WAVELET TRANSFORMS USE FOR SIGNAL DENOISING AND RESOLUTION ENHANCEMENT

I. Šindelářová[1], *J. Ptáček*[2], *and A. Procházka*[2]

[1] *University of Economics, Department of Econometrics*
[2] *Institute of Chemical Technology, Department of Computing and Control Engineering*

Abstract

The paper is devoted to description of selected properties of Wavelet transforms and their use for signal and image decomposition and reconstruction. Basic goals of this process are in signal denoising using estimated threshold limits and signal resolution enhancement. Resulting algorithms are verified for simulated signals at first and applied both for real one dimensional signal processing and analysis of biomedical images. Selected parts of the paper are based upon Wavelet and Image processing toolboxes.

1 Introduction

Wavelet transform represents a mathematical tool for one-dimensional or multi-dimensional signal analysis and processing. The paper is devoted to a brief description of Wavelet functions used for signal analysis at first. The main part of the paper provides selected algorithms for signal and image decomposition and reconstruction applied for their denoising at first. The final part of the paper presents the use of signal decomposition for its resolution enhancement resulting in different sampling period comparing to the original one.

Mathematical analysis and numerical experiments are devoted to the study of different Wavelet functions and threshold limits used during application of time scale signal analysis.

2 Principles of Signal Wavelet Analysis

Signal Wavelet decomposition using Wavelet transform (WT) provides an alternative to the short-time Fourier transform (STFT) for signal analysis [5, 3] resulting in signal decomposition into two-dimensional function of time and scale.

Wavelet functions used for signal analysis are derived from the initial function $W(t)$ forming basis for the set of functions

$$W_{m,k}(t) = \frac{1}{\sqrt{a}} W\left(\frac{1}{a}(t-b)\right) = \frac{1}{\sqrt{2^m}} W\left(2^{-m}t - k\right)$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\ 2^m$. Wavelet dilation closely related to its spectrum compression enables local and global signal analysis. An example of an analytically defined Wavelet function is presented in Fig. 1.

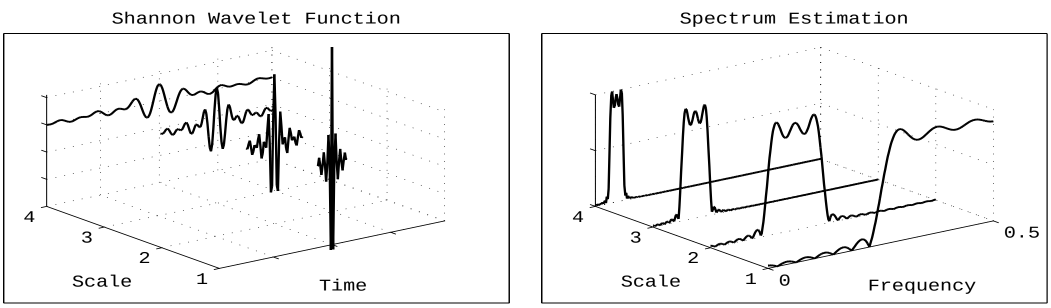

Figure 1: Shanon Wavelet function derived from the initial function defined in the form of relation $W(t)=\sin(\pi t/2)\ \cos(3\pi t/2)/(\pi t/2)$ and the effect of its dilation to spectrum compression

3 Signal Decomposition and Reconstruction

The principle of signal and image decomposition and reconstruction has various common features both in the case of discrete Fourier transform (DFT) and Wavelet transform. The use of discrete Fourier transform is presented in Fig. 2 for an image matrix $[g(n,m)]_{N,M}$ taking into account that a one-dimensional signal can be considered as a special case of an image having one column only. In the *decomposition stage* the discrete Fourier transform is applied to the original matrix column by column at first. Denoting values of a selected column of a matrix $[g(n,m)]_{N,M}$ having index c as $\{x(n)\}_{n=0}^{N-1} = \{g(n,c)\}_{n=0}^{N-1}$ it is possible to find its DFT $X(k)$. The set of indices $k = -N/2, -N/2+1, \cdots, N/2-1$ imply normalized frequencies $f(k) = k/N \in \langle -0.5, 0.5)$. The inverse discrete Fourier transform (IDFT) applied to the sequence $X(k)$ results in the original sequence again. Signal enhancement can be achieved by symmetric extension of the original sequence $X(k)$ by zeros resulting in the sequence

$$[Z(-R/2),\cdots,Z(R/2-1)]^T = [0,\cdots,0,X(-N/2),\cdots,X(N/2-1),0,\cdots,0]^T \quad (1)$$

for even values of $R > N$ using fundamental properties of the discrete Fourier transform. The IDFT of sequence $Z(k)$ results in enhanced sequence $z(n)$ having the sampling period of the length N/R in comparison with the original sampling period equal to one.

