COMPUTER ANALYSIS OF INTERFERENCE FIELDS USING MATLAB

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1. Introduction

Quantitative evaluation of interference fields for interferometric measurements consists of the pointwise determination of the phase difference $\Delta\varphi$ between the reference and object beam that interfere. Phase values are closely related to the optical path difference between these two beams. The physical quantity to be measured, e.g. displacement, can be then determined from the phase distribution [1-3]. The assumed distribution of the intensity of the interference field in the detector plane can be expressed as

$$I(x,y) = a(x,y) + b(x,y)\cos[\Delta\varphi(x,y)], \quad (1)$$

where $I(x,y)$ is the intensity of the interference field in the point (x,y), $a(x,y)$ is the function that characterizes the mean intensity of the interference pattern and $b(x,y)$ is the function that determines the modulation of the interference signal. From this intensity distribution phase values $\Delta\varphi$ can be determined by several different methods. This work focuses on main techniques for phase evaluation in practice: *fringe tracing method*, *Fourier transform method* and *phase shifting method*. All mentioned methods has their advantages and disadvantages. The choice of a specific method for phase evaluation depends on the particular measurement problem to be solved.

2. Phase evaluation techniques

The fringe tracing method [2,4] is based on the assumption that the local extrema of the intensity distribution in the interferogram correspond to the extrema of the assumed harmonic function of the detected interference signal. In this case the phase difference of the interference field at pixels, where an intensity maximum or minimum is located, equals an even or odd integer multiple of π. The main problem in the presented technique is to determine points lying on fringe centres, where the phase difference $\Delta\varphi$ is known, and from these points phase values can be approximated [5,6]. The evaluation process with fringe tracing method is schematically shown in **Fig.1**.

