# DECENTRALIZED ROBUST $H_\infty$ CONTROL OF MECHANICAL STRUCTURES

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## 1. Introduction

The results contributed by this paper concern simulation experience with decentralized control design for the building structure under strong wind or earthquake excitations such that the effect of disturbances on the overall system is reduced to a required acceptable level. The Riccati equation approach with paramater uncertainties is used to design decentralized memoryless $H_\infty$ controllers. We suppose unknown but constant parameter uncertainties in damping and stiffness matrices with known upper bounds. The following simulation cases are considered:

1. Robust decentralized control design with neglected couplings.
2. Robust decentralized control design with included only matched couplings.
3. Robust decentralized control design with included both matched and mismatched couplings.
4. Centralized control design serving as a reference.

The results are given in the form of figures of the overall system responses and corresponding control effort for these cases. Selected relevant references are supplied [1]–[11].

## 2. The problem

Consider the system to be controlled, including uncertainties, in the form:

$$\dot{x}_i(t) = (A_i + \Delta A_{ii}(t))x_i(t) + \sum_j A_{ij}(t)x_j(t) + B_i u_i(t) + B_{wi} w_i, \tag{1}$$

where $x_i(t) = 0$ if $t < 0$, $i = 1, ..., N$. $x_i(t), u_i(t), w_i(t)$ is $n_i-, m_i-, p_i-$ dimensional state, input, disturbance vector of the $i-$th subsystem, respectively. $N$ denotes the number of susystems. Suppose that all matrices in (1) satisfy the dimensionality requirements. $A_i, B_i, B_{wi}$ are constant matrices. Uncertainty matrices and interconnections are defined as follows:

$$\Delta A_{ii}(t) = D_{ii} F_{ii}(t) E_{ii}, \qquad A_{ij} = D_{ij}(t) E_{ij}, \tag{2}$$

where $D_{ii}, D_{ij}, E_{ii}, E_{ij}$ are known real matrices of appropriate dimensions. These matrices specify the locations of uncertainties and interconnections. Note only it is well known that the decomposition (2) always exists. $F_i(t) = (F_{i1} \; ... \; F_{iN})$ is an unknown matrix with Lebesgue measurable functions representing admissible uncertainties and satisfying the relation

$$F_i(t)^T F_i(t) \leq I. \tag{3}$$

We are interested in designing a state memoryless feedback controller

$$u_i(t) = -\frac{1}{\varepsilon_i} B_i P_i x_i(t) = -K_i x_i(t) \tag{4}$$

for all $i$, such that the overall closed-loop system

$$\dot{x}(t) = (A - BK + \Delta A)x(t) + B_w w \tag{5}$$

satisfies, with the zero-initial condition for $x(t)$, the relation

$$\|z\|_2 \leq \gamma \|w\|_2, \quad z(t) = \begin{bmatrix} Hx(t) \\ Rx(t) \end{bmatrix} \tag{6}$$

for all admissible uncertainties and all nonzero $w \in L_2[0,\infty]$. $\varepsilon_i$ is a positive scalar, $\|.\|_2$ denotes the usual $L_2[0,\infty]$ norm. $x = (x_1^T, ..., x_N^T)^T, w = (w_1^T, ..., w_N^T)^T$ denotes the overall $n-, p-$dimensional state, disturbance vector, respectively. $H, R$ are constant weighting selection matrices. The matrices in the overall system description given by (5) are defined as follows:

$$\begin{array}{rl} A = [A_{ij}(t)], & \Delta A = diag(\Delta A_{11}, ..., \Delta A_{NN}), \\ B = diag(B_1, ..., B_N), & B_w = diag(B_{w1}, ..., B_{wN}), \\ P = diag(P_1, ..., P_N), & K = diag(K_1, ..., K_N). \\ H = diag(H_1, ..., H_N), & R = diag(R_1, ..., R_N). \end{array} \tag{7}$$

$\gamma$ is a given scalar which defines the prescribed level of disturbance attenuation.

The goal is to present the simulation experience with the control design for structural systems when applying of the solution of the state-space $H_\infty$ decentralized stabilization problem for the system (1)-(2) by using state feedback (4) and interpreting the first order system result for structural systems.

## 3. Background - Theory

Let us introduce first some basic necessary concepts. Consider the following system

$$\dot{x} = (A + \Delta A)x + B_w w, \tag{8}$$

where $A, \Delta A$ have the same meaning as the matrices in (5).

