DECOMPOSITION AND RECONSTRUCTION METHODS IN BIOMEDICAL IMAGE DE-NOISING

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Abstract. Discrete transforms represent an efficient tool for signal and image analysis and further processing. The paper is devoted to the application of Wavelet transform using dilated and translated time limited functions for image analysis and its de-noising presenting comparison of the application of Wavelet transform with image filtering in frequency domain using two-dimensional discrete Fourier transform.

1 Introduction

Discrete one-dimensional or multi-dimensional transforms represent mathematical tools for efficient signal analysis and processing. The paper is devoted to the study of (i) Wavelet transform in connection with image decomposition and reconstruction using selected threshold methods and (ii) analysis of two-dimensional discrete Fourier transform and the use of window function in the frequency domain to reject selected image parts.

2 Decomposition and De-Noising

Discrete Wavelet transform (WT) stands for one of possibilities of signal analysis [1] allowing signal decomposition into two-dimensional function of time and scale. Wavelet functions used for signal analysis are derived from the initial function $W(t)$ forming basis for the set of functions

$$W_{m,k}(t) = \frac{1}{\sqrt{a}}\, W\,(\frac{1}{a}\,(t-b)) = \frac{1}{\sqrt{2^m}}\, W\,(2^{-m}t - k) \qquad (1)$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\ 2^m$. Wavelet dilation closely related to its spectrum compression enables local and global signal analysis. An example of an analytically defined Wavelet function is presented in Fig. 2.

Image Wavelet decomposition presented in Fig. 1 assumes that each column of image matrix is convolved with high-pass and low-pass filter followed by down-sampling at first and then the same process is applied to image matrix rows. Each step of image decomposition results in four image matrices of the number of rows and columns reduced to the half of that of the original matrix.

Both in the case of one-dimensional and two-dimensional signal decomposition it is possible to modify resulting coefficients **c** before the following signal reconstruction to eliminate undesirable signal components. Methods of such a process assume estimation of appropriate threshold limits studied in various books and papers [3] and they include

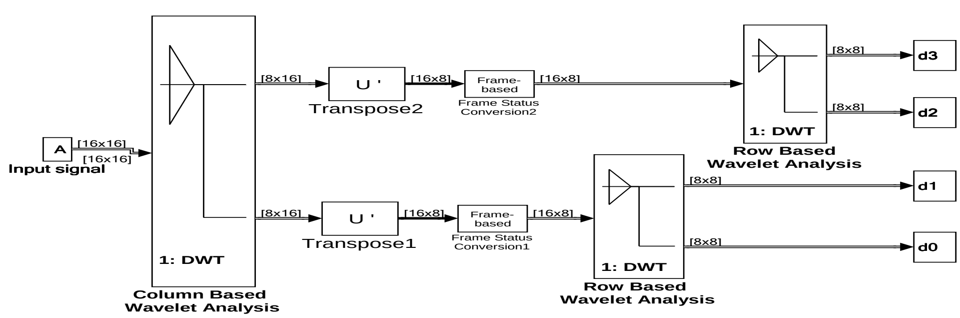

Figure 1: Wavelet use for image decomposition and subsampling by its columns and rows

- Signal decomposition using a selected Wavelet function up to the given level and evaluation of Wavelet transform coefficients $\{c(k)\}_{k=0}^{N-1}$
- The choice of threshold limits δ for each decomposition level and modification of its coefficients using for instance soft thresholding for evaluation of values

$$\overline{c}_s(k) = \begin{cases} \text{sign } c(k) \; (|\, c(k)\,| - \delta) & \text{if } |\, c(k)\,| > \delta \\ 0 & \text{if } |\, c(k)\,| \leq \delta \end{cases} \tag{2}$$

- Signal reconstruction from modified Wavelet transform coefficients

Results of this process depend on the choice of Wavelet functions and threshold limits.

The *Two-dimensional discrete Fourier transform* represent another efficient tool for image decomposition, analysis and reconstruction. In case of spectrum modification it can be used for rejection of selected image components as well. This procedure presented in Fig. 3 for a simulated image assumes the use of frequency window function before the inverse transform application.

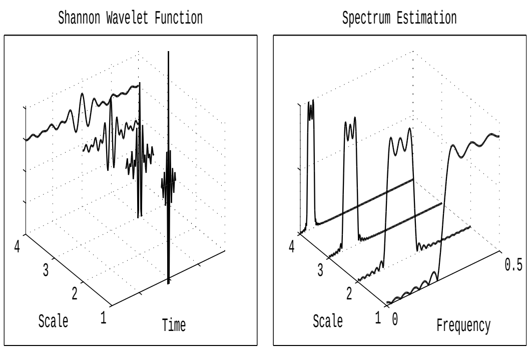

Figure 2: Shanon Wavelet function derived from the initial function defined in the form of relation $W(t) = \sin(\pi t/2)\cos(3\pi t/2)/(\pi t/2)$ and effect of its dilation to spectrum compression