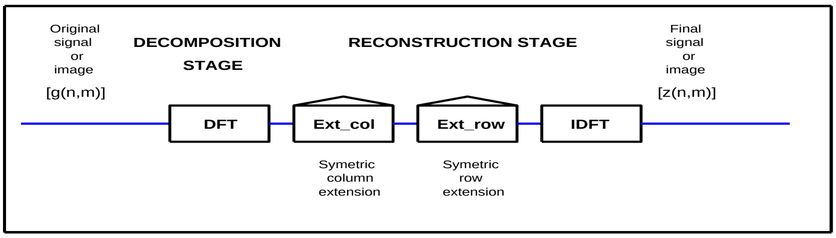

Figure 2: Principle of signal and image resolution enhancement by DFT

The principle of signal and image decomposition and reconstruction using Wavelet transform is presented in Fig. 3 for an image matrix $[g(n,m)]_{N,M}$. The decomposition stage includes

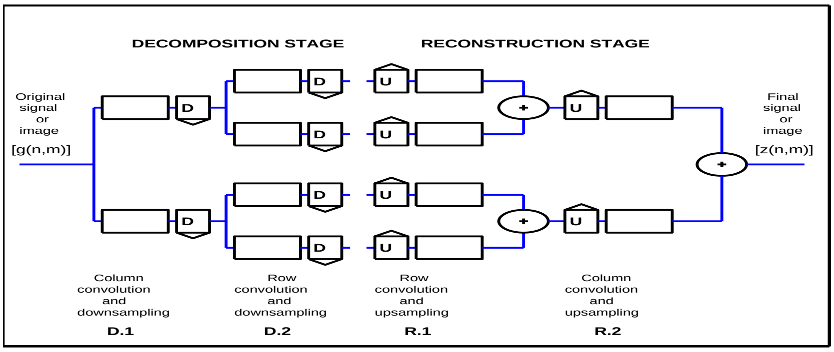

Figure 3: The principle of signal and image resolution enhancement by WT

the processing the image matrix by columns at first using Wavelet (high-pass) and scaling (low-pass) functions followed by row downsampling by factor D in stage $D.1$. In the following decomposition stage $D.2$ the same process is applied to rows of the image matrix followed by column downsampling. The decomposition stage results in this way in four images representing all combinations of low-pass and high-pass initial image matrix processing. The reconstruction stage includes row upsampling by factor U at first and row convolution in stage $R.1$. The corresponding images are then summed. The final step $R.2$ assumes column upsampling and convolution with reconstruction filters followed by summation of the results again.

In the case of one-dimensional signal processing, steps $D.2$ and $R.1$ are omitted. The whole process can be used for

1. Signal/image decomposition and perfect reconstruction using D=2 and U=2
2. Signal/image resolution enhancement in the case of D=1 and U=2

4 Signal and Image De-Noising

Both in the case of one-dimensional and two-dimensional signal Wavelet decomposition it is possible to modify resulting coefficients **c** before the following signal reconstruction to eliminate undesirable signal components. Methods of such a process assume estimation of appropriate threshold limits [6] and their application to Wavelet transform coefficients.

In the case of soft thresholding it is possible to evaluate new coefficients $\overline{c}(k)$ using original coefficients $c(k)$ for a chosen threshold limit δ by relation

$$\overline{c}(k) = \begin{cases} \text{sign } c(k) \ (\mid c(k) \mid - \delta) & \text{if } \mid c(k) \mid > \delta \\ 0 & \text{if } \mid c(k) \mid \leq \delta \end{cases}$$

Results of this process applied to selected real signals are presented in Figs. 4 and 5.

