PARALLEL ROBOTIC STRUCTURES AND THEIR CONTROL IN SIMULINK ENVIRONMENT

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Abstract: Machine tools and centers for industrial purposes should be accurate and proportionally flexible. One possible direction solving mentioned requirements is utilization of robots based on parallel structures. In proposed paper, modeling and control of such robots are discussed. The control is demonstrated by comparative simulation of classical PID/PSD controllers and discrete model-based predictive controllers. Predictive controllers use for design of control action either mechanical or global model. The global model, in comparison with mechanical, includes additionally model of drives (in this paper: DC motors). All together, i.e. classical and predictive control, was parallelly simulated in one block scheme, created under SIMULINK environment.

1. Introduction

Topically, the next development in industrial area is constrained by deficit of powerful machines with proportional dynamics and stiffness. Utilization of parallel robots controlled by suitable control algorithms seems to be promising way to improve dynamics, stiffness, accuracy and productivity of machine tools and their centers, which together with assembly lines largely constitutes the backbone of mass production.

The parallel robots – parallel structures can be simply understood as movable truss constructions or as a movable work platform supported by several parallel arms. Simple comparison of serial and parallel structures is shown in Fig. 1.

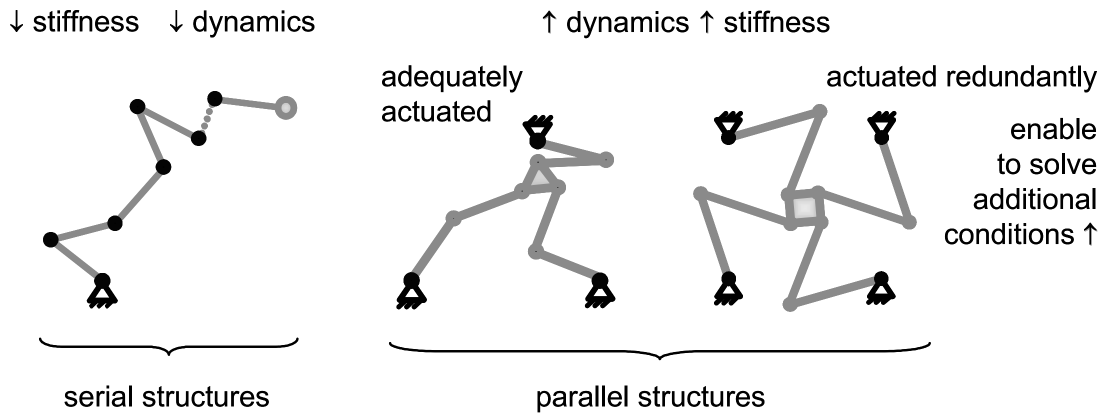

Fig. 1. Comparison of serial and parallel structures.

This paper briefly introduces modeling and control of such robots. After introduction of the theoretical background, the control is demonstrated by comparative simulation of classical PID/PSD controllers and discrete model-based predictive controllers. Predictive controllers are explained for mechanical and global model. The global model, in comparison with mechanical, includes additionally model of drives (in this paper: DC motors).

The fundamental task of control of mechanical structures driven by electromotors is to design appropriate control actions, which accomplish required movement of the movable platform. In general, we must solve the scheme in Fig. 2.