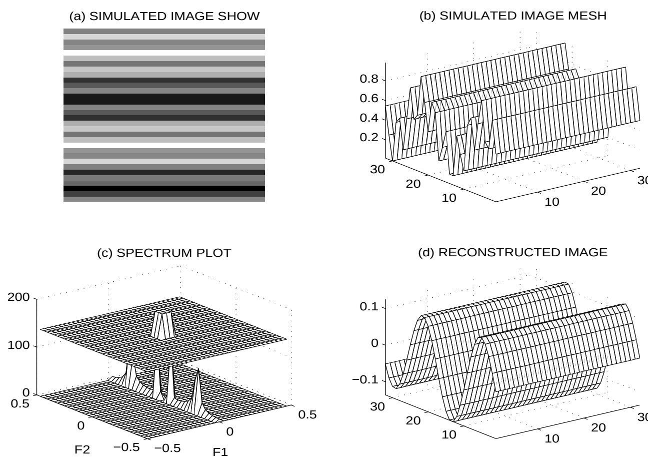

Figure 3: Simulated image analysis and the use of the 2D discrete FT for its de-noising

3 Biomedical Images

Image analysis used in biomedicine is commonly applied to two dimensional signals of magnetic resonance, computer tomography and sonography for investigation of brain, liver, kidneys and other organs. Even though these signals are different their mathematical processing is similar. Fig. 4 presents an example of an image standing for the magnetic resonance (MR) of brain.

Discrete transforms of observed one-dimensional or two-dimensional signals provide an efficient tool for signal analysis. In the case of *Wavelet decomposition* of images it is possible to use a tree structure [2] presented for one step of image decomposition in Fig. 1. The image is decomposed in this way into 4 subimages in the first stage. The low-pass image component evaluated both for image columns and rows is then decomposed in the same way again. Results of such a two level noisy image decomposition are presented in Fig. 5 for the biomedical image obtained by the magnetic resonance. The following image reconstruction assumes the use of selected image components. In the case of *two dimensional Wavelet transform* it is necessary to find proper threshold limits for image de-noising. Selected results are presented in Fig. 1.

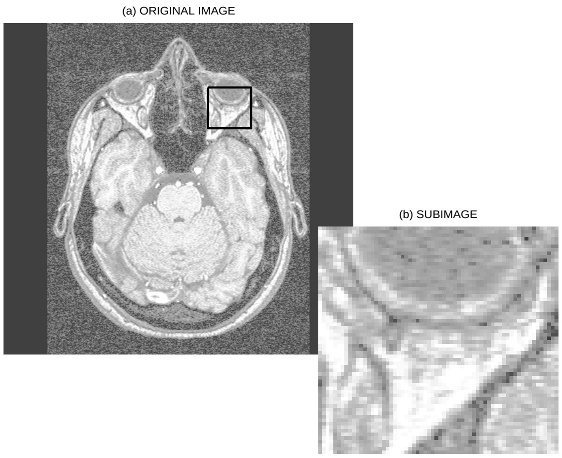

Figure 4: Magnetic resonance image visualization presenting (a) selected slice of brain and (b) its subimage chosen for the following digital processing

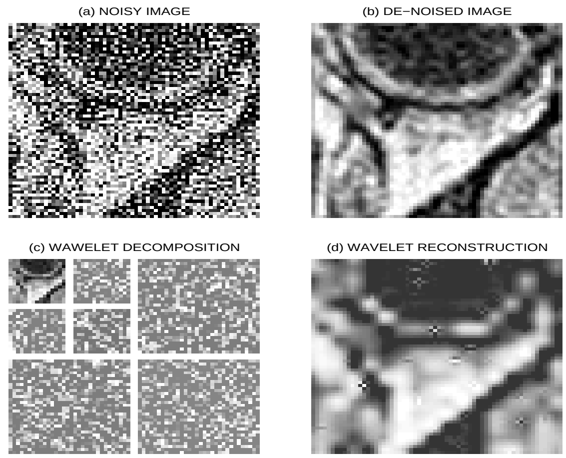

Figure 5:MR image processing:(a)noisy MR image, (b) de-noising by 2D discrete Fourier transform, (c) two level wavelet image decomposition, (d) wavelet image de-noising

4 Results

The mean square errors evaluated by various de-noising algorithms are compared in Table 1 for different types of added noise. It is obvious that for the normal noise distribution both Fourier transform and Wavelet transform provide good results. The advantage of the Wavelet transform is in its flexibility caused by the choice of various Wavelet functions including Daubechies functions used in this case. The last column of Table 1 provides results of the shot noise elimination. It can be seen that the image corrupted by this type of noise can be efficiently processed by median filtering to achieve its rejection.

Table 1: Mean square error between the original and de-noised image

	Mean Square Error	
	Normal Noise	*Shot Noise*
Noisy image	0.2604	0.0237
DFT denoising	0.0012	0.0074
WT denoising (Daubechies function)	0.0016	0.0036
Median filtering	0.0260	0.0006

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References

[1] D. E. Newland. *An Introduction to Random Vibrations, Spectral and Wavelet Analysis.* Longman Scientific & Technical, Essex, U.K., third edition, 1994.

[2] N.G. Kingsbury and J.F.A. Mugarey. Wavelet Transforms in Image Processing. In A. Procházka, J. Uhlíř, P. J. W. Rayner, and N. G. Kingsbury, editors, *Signal Analysis and Prediction*, Applied and Numerical Harmonic Analysis, chapter 2. Birkhauser, Boston, U.S.A., 1998.

[3] G.Strang and T.Nguyen. *Wavelets and Filter Banks.* Wellesley-Cambridge Press,1996.