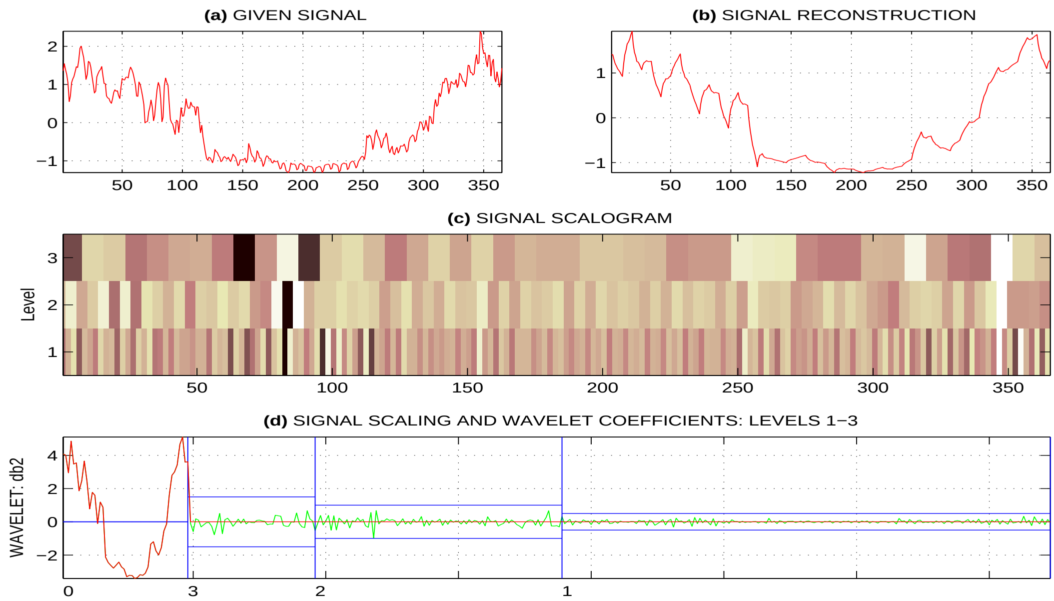

Figure 4: Principle of a gas consumption signal decomposition, thresholding and reconstruction

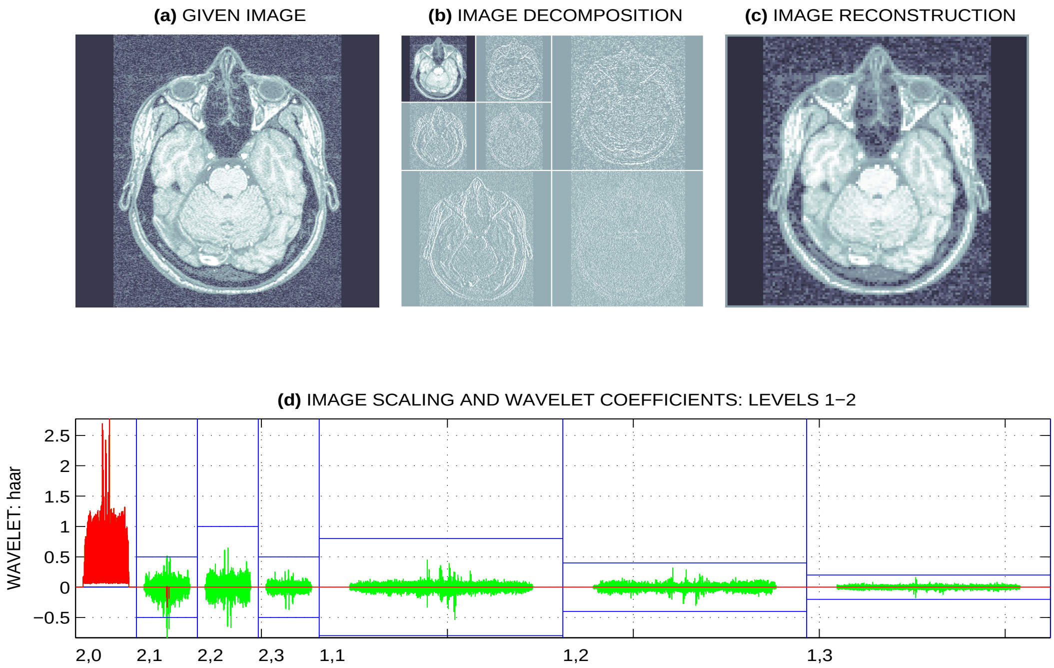

Figure 5: Principle of magnetic resonance image decomposition, thresholding and reconstruction

5 Signal Resolution Enhancement

Fig. 6 presents the main parts of algorithm for one-dimensional signal resolution enhancement in Matlab notation using both discrete Fourier transform and Wavelet transform. Similar approach can be used for image resolution enhancement.

```
% DFT in Signal Resolution Enhancement
% N - length of the sequence
% R - new sequence length
% x - given sequence
  X=fftshift(fft(x));
  Y=wextend('1D','zpd',X,(NZ-N)/2);
  y=R/N*ifft((ifftshift(Y)));
% WT in Signal Resolution Enhancement
% wavelet - definition of Wavelet function
% l - decomposition level
  [c,l]=wavedec(x,level,wavelet);
  [Lo_D,Hi_D,Lo_R,Hi_R]=wfilters(wavelet);
  XL=wconv('1D',x,Lo_D);
  XH=wconv('1D',x,Hi_D);
    XL2=dyadup(XL);
    XH2=dyadup(XH);
  XLL=wconv('1D',XL2,Lo_R);
  XHH=wconv('1D',XH2,Hi_R);
    z=XLL+XHH;
```