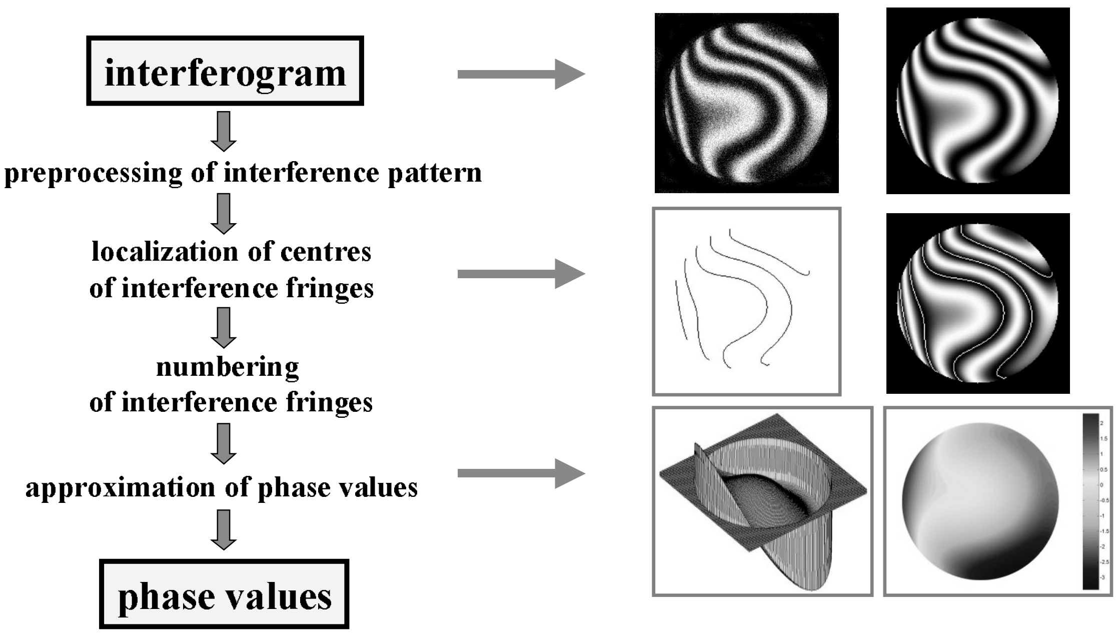

Fig.1: Process of phase evaluation using fringe tracking

The Fourier transform method [1,7] is based on the Fourier transform of the distribution of the intensity of the interference field. The two-dimensional Fourier transform of the interference pattern (1) is a Hermitean function and the amplitude spectrum thus looks symmetric with respect to the dc-term. The spectral peak at zero frequencies represents low frequency spectral component that arises from the modulation of the background intensity of the interferogram. Two symmetric spectral sidelobes carry the same information about the phase values $\Delta\varphi$. Using appropriate adaptive bandpass filters in the spatial frequency domain we can extract one of sidelobes. The unwrapped phase values can be then calculated by the inverse Fourier transform of the filtered spectrum (**Fig.2**). Discontinuities of phase values must be correctly unwrapped by appropriate mathematical techniques [8,9].

A disadvantage of the described method as well as the fringe tracing technique is the ambiguity of the sign of calculated phase values. The bandpass filtering of the Fourier spectrum is the basic step in the described method. Using a proper filter noise and nonuniform background intensity of the interference pattern can be partially removed. However, the badly chosen filter may cause a coarse disturbance of resulting data. The problem of the difficult separability of spectral sidelobes from the spectrum can be solved using the *spatial carrier frequency* in the recorded interference field.