*Definition.* Given a scalar $\gamma > 0$. The system (8) is said to be *quadratically stable with an $H_\infty$-norm bound* $\gamma$ (QSH) if it satisfies for any admissible parameter uncertainty $\Delta A$ the folllowing conditions:

1. The system is asymptotically stable.
2. Subject to the assumption of the zero-initial condition, the state $x$ satisfies the inequality (6).

The main result is given in the form of the following theorem.

*Theorem.* Given a scalar $\gamma > 0$. Consider the system (1)-(2). Suppose that for some $\varepsilon_i > 0$ and $\eta_i > 0$ there exists a positive definite solution $P_i$ of the following equation

$$\begin{array}{c} P_i A_i + A_i^T P - \frac{1}{\varepsilon_i} P_i B_i R_i^{-2} B_i^T P_i + \gamma^2 P_i B_{wi} B_{wi}^T P_i \\ + H_i H_i^T + \eta_i P_i D_i P_i + \frac{1}{\eta_i} E_i = -\varepsilon_i Q_i, \quad i = 1, \ldots, N, \end{array} \tag{9}$$

where $Q_i > 0,\ D_i = \sum_j D_{ij} D_{ij}^T,\ E_i = \sum_j E_{ij}^T E_{ij}$. Then the closed-loop system (1)–(4) is QSH.

*Proof.* See [4] for details.

Now, let us deal with the interpretation of (1) for structural systems. We follow its derivation presented in [7].

Consider the structural dynamic system in the form

$$M\ddot{x}_m + (C + \Delta C)\dot{x}_m + (K + \Delta K)x_m = B_m u(t) + B_{wm} w = f(t), \tag{10}$$

where $x_m$ is a generalized position vector, $f(t)$ is a force vector. $M = M^T$ is a mass matrix, $C = C^T \geq 0$ is a damping matrix, $K = K^T \geq 0$ is a stiffness matrix. $\Delta C(t) = \Delta C^T(t) \geq 0$, $\Delta K(t) = \Delta K^T(t) \geq 0$, for all $t \geq 0$, are the uncertainty matrices corresponding with $C,\ K$, respectively. $B_m,\ B_w$ are constant matrices. Suppose that

$$\Delta C(t) = D_c F_c(t) E_c, \quad \Delta K(t) = D_k F_k(t) E_k, \tag{11}$$

where $F_c(t),\ F_k(t)$ satisfy the same inequality as $F_i(t)$ in (3). Denoting $x=(x_m^T,\dot{x}_m^T)^T$, we get

$$\dot{x}=\left[\begin{array}{cc} 0 & I \\ -M^{-1}(K+\Delta K) & -M^{-1}(C+\Delta C) \end{array}\right]x+\left[\begin{array}{c} 0 \\ M^{-1} \end{array}\right]f(t). \tag{12}$$

The first order system description, when relating the matrices in (7) with (12), has the form

$$A=\left[\begin{array}{cc} 0 & I \\ -M^{-1}K & -M^{-1}C \end{array}\right],\quad \Delta A=\left[\begin{array}{cc} 0 & I \\ -M^{-1}\Delta K & -M^{-1}\Delta C \end{array}\right],$$
$$B=\left[\begin{array}{c} 0 \\ M^{-1} \end{array}\right]B_m,\ \ B_w=\left[\begin{array}{c} 0 \\ M^{-1} \end{array}\right]B_{wm}. \tag{13}$$

The interconnected system decomposition corresponding to (1) has the form

$$M_i\ddot{x}_{mi}+(C_i+\Delta C_i)\dot{x}_{mi}+(K_i+\Delta K_i)x_m+\sum_{j=0,j\neq i}^{N}[M_{ij}\ddot{x}_{mj}+\Delta C_{ij}\dot{x}_{mj}$$
$$+\Delta K_{ij}x_{mj}]=B_{mi}u_i(t)+B_{wmi}w_i, \tag{14}$$

where $x_i=(x_{mi}^T,\dot{x}_{mi}^T)^T$. The relation between the matrices in (1) and (14) is straightforward. Only the corresponding indices must be supplied.

