ANALYSIS OF BIPOLAR PWM FUNCTIONS USING DISCRETE COMPLEX FOURIER TRANSFORM IN MATLAB

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Abstract*. The paper deals with the discrete complex Fourier transform which has been considered for both three- and two phase orthogonal voltages and currents of systems. The investigated systems are power electronic converters supplying alternating current motors. Output voltages of them are strongly non-harmonic ones, so they must be pulse-modulated due to requested nearly sinusoidal currents with low total harmonic distortion. Modelling and simulation experiment results of half-bridge matrix converter for both steady- and transient states are given under substitution of the equivalence scheme of the electric motor by resistive-inductive load and back induced voltage. The results worked-out in the paper confirm a very good time-waveform of the phase current and results of analysis can be used for fair power design of the systems.*

1. Introduction

Time domain waveforms of electrical quantities can be either *continuous* or *discrete*, and they can be either *periodic* or *aperiodic*. This defines four types of Fourier transforms: the Fourier series (continuous, periodic), and the Fourier transform (continuous, aperiodic) and discrete versions: the Discrete Fourier Transform - DFT (discrete, periodic), the Discrete Time Fourier Transform (discrete, aperiodic) [1]-[2]. All four members of the Fourier transform family above can be carried out with either real- or complex input data. In spite of complex amplitudes of harmonic components is notation of Fourier series in complex form more compact and easier than pure real expressions. This holds true also for complex Fourier transform which is very close to complex Fourier series [2]. Both of them are usually handling with real time functions [3], [4]. Method of complex conjugated amplitudes has been used for solving of electrical circuits, and electrical machines, too

However, the output quantities of real power electronic converters can be transformed into complex time functions using Park or Clarke transform, respectively, as vectors rotating in complex Gauss plain. The most advantage of this form of presentation is – in case of symmetrical system - that periodicity of the waveforms in complex plain is *2m*-times higher then in real time domain. So, the Fourier analysis, also integral values calculation, can be done more quickly. Another benefit is possibility of direct using of complex Fourier transform/series because of quantity functions present complex input data for continuous or digital processing.

2. Using orthogonal output voltages and complex Fourier analysis

Applying Park/Clarke transform the complex time function of output phase voltage in three-phase system is, Fig. 1

$$\boldsymbol{u}(t) = \frac{2}{3}\left[u_1(t) + \mathbf{a}\cdot u_2(t) + \mathbf{a^2}\cdot u_3(t)\right] = u_\alpha + \mathrm{j}\cdot u_\beta \qquad (1)$$

where after adapting

$$u_\alpha = \frac{1}{3}\left[2u_\mathrm{a}(t) - u_\mathrm{b}(t) - u_\mathrm{c}(t)\right] \qquad u_\beta = \frac{\sqrt{3}}{3}\left[u_\mathrm{b}(t) - u_\mathrm{c}(t)\right] \qquad (2a,b)$$

It deals with the voltage vectors rotating in Gauss *α,β*-plain by angular speed ω which can be also non-constant.

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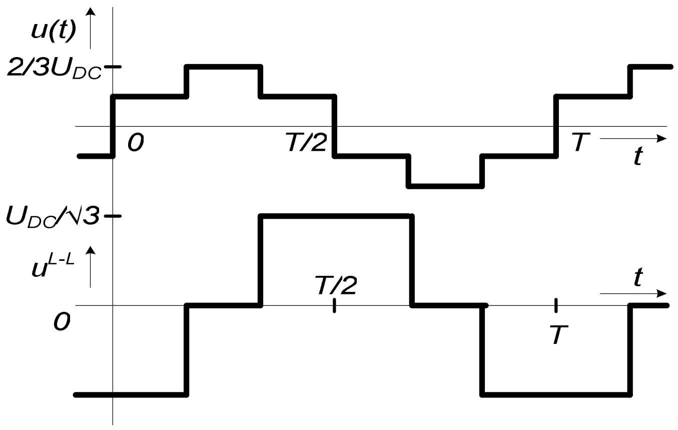


Fig. 1 Orthogonal voltage systems of three-phase inverter with full-width pulses: direct u_α (a)- and quadrature voltage u_β (b)