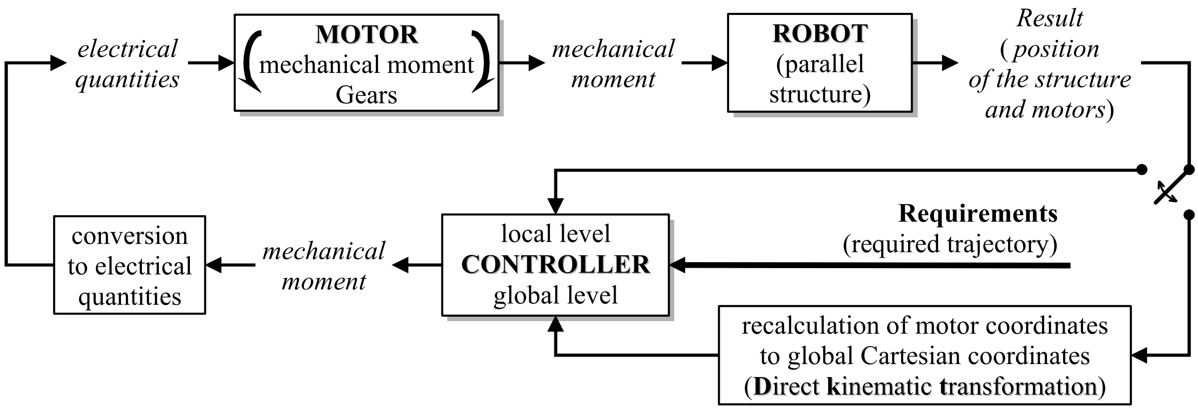

Fig. 2. Conceptual scheme of control circuit
for local (independent) and global (centralized) model-based level.

There are two levels of the control [5] shown in Fig. 2: local and global. Local (decentralized) level, represented by PID/PSD controllers, controls each drive independently, without any consideration of mutual relations. The global (centralized) level represented by predictive controllers uses for control design mathematical model of the robot structure. In control circuit, there are also drives, which represent as well certain dynamics to consideration. The model is used not only for control design, but also for simulation of a real object - - robot structure. Its forms may be different. In the following section, suitable model of the robot and its drives will be defined.

2. Model for simulation and for control design

The robot structure is, in ideal, a system of rigid bodies, thus the classical equations of motion can be composed e.g. by utilization of Lagrange's equations. For simulation and also for control design, the model, based e.g. on Lagrange's equations, is suitable to transform to independent coordinate system (here, it is Cartesian system) [2]:

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{R}^T\mathbf{T}\mathbf{u} \qquad (1)$$

The model (1) is a nonlinear system of ordinary differential equations (ODE), where $\mathbf{M}$ is a mass matrix, $\mathbf{g}$ is a vector of right sides, $\mathbf{T}$ is a redistributional matrix, $\mathbf{R}$ is a Jacobian matrix, $\mathbf{x}$ is a robot output (Cartesian coordinates of a movable work platform, $\mathbf{x} = [x_E, y_E, \psi]$) and $\mathbf{u}$ is an input vector.

This ODE model can be rewritten to different forms according to the requirements of appropriate control approach. Since the robot (multibody system) represents relatively time-consuming computation of elements of the model (1), the discrete approach for global model-based strategies is used. It simply respects the time requirements.

The model in the form (1) can be used for simulation of mechanical structure on simple local level. Thus, let us continue with preparation of the model for global level - - for predictive controllers. As mentioned previously, we choose the discrete approach. To easy describe the structure, we modify the equation system (1):

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{x}} + \mathbf{R}^T\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{x}} = \mathbf{R}^T\mathbf{g} + \mathbf{h}\ , \qquad \mathbf{h} = \mathbf{R}^T\mathbf{T}\mathbf{u} \qquad (2\text{ a, b})$$

and simplify it to the form

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x})\mathbf{h} \qquad (3)$$

The simplification (3), transformed to the state-space formulation, is in a form (4)

$$\begin{aligned}\dot{\mathbf{X}} &= \mathbf{f}(\mathbf{X}) + \mathbf{g}(\mathbf{X})\mathbf{h} \\ \mathbf{x} &= \mathbf{C}\,\mathbf{X}\end{aligned} \tag{4}$$

where the state vector consists of positions and velocities $\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}]^T = [\mathbf{x}_1, \mathbf{x}_2]^T$.

(Note: To preserve the traditional control notation in the following text, the symbol **u** is used instead of **h**.)