## 4. Simulation experience

Consider a six-degree-of-freedom (DOF) building structure subject to a horizontal acceleration, as an external disturbance by Bakule *et al.* [2] in Example 3. This structure is decomposed into two disjoint subsystems (floors 1-3 and 4-6) with two actuators supplying active control forces, which are initially located at the 2nd and the 5th floors. The experimentally identified damping and stiffness matrices in this structure have been reduced to tridiagonal matrices by neglecting off-tridiagonal terms. The open-loop system eigenvalues have not been changed by this neglection, but the simplified model does not satisfy standard modelling requirements, i.e. the sum of off-diagonal terms in $C$ and $K$ is not equal to the diagonal element with the opposite sign for the floors 2-5. To satisfy these requirements, the sum is considered as a nominal value and the difference as uncertainties, i.e. $F(t)$ is considered as a constant matrix. The magnitude of uncertainties in stiffness matrix grows when moving from the bottom to the upper floors. It confirmes the physical expectations. The presented simulation study includes 4 cases as follows:

*Case 1:* Decentralized control design for individual subsystems with neglected couplings. It means to solve (9) with $D_{ij}=0, E_{ij}=0$ and consequently to check the overall systems stability only. Actuator locations remain in the initial positions at the floors 2 and 5. Two figures are supplied: Fig.1 (denoted as case 1a) with $\gamma_1=0.104,\gamma_2=0.16,\eta_1=1,\ \eta_2=1,H_1=H_2=1,R_1=R_2=1$ and Fig.2 (denoted as case 1b) with $\gamma_1=1.68,\gamma_2=.611,\eta_1=1,\eta_2=1,H_1=H_2=100,R_1=R_2=0.1$.

Note that the influence of uncertainties in damping and stiffness matrices has been also tested in this way. Though the maximal magnitude of parameter uncertainty reached about 30%, it has been verified that it has a minimal influence on the system behaviour. It means that the system responses and the closed-loop system eigenvalues remain without essential changes.

*Case 2:* Decentralized control design according to Theorem. It means that the interconnections have been incorporated into (9). Because the actuator locations remain in the initial positions. i.e at the floors 2 and 5, the interconnections are mismatched. We did not found the solution in this case. Because of this, we have changed the actuator locations to the floors 3 and 4 to get matched couplings. It is well known that the solution always exists in this case and that it guarantees the overall closed-loop system stability. Because the couplings are strong, the solution results in a high gain as shown in Fig.3. In this case $\gamma_1=\gamma_2=0.025,\eta_1=10^6,\eta_2=10^6H_1=H_2=1,R_1=R_2=0.1$.

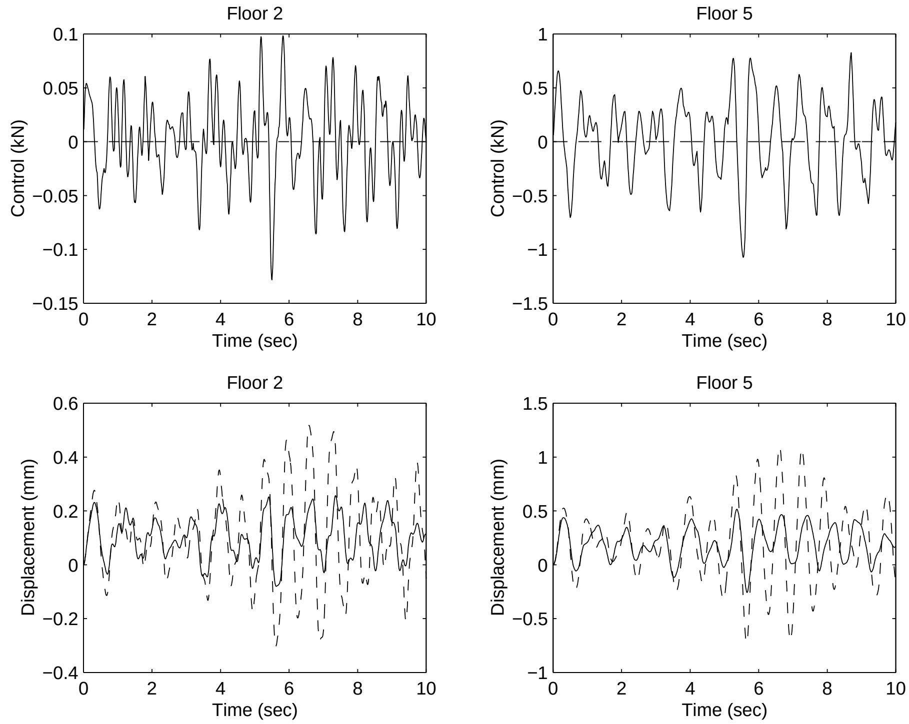


*Figure 1.* Decentralized control - case 1a, (- - -) without control, (——) with control.