Now, the voltages u_α and u_β create orthogonal system, and complex Fourier transform can be used. So, then the complex Fourier transform or/and complex Fourier coefficients can be calculated

$$\boldsymbol{U}(\nu\omega t) = \int_0^T \boldsymbol{u}(t)\cdot\exp(-\mathrm{j}\,\omega t)dt \tag{3}$$

or, respectively

$$\boldsymbol{C}_\nu = \frac{1}{T}\int_0^T \boldsymbol{u}(t)\cdot\exp(-\mathrm{j}\,\nu\frac{2\pi}{T}t)dt \tag{3a}$$

whereby their mutual relation is $\boldsymbol{C}_\nu = \frac{1}{T}\boldsymbol{U}(\nu\omega t)$ (3b)

where $\omega = 2\pi/T$.

The discrete Fourier transformation has been used for calculation of individual harmonics coefficients [2]:

$$\boldsymbol{U}[\nu] = \frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{u}[n]\cdot\exp(-\mathrm{j}\,\nu\frac{2\pi}{N}n) \tag{4}$$

Alternatively, Euler's relation can be used to rewrite the forward transform in rectangular form:

$$\boldsymbol{U}[\nu] = \frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{u}[n]\cdot\left(\cos(2\pi\nu n/N) - \mathrm{j}\cdot\sin(2\pi\nu n/N)\right) \tag{4a}$$

Real and imaginary part of $\boldsymbol{U}(\nu)$ can also be expressed:

$$\mathrm{Re}\{U(\nu)\} = \frac{2}{N}\sum_{n=0}^{N-1}u(n)\cos\left(2\pi\frac{n\nu}{N}\right);\ \mathrm{Im}\{U(\nu)\} = \frac{-2}{N}\sum_{n=0}^{N-1}u(n)\sin\left(2\pi\frac{n\nu}{N}\right) \tag{5a,b}$$

Based of above definition the relation for complex Fourier coefficients of complex voltage function yields:

$$\boldsymbol{C}_\nu = \frac{1}{T}\int_0^T\left[u_\alpha(t) + \mathrm{j}u_\beta(t)\right]\cdot\exp(-\mathrm{j}\,\nu\frac{2\pi}{T}t)\,dt \tag{6}$$

Eq. (6) can be decomposed into two scalar equations for C_ν^α and C_ν^β, if needed:

$$C_{\nu\alpha} = \frac{2}{T}\int_0^T u_\alpha(t)\cdot\exp(-\mathrm{j}\,\nu\frac{2\pi}{T}t)dt \tag{6a}$$

$$C_{\nu\beta} = \frac{2}{T}\int_0^T u_\beta(t)\cdot\exp(-\mathrm{j}\,\nu\frac{2\pi}{T}t)\,dt \tag{6b}$$

Such a Fourier series is developing on system of orthogonal functions exp(*j.n.*2π*.t/T*), *n* = 0, ±1, ±2.., for which the integral

$$\int_0^T \exp(-\mathrm{j}n\frac{2\pi}{T}t)\exp(-\mathrm{j}m\frac{2\pi}{T}t)dt \qquad (6c)$$

is equal to *0* for $m \neq -n$, and equal to *T* for $m = -n$.

The system of voltages is ortho-normal one, too. Since u_α voltage will contain sin-terms only, the second one u_β cos-terms.

3. Complex Fourier analysis of the voltage of AC/AC half-bridge matrix converter system

Matrix converter system DC/HF_AC/2AC with high frequency AC interlink can generate two-phase orthogonal output with both variable voltage and frequency [4] and others. Usually, the switching frequency of the converter is rather high (~tens kHz). Equivalent circuit diagram of one half-bridge single phase converter (one of two-phase orthogonal systems) is depicted in Fig. 2. Since the voltages of the matrix converter system are orthogonal, the second phase converter is the same and its voltage is shifted by 90 degree.