Figure 6: Signal resolution enhancement by discrete Fourier transform and Wavelet transform

Fundamental functions used in this program segment include the following:

$X = fft(x)$ – fast Fourier transform of a given sequence $[x(0), x(1), ..., x(N-1)]^T$

$X = fftshift(X)$ – shift of the zero frequency value to the center of the spectrum

$Y = wextend('1D', 'zpd', X, L)$ – symetric extension of a given one dimensional signal by L zeros on both sides

$[c, l] = wavedec(x, level, wavelet)$ – Wavelet decomposition of the signal x at a specified $level$ using a selected $wavelet$ to provide vector c of approximate and detailed wavelet coefficients which are the result of the low-pass and high-pass filtering of the signal and vector l of their lengths

$[Lo\_D, Hi\_D, Lo\_R, Hi\_R] = wfilters(wavelet)$ – definition of four filters associated with the orthogonal or biorthogonal $wavelet$

$XX = wconv('1D', x, f)$ – one dimensional convolution of a signal specified by a vector $[x(0), x(1), ..., x(N-1)]^T$ and a selected filter f

$XX = dyadup(X)$ – extended copy of vector X obtained by inserting zeros after each element of the given vector

Selected results of signal resolution enhancement by Wavelet transform applied to gas consumption data are presented in Fig. 7. Numerical comparison of application of different Wavelet functions is presented in Table 1. Fig. 8 presents selected results of using the Wavelet transform for a magnetic resonance subimage resolution enhancement. Numerical comparison of application of different Wavelet functions is presented in Table 2.

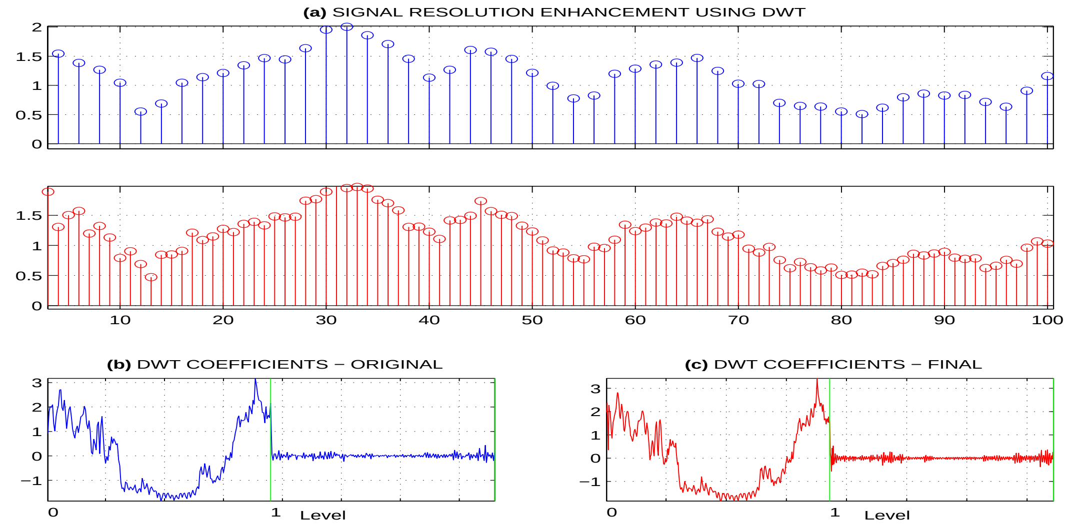

Figure 7: Gas consumption resolution enhancement using Wavelet transform resulting in halving the sampling period

Table 1: Mean square errors (MSE) of selected methods of Wavelet enhancement applied to a gas consumption using different sampling periods

Method	*MSE*	*Method*	*MSE*
Haar Wavelet	0.3296	*Wavelet SYM2*	0.4162
Daubechies Wavelet DB3	0.5661	*Wavelet SYM4*	0.2654
Daubechies Wavelet DB4	0.7919	*Wavelet SYM8*	0.2692