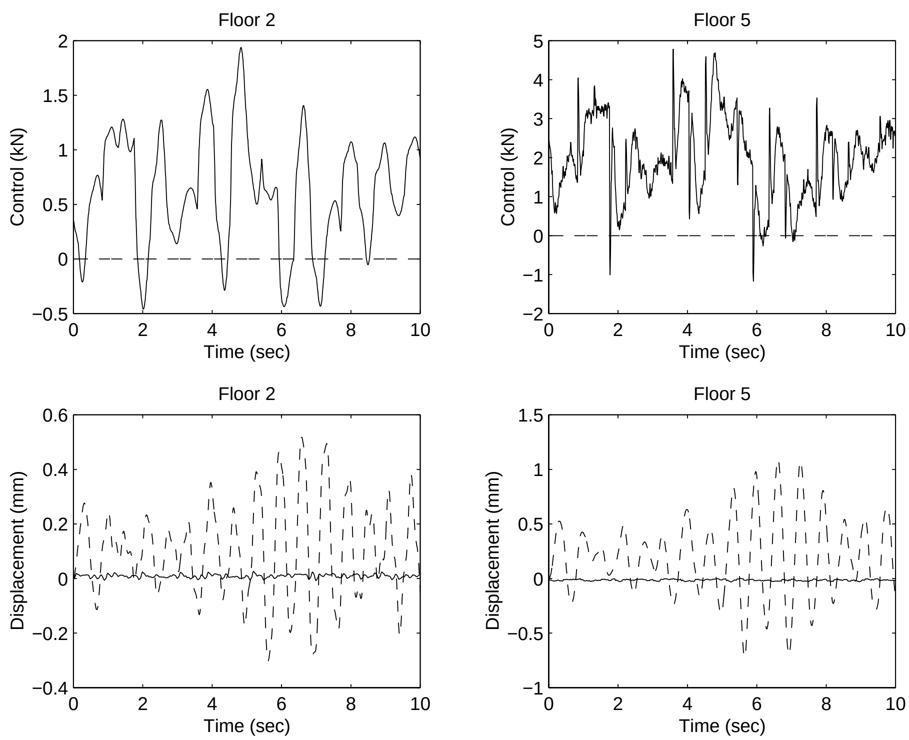


*Figure 2.* Decentralized control - case 1b, (- - -) without control, (——) with control.

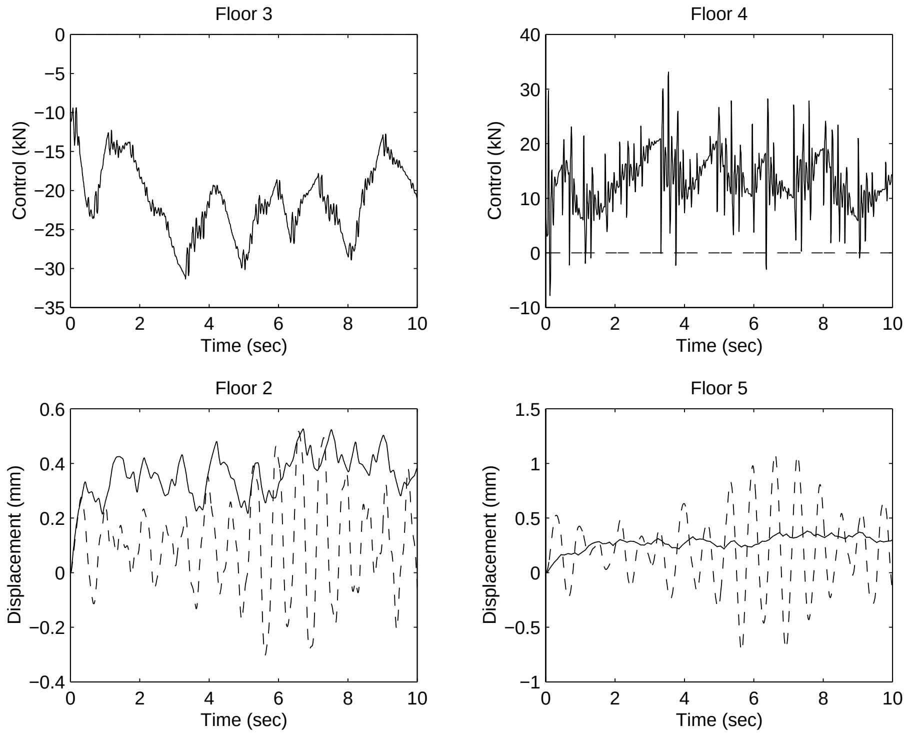


*Figure 3.* Decentralized control - case 2, (- - -) without control, (——) with control.