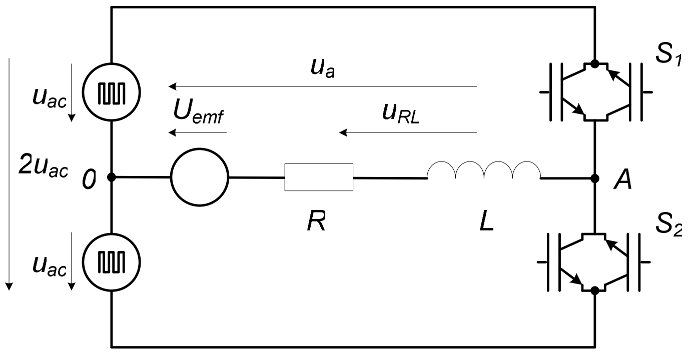


Fig. 2 Circuit diagram of single-phase half-bridge matrix converter

Contrary to bridge matrix converter the half-bridge connection doesn't provide unipolar PWM control, so the bipolar pulse switching technique should be used. The orthogonal voltages with bipolar PWM control are depicted in Figs. 3a and 3b.

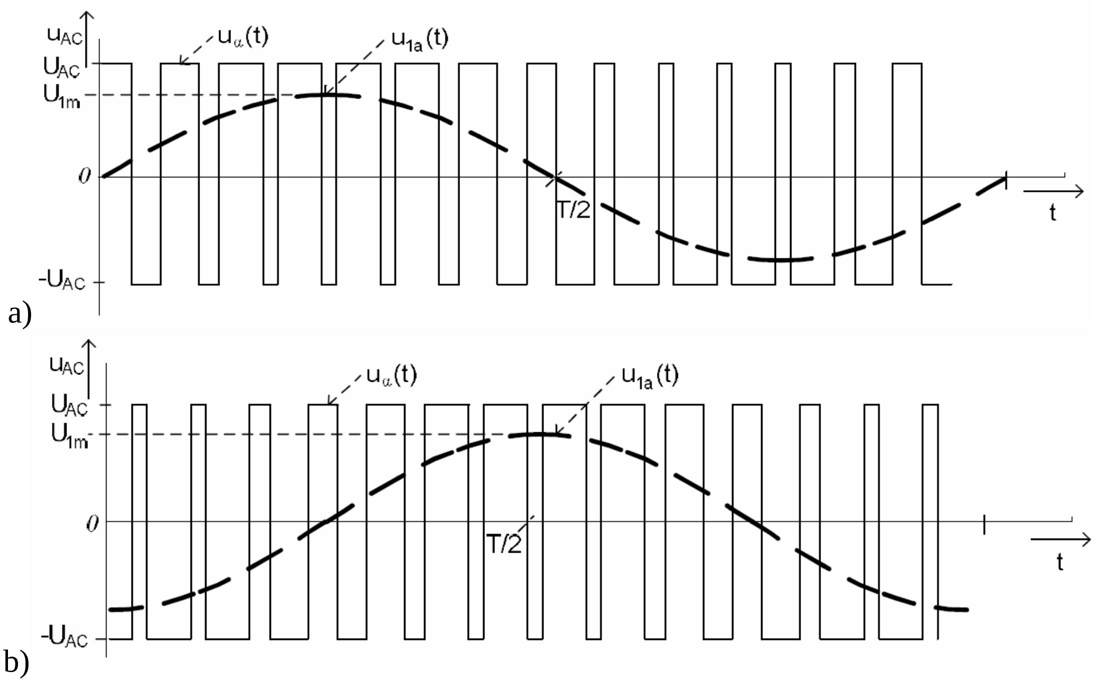


Fig. 3 Output orthogonal voltages of the half-bridge matrix converter system with bipolar PWM

Considering bipolar PWM with switching frequency equal to odd multiply of fundamental frequency.