For discrete approach, the formulation (4) must be linearized [6] and converted from continuous to discrete domain – for absolute algorithm of predictive controller:

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}\,\mathbf{X}(k) + \mathbf{B}\,\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\,\mathbf{X}(k)\end{aligned} \tag{5}$$

and for incremental algorithm of predictive controller ($\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta\mathbf{u}(k)$):

$$\begin{aligned}\mathbf{X}(k+1) &= \mathbf{A}\mathbf{X}(k) + \mathbf{B}\mathbf{u}(k-1) + \mathbf{B}\Delta\mathbf{u}(k) \\ \mathbf{x}(k) &= \mathbf{C}\mathbf{X}(k)\end{aligned} \tag{6}$$

The base of all predictive controllers is a prediction of new unknown output values **x** from actual topical state **X** in considered horizon of prediction *N*. The prediction of **x** can be defined for both algorithms in one expression:

$$\hat{\mathbf{x}} = \mathbf{f} + \mathbf{G}\bar{\mathbf{u}} \tag{7}$$

where

$$\hat{\mathbf{x}} = [\hat{\mathbf{x}}(k+1), \hat{\mathbf{x}}(k+2), \cdots, \hat{\mathbf{x}}(k+N)]^T \tag{8}$$

for absolute algorithm

$$\bar{\mathbf{u}} = [\mathbf{u}(k), \mathbf{u}(k+1), \cdots, \mathbf{u}(k+N-1)]^T \tag{9}$$

$$\mathbf{f} = \begin{bmatrix}\mathbf{CA} \\ \vdots \\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k), \quad \mathbf{G} = \begin{bmatrix}\mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & & \cdots & \mathbf{CB}\end{bmatrix} \tag{10}$$

and for incremental algorithm with $\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta\mathbf{u}(k)$

$$\bar{\mathbf{u}} = \Delta\mathbf{u} = [\Delta\mathbf{u}(k), \Delta\mathbf{u}(k+1), \cdots, \Delta\mathbf{u}(k+N-1)]^T \tag{11}$$

$$\mathbf{f} = \begin{bmatrix}\mathbf{CA} \\ \vdots \\ \mathbf{CA}^N\end{bmatrix}\mathbf{X}(k) + \begin{bmatrix}\mathbf{C}\,\mathbf{B} \\ \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B}\end{bmatrix}\mathbf{u}(k-1), \quad \mathbf{G} = \begin{bmatrix}\mathbf{C} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{C}(\mathbf{A}^{N-1} + \cdot + \mathbf{A} + \mathbf{1})\mathbf{B} & & \cdots & \mathbf{CB}\end{bmatrix} \tag{12}$$

To obtain the global model of the robot, a model of the drives (electromotors) must be added to the model (1) or to its discrete forms (5) or (6) respectively.

In this paper, the simple brushes DC motors having permanent magnets in stator are considered. Such motors can be described by ordinary differential equation of second order [7]:

$$\ddot{M}_k + \frac{R}{L}\dot{M}_k + \frac{k_{m_1}k_{m_2}}{JL}M_k = \frac{k_{m_1}}{L}\dot{u} \qquad (13)$$

where k_{m_1} and k_{m_2} are torque and speed constants, R and L is terminal resistance and inductance and J is rotor inertia. Equation (13) corresponds with the scheme in Fig. 3.

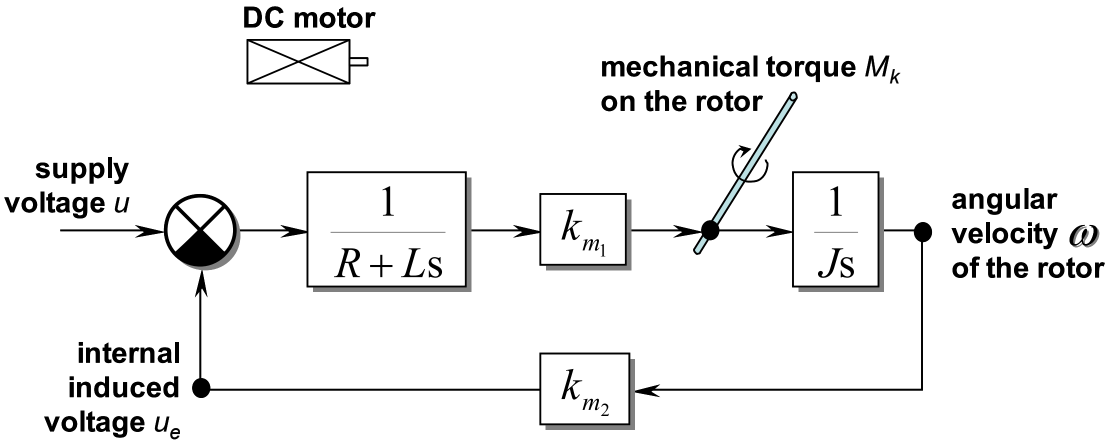

Fig. 3. Block scheme of brushes DC motor
with permanent magnet in the stator.