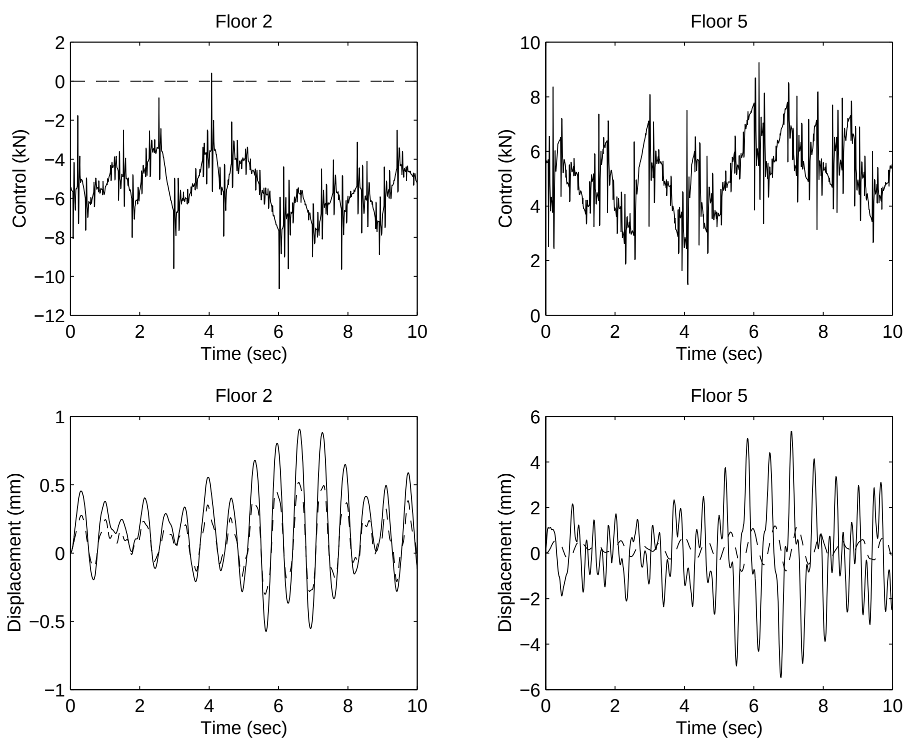


*Figure 4.* Decentralized control - case 3a, (- - -) without control, (——) with control.

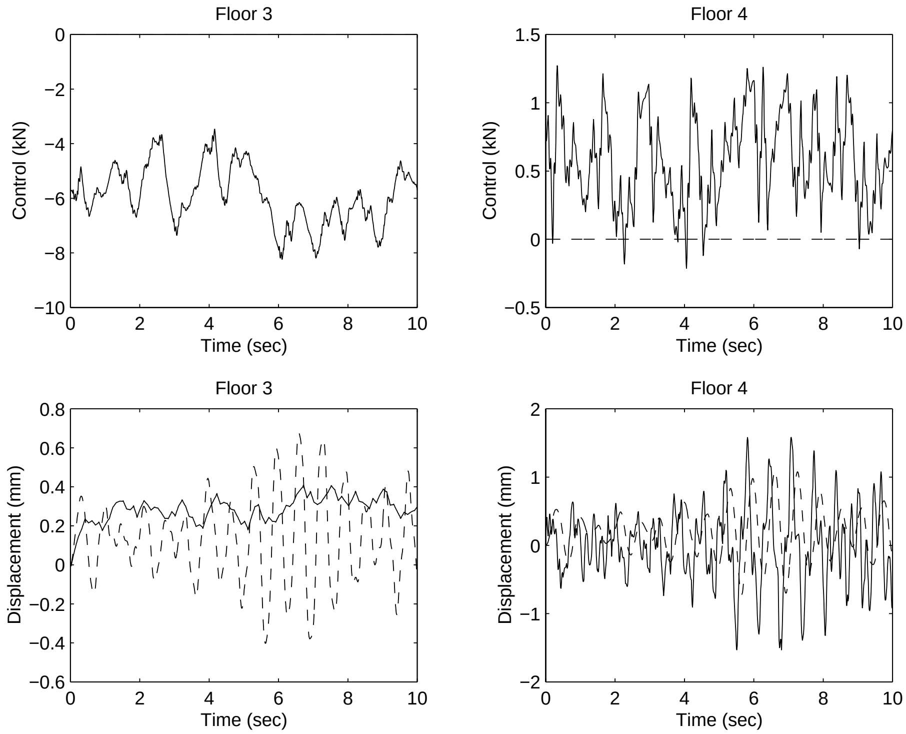


*Figure 5.* Decentralized control - case 3b, (- - -) without control, (——) with control.