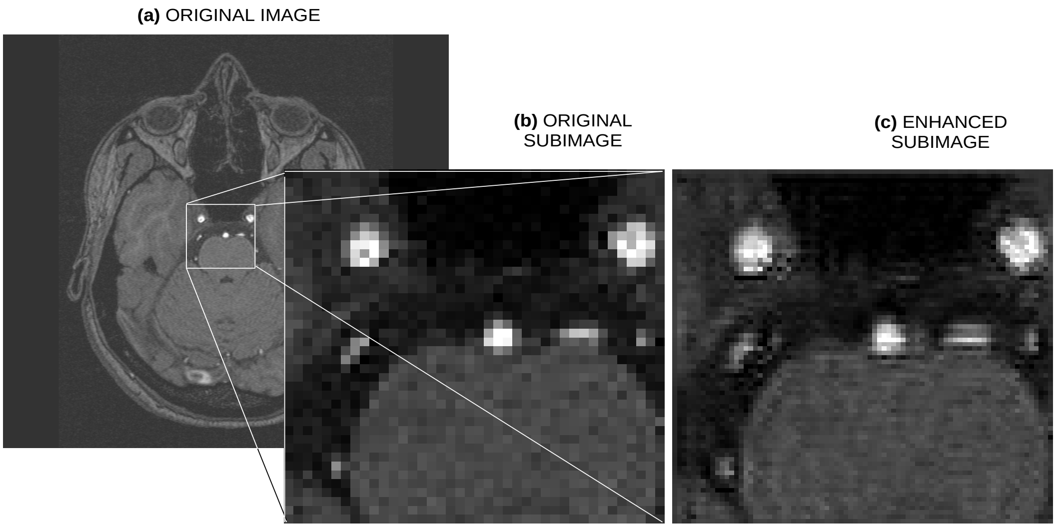

Figure 8: Resolution enhancement of a magnetic resonance subimage using Wavelet transform

Table 2: Mean square errors (MSE) of selected methods of Wavelet enhancement applied to a magnetic resonance subimage

Method	*MSE*	*Method*	*MSE*
Haar Wavelet	0.1206	*Wavelet SYM2*	0.0865
Daubechies Wavelet DB3	0.2072	*Wavelet SYM4*	0.0867
Daubechies Wavelet DB4	0.2655	*Wavelet SYM8*	0.0873

6 Conclusion

The paper presents similar approach to signal and image denoising [7] and resolution enhancement using Wavelet transform and providing comparison of different Wavelet functions. Resulting signal can be then used for its prediction based upon modified sampling period [2]. In the case of the image this approach can be used for reconstruction of missing parts of images and to more precise classification of their regions [4, 1].

Acknowledgments

The work has been supported by the research grant of the Faculty of Chemical Engineering of the Institute of Chemical Technology, Prague No. MSM 223400007. All real MR images were kindly provided by the Faculty Hospital Královské Vinohrady in Prague.

References

[1] S. Armstrong, A.C. Kokaram, and P. Rayner. Reconstructing missing regions in colour images using multichannel median models. In *IXth European Signal Processing Conference EUSIPCO-98*, pages 1029–1032. European Association for Signal Processing, 1998.

[2] D. P. Mandic and J. A. Chambers. *Recurent Neural Networks for Prediction.* John Wiley, 2001.

[3] D. E. Newland. *An Introduction to Random Vibrations, Spectral and Wavelet Analysis.* Longman Scientific & Technical, Essex, U.K., third edition, 1994.

[4] M. Petrou and P. Bosdogianni. *Image Processing, The Fundamentals.* John Wiley & Sons, New York, 1999.

[5] G. Strang. Wavelets and Dilation Equations: A brief introduction. *SIAM Review*, 31(4):614–627, December 1989.

[6] G. Strang and T. Nguyen. *Wavelets and Filter Banks.* Wellesley-Cambridge Press, 1996.

[7] Saeed V. Vaseghi. *Advanced Signal Processing and Digital Noise Reduction.* Wiley & Teubner, West Sussex, U.K., first edition, 1996.

Mgr Irena Šindelářová
University of Economics, Prague
Department of Econometrics
W. Churchill Sq. 4, 130 00 Prague 3
Phone.: 00420-2240 95 443, Fax: 00420-2240 95 432
E-mail: isin@vse.cz

Prof. Aleš Procházka, Ing. Jiří Ptáček
Institute of Chemical Technology, Prague
Department of Computing and Control Engineering
Technická 1905, 166 28 Prague 6
Phone.: 00420-22435 4198, Fax: 00420-22435 5053
E-mail: {J.Ptacek, A.Prochazka}@ieee.org