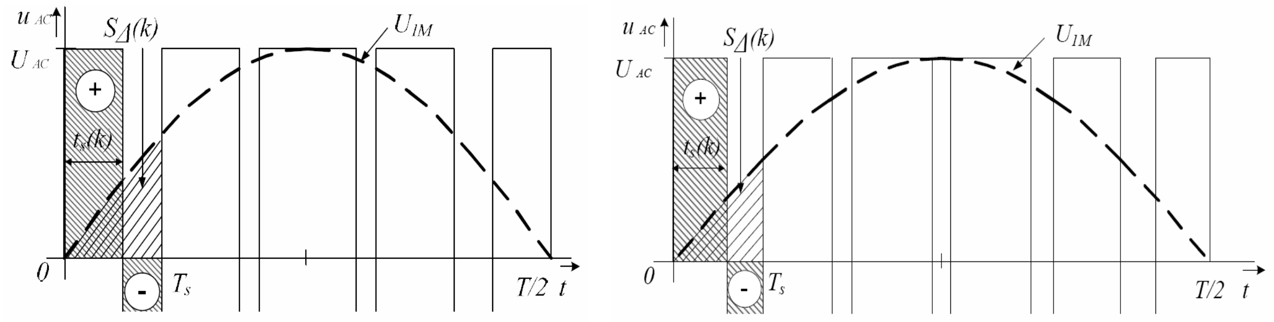
It deals with sinusoidal bipolar pulse-width-modulation contrary to unipolar regular PWM [5], [6]. Switching-pulse-width can be determined based on equivalence of average values of reference waveform and resulting average value of positive and negative switching pulses area during switching period (see Figs. 5a,b).

First, let’s define both amplitude- and frequency modulation ratios m_a and m_f as:

$$m_a = \frac{U_{1m}}{U_{AC}}; \qquad m_f = \frac{f_s}{f_1}, \tag{7a,b}$$

where U_{1m} is reference amplitude of fundamental harmonic,
- U_{AC} magnitude of supply voltage,
- f_s switching frequency,
- f_1 fundamental frequency.

Choosing frequency modulation ratio m_f as odd integer results in an odd symmetry [$u(-t) = - u(t)$] as well as half-wave symmetry [$u(-t) = - u(t+T_s/2)$] with the time origin shown in Fig. 4. Therefore, only odd harmonics are present and the even harmonics disappear from the wave form of u_a. Moreover, only the coefficients of the sine series in Fourier analysis are finite; those for the cosine series are zero.



a) b)

Fig. 5 Output voltages of single-phase bridge matrix converter with odd- a) and even m_f b)

Then total voltage time waveform will be:

$$\begin{aligned} u(t) = \sum_{\nu=1}^{\infty} \frac{4 \cdot U_{DC}}{\nu \cdot \pi} \sum_{k=0}^{\frac{(m_f-2)}{4}-1} \Bigg\{ & \left[\cos\left(\nu \cdot k \cdot \frac{2\pi}{m_f}\right) - \cos\left(\nu \cdot k \cdot \frac{2\pi}{m_f} + \nu \cdot \omega \cdot t_s(k)\right)\right] - \\ & - \left[\cos(\nu \cdot (k+1) \cdot \frac{2\pi}{m_f} - \nu \cdot \omega \cdot t_s(k)) - \cos(\nu \cdot k \cdot \frac{2\pi}{m_f}) \right] + \\ & + \left[\cos\left(\nu \cdot \frac{2\pi}{m_f} \cdot \frac{m_f - 2}{4}\right) - \cos\left(\nu \cdot \left(\frac{2\pi}{m_f} \cdot \frac{m_f-2}{4} + \frac{\pi}{m_f}\right)\right)\right]\Bigg\} \cdot \sin(\nu \cdot \omega t) \end{aligned} \tag{8}$$

where switching instant is equal to:

$$t_s(k) = \frac{1}{2U_{DC}} \cdot S_\Delta(k) + \frac{T_s}{2} \quad \text{and} \quad t_s\left(\frac{m_f - 2}{4}\right) = \frac{T_s}{2} \tag{9a,b}$$

and $S_\Delta(k)$ is area under sinewave during k-switched interval:

$$S_\Delta(k) = U_m \cdot \frac{m_\mathrm{f}}{2\pi} \cdot \left[\cos\left(\frac{2\pi}{m_\mathrm{f}} \cdot k \right) - \cos\left(\frac{2\pi}{m_\mathrm{f}} \cdot (k+1) \right) \right] \tag{10a}$$

and

$$S_\Delta\left(\frac{m_\mathrm{f}-2}{4}\right) = U_m \cdot \frac{m_\mathrm{f}}{2\pi} \cdot \left[\cos\left(\nu \cdot \frac{2\pi}{m_\mathrm{f}} \cdot \frac{m_\mathrm{f}-2}{4} \right) - \cos\left(\nu \cdot \left(\frac{2\pi}{m_\mathrm{f}} \cdot \frac{m_\mathrm{f}-2}{4} + \frac{\pi}{m_\mathrm{f}} \right) \right) \right] \tag{10b}$$