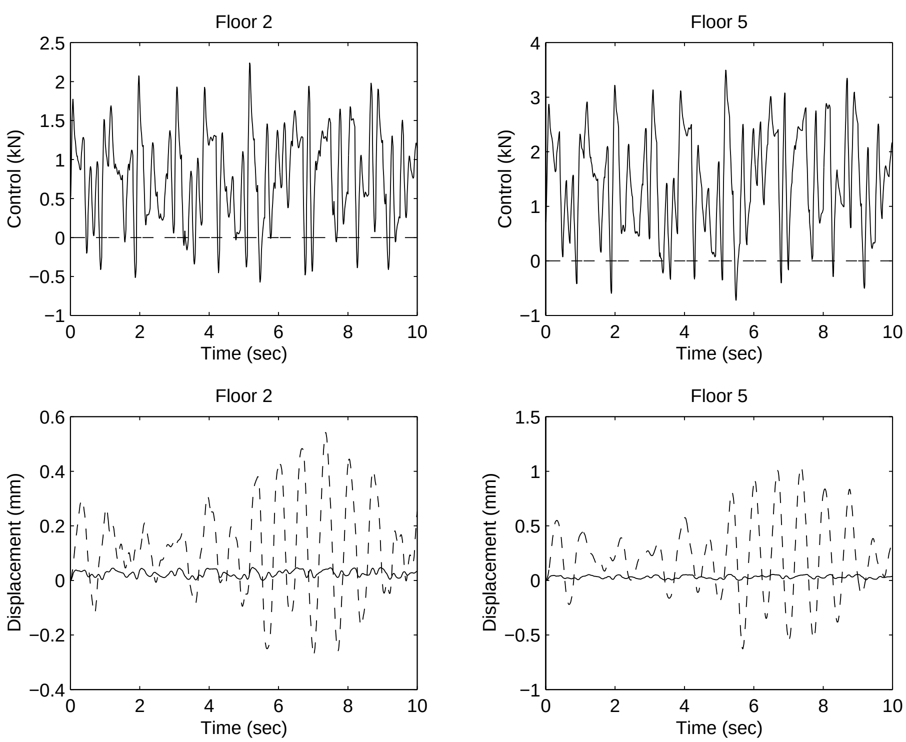


*Figure 6.* Centralized control - case 4, (- - -) without control, (——) with control.

*Case 3:* Decentralized control design according to Theorem, but with actuators locations at the floors 2, 3, 4 and 5 to reduce the gain in the feedback. Thereby, we have created the situation with both matched and mismatched couplings. This case it is included in Figs.4 and 5 with $\gamma_1 = \gamma_2 = 0.025, \eta_1 = 10^6, \eta_2 = 10^6, H_1 = H_2 = 1, R_1 = R_2 = 1$. Fig.4 (case 3a) shows the responses at the floors 2 and 5, while Fig.5 (case 3b) shows the same at the floors 3 and 4. The inclusion of matched case essentially reduces the gain when comparing it with the case 2.

*Case 4:* Centralized overall control system design with actuator positions at the floors 2 and 5. This case is shown in Fig.5 with $\gamma = 65, H_1 = H_2 = 10^4, R_1 = R_2 = 1$ and serves as a reference case.

## 5. Conclusion

The presented control design approach for mechanical systems described in a standard way by using tridiagonal damping and stiffness matrices may be considered as convenient for realatively large parameter uncertanties in subsystem matrices. Unknown but constant uncertainties with known upper bounds are supposed. However, the control design approach given by Theorem is convenient for weakly coupled systems. It means that when considering mechanical systems, the decomposition must be carefully selected at the places with a weak coupling in damping and stiffness matrices. Moreover, actuator locations should be located at such places to enable matching in interconnections to eliminate their influence by high gain in the feedback matrices. Only disjoint decomposition has been considered in our cases.

## 6. Acknowledgment

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