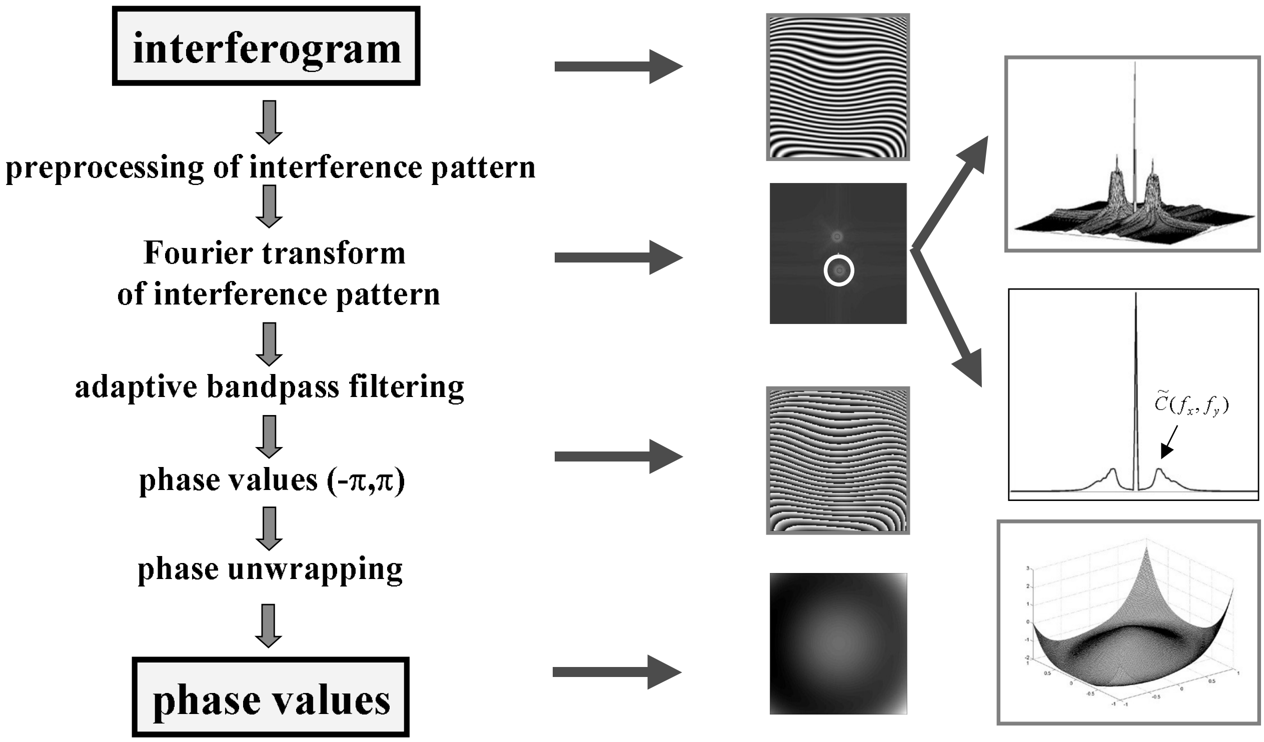

Fig.2: Process of phase evaluation using the Fourier transform method

The phase shifting method [1,2,10] is based on evaluation of the interference field (1) from several phase shifted measurements of the intensity. Phase values can be unambiguously determined from several phase shifted patterns with high accuracy in all pixels of the detector due to the specific process of phase evaluation. For unambiguous determination of phase values it is necessary to carry out at least $N = 3$ measurements with known phase shift values. The phase evaluation process in case $N = 3$ phase shifted measurements is shown in **fig.3**.

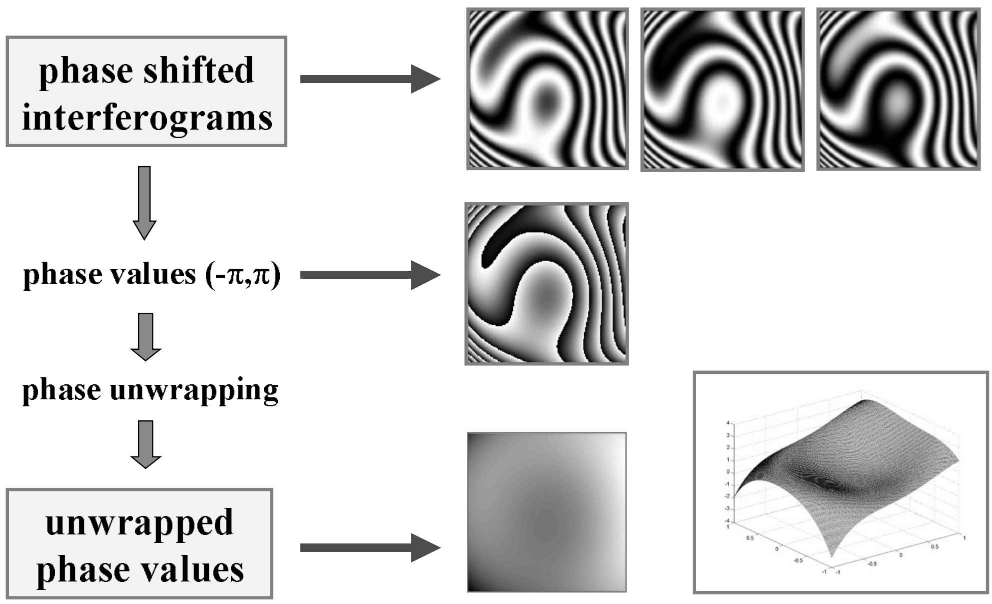

Fig.3: Process of phase evaluation using phase shifting

3. Comparison of evaluation techniques

Particular phase evaluation techniques were compared on examples of interference patterns using a computer simulation in Matlab. For reliable comparison of properties of individual described methods interference patterns with the same phase information were simulated. These interferograms were evaluated with mentioned methods. The phase difference $\Delta\varphi(x,y)$ was modelled by chosen Seidel aberration polynomials [6]

$$\Delta\varphi(x,y) = \frac{2\pi}{\lambda}(W_{20}r^2 + W_{40}r^4 + W_{60}r^6 + W_{1x}x + W_{1y}y + W_{31}r^2x + W_{51}r^4x + W_{22}x^2), \qquad (2)$$

where λ is the wavelength of light, W_{ij} are coefficients of aproximating polynomials and $r^2 = x^2 + y^2$. The interference patterns were then simulated from these modelled phase values due to following expressions:

A) *Fringe tracing method* (method A):

$$I(x,y) = \left(a(x,y) + b(x,y)\cos[\Delta\varphi(x,y)]\right)N_m + N_a, \qquad (3a)$$