For the parameters (the same as in [11] to be compared):

2 x $U_{DC} = 300$ V – input voltage,
$f_\mathrm{IN} = f_\mathrm{S}$ = 39 kHz – switching frequency,
f_OUT = 50 Hz – fundamental output frequency,
m_a = 1; m_f = 39 – amplitude and frequency ratios,

The discrete complex Fourier transformation has been considered for both single- and two phase orthogonal systems. Based on discrete formulas (4)-(5a) the amplitudes of the first 30 voltage harmonics (by 165$^{\mathrm{th}}$-harmonic) have been calculated:

$A_1 = 150.m_\mathrm{a}$ = 150 V; A_{39} = 90.16 V; $A_{39-2} = A_{39+2}$ = 47.70 V; $A_{39-4} = A_{39+4}$ = 2.70 V; $A_{78-1} = A_{78+1}$ = 27.15 V; $A_{78-3} = A_{78+3}$ = 31.80 V; $A_{78-5} = A_{78+5}$ = 4.95 V; A_{117} = 16.95 V; $A_{117-2} = A_{117+2}$ = 9.30 V; $A_{117-4} = A_{117+4}$ = 23.55 V; $A_{117-6} = A_{117+6}$ = 6.60 V; $A_{156-1} = A_{156+1}$ = 10.20 V; $A_{156-3} = A_{156+3}$ = 1.35 V; $A_{156-5} = A_{156+5}$ = 17.85 V; $A_{156-7} = A_{156+7}$ = 7.50 V;

The harmonic spectrum is plotted in Fig. 6, which is plotted for m_f = 39.

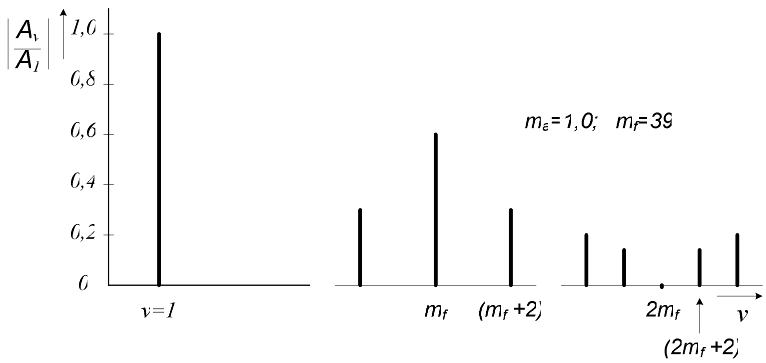


Fig. 6 Harmonic amplitude spectrum of bipolar PWM with odd m_f

Note: The carried-out results are almost identical ones compared with those given in [5] for DC/AC inverter with bipolar sinusoidal PWM.

Considering bipolar PWM with switching frequency equal to even multiply of fundamental frequency.

The voltage waveform is depicted in Fig. 5b above. Then total voltage time waveform will be:

$$u(t) = \sum_{\nu=1}^{\infty} \frac{4 \cdot U_\mathrm{DC}}{\nu \cdot \pi} \sum_{k=0}^{(m_f/4)-1} \left\{ \left[\cos\left(\nu \cdot k \cdot \frac{2\pi}{m_\mathrm{f}} \right) - \cos\left(\nu \cdot k \cdot \frac{2\pi}{m_\mathrm{f}} + \nu \cdot \omega \cdot t_s(k) \right) \right] - \left[\cos(\nu \cdot (k+1) \cdot \frac{2\pi}{m_\mathrm{f}} - \nu \cdot \omega \cdot t_s(k)) - \cos(\nu \cdot k \cdot \frac{2\pi}{m_\mathrm{f}}) \right] \right\} \cdot \sin(\nu \cdot \omega \cdot t) \tag{11}$$

where switching instant is equal to:

$$t_s(k) = \frac{1}{2U_{DC}} \cdot S_\Delta(k) + \frac{T_s}{2} \tag{11a}$$

and $S_\Delta(k)$ is area under sinewave during k-switched interval:

$$S_\Delta(k) = U_m \cdot \frac{m_f}{2\pi} \cdot \left[\cos\left(\frac{2\pi}{m_f} \cdot k \right) - \cos\left(\frac{2\pi}{m_f} \cdot (k+1) \right) \right] \qquad (11b)$$