In equilibrium case ($\omega = 0$), the order of equation (13) is reduced to first order:

$$L\dot{M}_k + RM_k = k_{m_1}u \qquad (14)$$

In transient process, the necessary supply voltage u is a function of load (required) torque $u_r(M_k)$ and angular velocity $u_e(\omega)$ of the rotor.

The global model of whole robot can be composed as follows:

description of mechanical structure:

$$\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{g}(\mathbf{x})\mathbf{M_k} \qquad (15)$$

model of the motors:

$$\dot{\mathbf{M}}_\mathbf{k} = -\frac{R}{L}\mathbf{M_k} + \frac{k_{m_1}}{L}\mathbf{u_r} \qquad (16)$$

the compensation of internal induced voltage u_e:

$$\mathbf{u} = \mathbf{u_r}(\mathbf{M_k}) + \mathbf{u_e}(\boldsymbol{\omega}) = \mathbf{u_r}(\mathbf{M_k}) + k_{m_2}\dot{\boldsymbol{\varphi}} \qquad (17)$$

In general, state formulations (5) and (6) stay unchanged. In detail, the system of equations is only extended by equations of motors and the state vector will include one internal state more

$$\mathbf{X} = [\mathbf{x}, \dot{\mathbf{x}}, \mathbf{M}_k]^T = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3]^T \qquad (18)$$

Finally, the expression for prediction is also the same as written in (7). Now, we can describe the control design.

3. Simple local and model-based predictive control

3.1 Local PID/PSD control

The simplest control approach considered means taking the robots and manipulators, powered by group of independent drives /actuators/, separately controlled, as a set of single input - single output systems (setSISO). Mutual interactions among all drives, caused by different positions during the robot movement, are included as disturbances entering each "single" system constituting the robot.

In that view, the classical PID/PSD feedback control scheme can be used. If it is applied, serious problem of mutual conflict of drives may occur [5]. It is indicated by unpredictable increase of integral/sum (I/S) channels in a controller, caused by the fact, that kinematic description is never perfect, i.e. it does not represent exactly the real kinematics of redundant parallel structure, given by production and partly by topical technological conditions.

Moreover, in case of drive redundancy, there exists no unique transformation between coordinates of drives $\boldsymbol{\varphi}$ and independent coordinates $\mathbf{x}$ here. The PID/PSD controllers try to achieve zero errors for all dependent drive coordinates $\boldsymbol{\varphi}$, but it is sometimes impossible. This fact causes the increase of I/S channels in controllers to saturation.

Idea of the solution is the following: local decentralized controllers compute magnitudes of actuators $\mathbf{u}$ for drives and then some operation as a certain projection is applied to these magnitudes [5]. The projection transforms the actuators to independent space (i.e. it computes so-called general force effects), where the undesirable effects are eliminated, and consecutively the inverse projection recomputes the effects back.

3.2 Model-based predictive control

The derivation of predictive control uses the prediction given by (7). In this paper, the root form is considered. In real computation, it needs matrixes with smaller dimensions and if the penalization λ is nonzero value, it keeps redundant properties (if they exist; see Fig. 1). Moreover, it can accomplish some additional control requirements (e.g. antibacklash condition, smoothing of torques etc.).