B) *Fourier transform method* (method B):

$$I(x,y) = \{a(x,y) + b(x,y)\cos[\Delta\varphi(x,y) + f_x x + f_y y]\}N_m + N_a, \qquad (3b)$$

C) *Phase shifting method* (method C):

$$I(x,y) = \{a(x,y) + b(x,y)\cos[\Delta\varphi(x,y) + \psi_i]\}N_m + N_a, \qquad (3c)$$

where $I(x,y)$ is the intensity distribution in the detection plane (x,y), $a(x,y)$ denotes the mean intensity of the interference field, $b(x,y)$ characterizes the modulation of the detected interference signal, $\boldsymbol{f} = (f_x,f_y)$ is the spatial carrier frequency, ψ_i is the phase shift, N_m and N_a characterize multiplicative and additive noise in the interference pattern. There were also considered possible measurement factors that can negatively affect the interferometric measurement techniques that use the phase shifting method for phase evaluation. [9].

The simulated interference field was influenced by noise and it was subsequently evaluated using all mentioned methods, i.e. the phase difference of the interference field $\Delta\varphi'(x,y)$ was calculated. Further the root-mean-square of phase values $\sigma_{\Delta\varphi}$ is given by

$$\sigma_{\Delta\varphi} = \sqrt{\frac{\sum[\delta(\Delta\varphi)]^2}{K}}, \qquad \delta(\Delta\varphi) = \Delta'\varphi - \Delta\varphi, \tag{4}$$

where $\delta(\Delta\varphi)$ is the deviation of calculated phase data from the original phase data and K is the total number of pixels, where the phase deviation is calculated. The mentioned value $\sigma_{\Delta\varphi}$ expresses the mean error of approximation with the particular phase evaluation method. Described methods can be compared using $\sigma_{\Delta\varphi}$. In this paper there are shown two examples of simulated interference patterns that differs in the shape of interference fringes. The examples were chosen to enable a reliable comparison of all mentioned methods.

1) Example of interference pattern A

The interferogram is defined on the rectangle area with the size 256x256 pixels and contains of open interference fringes (**Fig.4**).

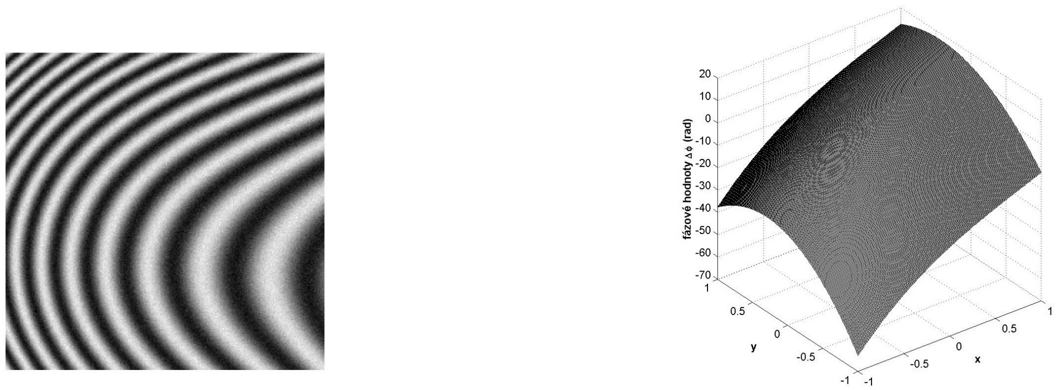

Fig.4: Interferogram and original phase values $\Delta\varphi$

2) Example of interference pattern B

The interferogram is defined on the circular area with the radius 240 pixels and contains of closed interference fringes (**Fig.5**).