4. PC simulation in MatLab programming environment

Simulation experiments have been done for the parameters: R = 10 Ohm, L = 25 mH, U = 150 V, f = 50 Hz at m_a = 1, m_f = 39, time increment Δt = 5 μs.

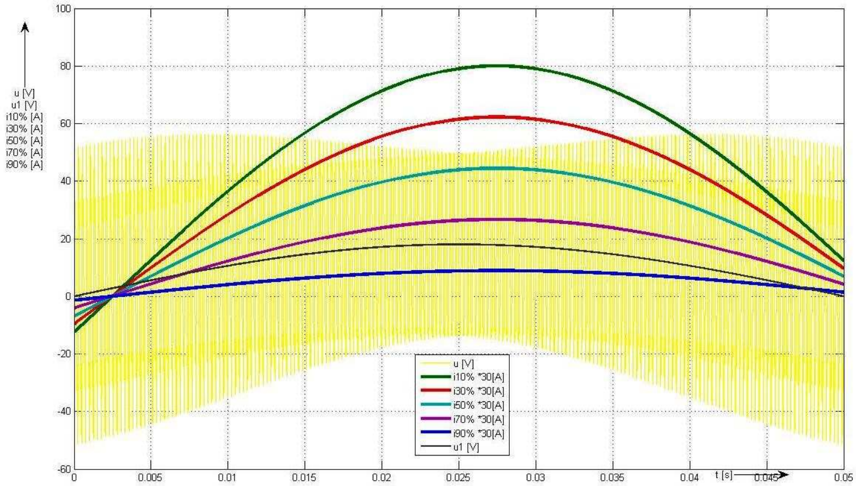


Fig. 7: Time waveform of voltage (1. harmonic component) and load current – with various counter-voltage and modulation index of bipolar PWM m_a=0.2-0.8 and m_f=39

Root-mean-square value of the total steady-state current of R-L load can be calculated as:

$$I = \sqrt{\sum \left(\frac{I_{\nu m}}{\sqrt{2}} \right)^2} = \sqrt{\sum I_\nu^{\,2}} = \sqrt{\sum \left(\frac{U_\nu}{|\underline{Z}_\nu|} \right)^2} = \sqrt{\sum \left(\frac{U_\nu}{\sqrt{R^2 + (\nu \cdot \omega \cdot L)^2}} \right)^2} \qquad (12)$$

The total harmonic distortion of the current is given by:

$$\sqrt{\left(\frac{I}{I_1} \right)^2 - 1} = \sqrt{\frac{\sum \left(\frac{U_\nu}{|\underline{Z}_\nu|} \right)^2}{\left(\frac{U_1}{|\underline{Z}_1|} \right)^2} - 1} = \sqrt{\frac{\sum \frac{U_\nu^{\,2}}{R^2 + (\nu \cdot \omega \cdot L)^2}}{\frac{U_1^{\,2}}{R^2 + (\omega \cdot L)^2}} - 1} \sim 2\ \% \qquad (12a)$$

5. Conclusions

The complex Fourier transformation has been considered for two phase orthogonal systems of converter output voltages, strongly non-harmonic ones. The solution given in the paper makes it possible to analyse more exactly effect of each harmonic component comprised in total waveform on resistive-inductive load or induction motor quantities. The proposed system with AC interlink in comparison with currently used conventional systems uses two single phase half bridge matrix converters with bipolar pulse-width modulation. The advantage is then less number of semiconductor devices of the converters. However, in practice, the necessary imposition of a dead-band, or blanking time, results in some distortion of the output voltage. Then the dead-band, its symptoms and related remedies, is necessary to take into account for solutions.

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