To start derivate Predictive Control in the square - root form, let us define quadratic cost function

$$J_k = \mathcal{E}\left\{(\hat{\mathbf{x}} - \mathbf{w})^T(\hat{\mathbf{x}} - \mathbf{w}) + \bar{\mathbf{u}}^T \lambda^T \lambda\, \bar{\mathbf{u}}\right\} \tag{19}$$

whose minimization leads to solving the system of algebraic equations:

$$\mathbf{A}\ \bar{\mathbf{u}}\ -\ \mathbf{b}\ =\ \mathbf{0} \tag{20}$$

For solution, the orthogonal triangular decomposition is used [8]. It reduces matrix $\mathbf{A}$ and vector $\mathbf{b}$ to upper triangular matrix $\mathbf{R}$ and vector $\mathbf{c}$ as follows:

$$\mathbf{A}\,\bar{\mathbf{u}} = \mathbf{b} \quad /\mathbf{Q}^T \tag{21}$$

$$\begin{bmatrix}\mathbf{A}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{u}}\end{bmatrix} = \begin{bmatrix}\mathbf{b}\end{bmatrix} \Rightarrow \begin{bmatrix}\mathbf{R}_1\\ \mathbf{0}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{u}}\end{bmatrix} = \begin{bmatrix}\mathbf{c}_1\\ \mathbf{c}_z\end{bmatrix} \tag{22}$$

Vector $\mathbf{c}_z$ is a residuum vector, whose Euclidean norm $|\mathbf{c}_z|$ is equal to the square root of cost function (19).

Solving the upper part of the system (22), we obtain either values of supply voltage $\mathbf{u}_r(\mathbf{M}_k)$ or fictitious general force effects $\bar{\mathbf{u}} = \mathbf{h}$ or its increments $\bar{\mathbf{u}} = \Delta\mathbf{h}$, respectively. The values of the voltage can be directly applied (Fig. 4). In the second case, when the incremental algorithm is used, the final force effects are given by expression

$$\mathbf{h}(k) = \mathbf{h}(k-1) + \bar{\mathbf{u}}(k) \Big|_{\bar{\mathbf{u}}(k) = \Delta\mathbf{h}(k)} \tag{23}$$

To obtain the real values of actuators **u**, the equation (2 b) must be solved based on fictitious force effects **h** for motor torques **u** (return to initial notation). The solution may not be unique. In general, it represents deficient rank system, where pseudo-inverse operation can be applied [8].

For computation, the values of torques, supply voltage, positions etc. are taken into account in basic units of the system SI (**Mk** [Nm], **u** [V], $\mathbf{x} = [\,x\,[\mathrm{m}], y\,[\mathrm{m}], \psi\,[\mathrm{rad}]]^T$).

(Note: In Fig. 4 and also in the following schemes of control circuit, the model of the robot includes also moment of inertia of the rotor of DC motor J [kg·m^{-2}].)

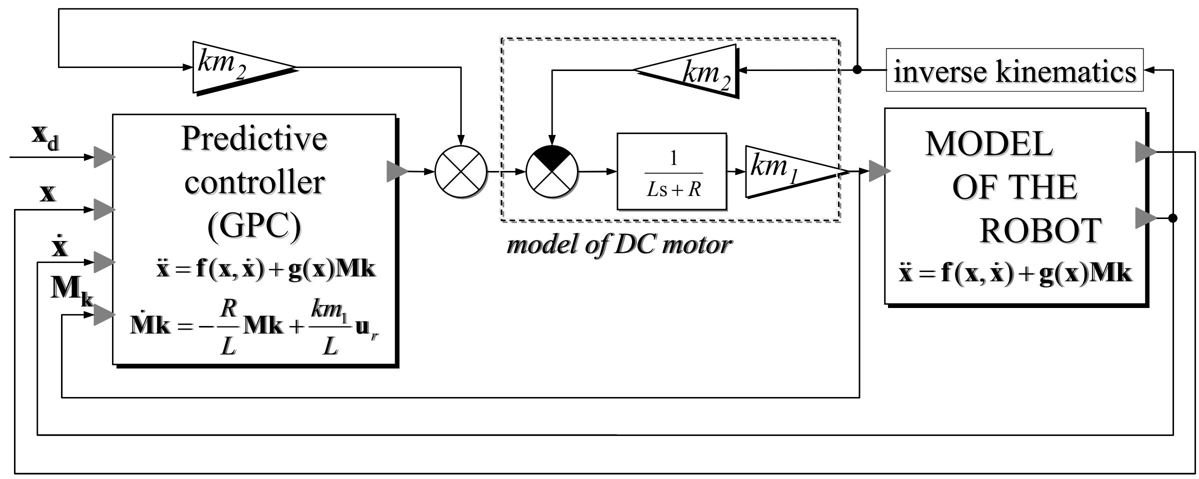

Fig. 4. Scheme of control circuit with predictive controller applied to global model.

