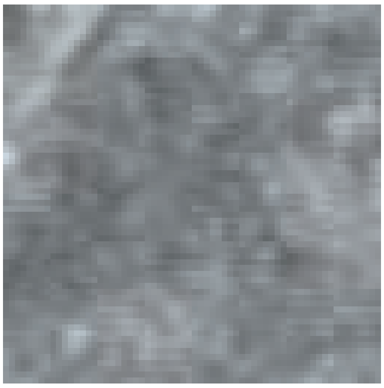

Figure 3: Dependent Bootstrap

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