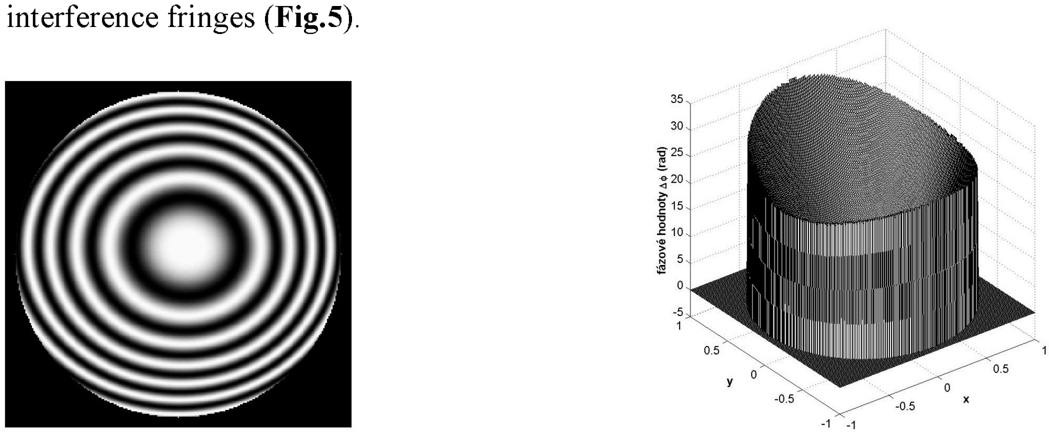

Fig.5: Interferogram and original phase values $\Delta\varphi$

Calculated root-mean-squares of phase data $\sigma_{\Delta\varphi}$ and P-V(peak-to-value) for particular phase evaluation methods and for the case A are shown in **tab.1**.

Table 1

Method	$\sigma_{\Delta\varphi}$ [rad]	P-V [rad]
A	0.18	0.8
B	0.07	0.2
C	0.03	0.09

Calculated root-mean-squares of phase data $\sigma_{\Delta\varphi}$ and P-V(peak-to-value) for particular phase evaluation methods and for the case B are shown in **tab.2**.

Table 2

Method	$\sigma_{\Delta\varphi}$ [rad]	P-V [rad]
A	0.31	1.4
B	0.11	0.37
C	0.03	0.09

From the results of the performed analysis we can see that the phase shifting method is more accurate than remaining evaluation methods in case when it is possible to use this method in practice.

4. Conclusion

The article deals with the techniques of computer analysis of interference fields for automatic evaluation of interferometric measurements in science and industry. Noncontact interferometric methods are used in many parts of industry for measurements of displacements, vibrations, the quality of optical surfaces, etc. A proper evaluation of the interference field that arises from the interference of the reference and object coherent wave field is very important for practical applications of mentioned measurement methods. The work describes an implementation of several different techniques for evaluation of interference fields and there is also carried out a comparison of described techniques using Matlab. The system Matlab was successfully used for programming mentioned automatic evaluation methods.

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References:

[1] Kreis T.: *Holographic interferometry: Principles and methods*, Akademie Verlag Berlin, 1996.

[2] Robinson D.W., Reid G.T.: *Interferogram analysis: Digital Fringe Pattern Measurement Technques,* Institute of Physics Publishing, Bristol, 1993.

[3] Mikš A.,Novák J.: *Application of multi-step algorithms for deformation measurement.* SPIE Proceedings vol.4398, Washington 2001.

[4] Yatagai T.: *Automatic fringe analysis using digital image processing techniques*. Optical Engineering 21, s.432, 1982.

[5] Novák J.: *Analýza a vyhodnocování interferenčních struktur.* MATLAB 2001, Praha.

[6] Malacara D.: *Optical Shop Testing*, John Wiley & Sons, N.Y. 1992.

[7] Takeda M., Ina H., Kobayashi S.: *Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry*, J.Opt.Soc.Am, Vol.72, No.1,1982.

[8] Ghiglia D.C., Pritt M.D.: *Two-dimensional phase unwrapping: Theory, algorithms and software*, John Wiley & Sons, Inc., New York 1998.

[9] Novák J.: *Methods for 2-D phase unwrapping in MATLAB.* MATLAB 2001, Prague.

[10] Novák J.: *Computer Simulation of Phase Evaluation Process with Phase Shifting Technique*. Physical and Material Engineering 2002, Prague.

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