Now, let us focus on structure of control shown in Fig. 4. There are several feedbacks there. Position feedback (state variable **x**) is the main; current (torque) and velocity feedbacks are inherent and auxiliary feedbacks respectively. Inherent current feedback represents channel of internal state variable **Mk** (**Mk** = *f*(current ***i*** [A])). Auxiliary velocity feedback (state variable $\dot{\mathbf{x}}$) serves the computation of elements of mechanical model and provides compensation of internal induced voltage $\boldsymbol{u_e}$. All mentioned state variables $\mathbf{x}$, $\dot{\mathbf{x}}$ and **Mk** are assumed to be available.

4. Description of the block scheme in SIMULINK environment

The Fig. 5 shows parallel simulation of the six different structures of controllers applied to mechanical model of the robot - equations of motion (1).

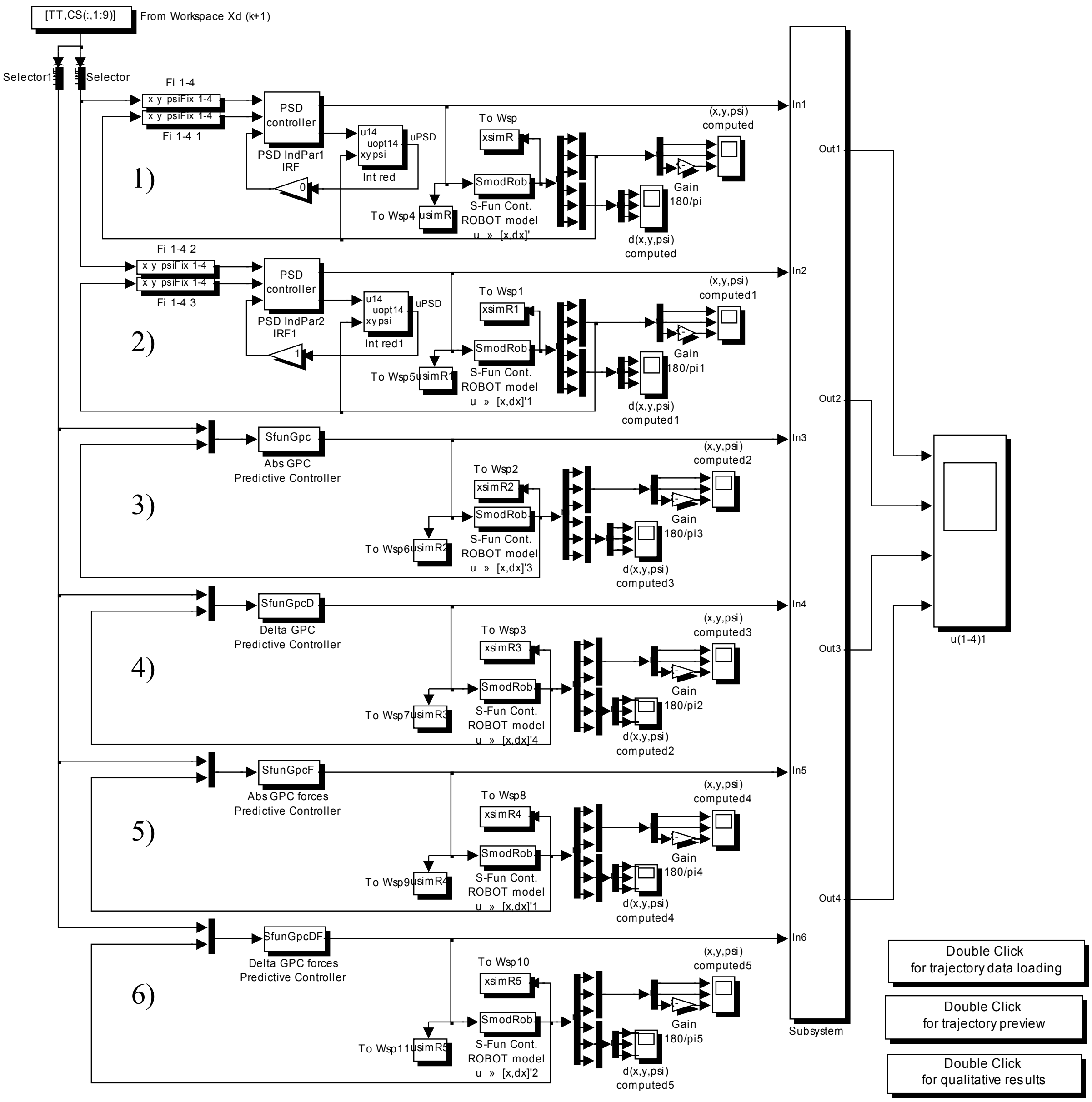

Fig. 5. SIMULINK scheme for parallel comparative simulation
of PID/PSD controllers and predictive controllers;

control circuit 1) PID/PSD without compensation of I/S channels;
2) PID/PSD with compensation of I/S channels;
3) and 4) predictive controllers generating the torques directly;
5) and 6) predictive controllers generating control torques via generalized force effects
(3) and 5) absolute algorithms; 4) and 6) incremental algorithms).

The following figure - Fig. 6 demonstrates predictive controller, which considers global robot model (15)-(17), i.e. it takes into account the model of the drives (DC motors).

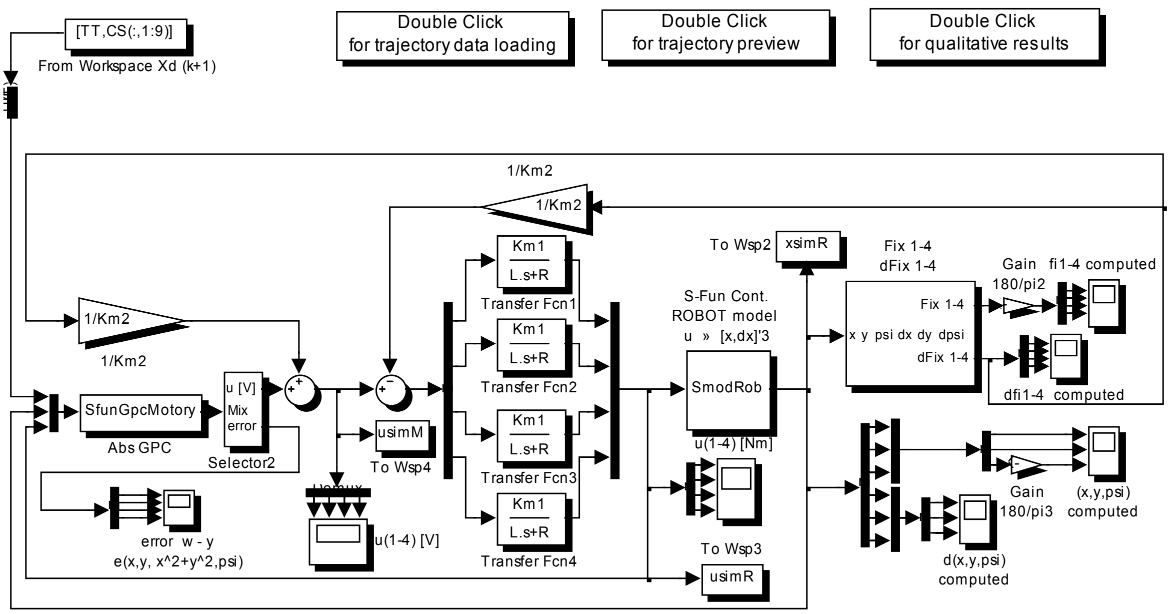

Fig. 6. Control circuit with global robot model.

The simple implementation in SIMULINK environment is provided by classically structured M S-functions (predictive controllers - `SfunGpc*.m`, model of the robot - `SmodRob.m`).

5. Comparative example from simulation

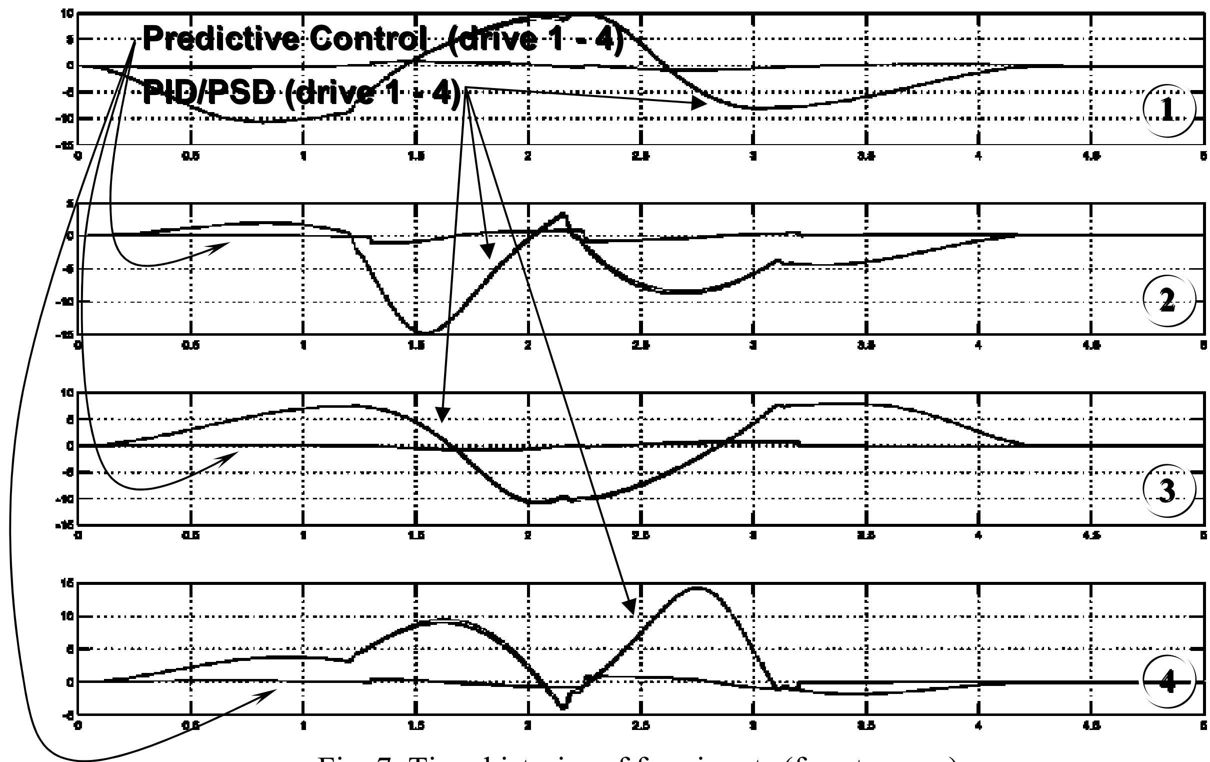

Fig. 7. Time histories of four inputs (four torques)
during parallel simulation of the block scheme shown in Fig. 4.

The time histories (Fig. 7) were recorded during the motion along “*S*- shaped” trajectory. Fig. 7 illustrates all predictive control approaches, with both mechanical and global model. The results of predictive controllers look similar, but in real implementation, are different.

6. Conclusion

The paper deals with a set of possible ways of control and their comparison. It shows that the classical approaches are energetically exigent. They should be, in a future, replaced by such strategies e.g. strategies based on predictive controllers, which consider most of the available information on given controlled object, in this paper, robotic structure driven by DC motors.

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