NON-LINEAR MEDIAN FILTERING
OF BIOMEDICAL IMAGES

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Abstract

The paper presents basic principles of median filtering on two and three dimensional biomedical images. The proposed algorithm of image de-noising is applied to magnetic resonance (MR) images. Results of such a non-linear filtering are compared with that achieved by linear filtering methods. Algorithms are at first verified for simulated images and then applied to real MR images.

1 Introduction

Noise elimination forms a fundamental problem in image processing. Usually noise is encountered during image transmission as pixel drop-outs. Nonlinear median filtering is technique commonly used in the processing of images corrupted with impulse noise. In this paper we will discuss the application of 2D and 3D median filter masks in the de-noising of MR images. Median filtering is very simple in operation, effective in filtering isolated impulses and has good edge preserving properties.

2 Median Filtering

Median filtering is a a non-linear method used for the removal of impulsive noise. It is implemented to an image using a mask of odd length, the mask moves over the image and at each center pixel the median value of the data within the window is taken as the output. When the filter window is centered at the beginning or at the end of the input image some values must be assigned to empty window positions thus the first and the last value carry-on appending strategy can be applied which means the borders of the image can be filtered by duplicating the outmost values.

Let us denote the original image matrix as $X2(1 : M, 1 : N)$. Having a moving two-dimensional array mask with size of 3×3, the median algorithm can be explained using the MATLAB notation presented in Fig. 1.

```
for i=1:M-2
   for j=1:N-2
      B(1:3,1:3)= X2(i-1:i+1,j-1:j+1);
      y(i,j)= med(B(:));
   end
end
```
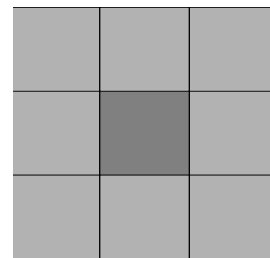

Figure 1: The two-dimensional median filter mask

where $y_{i,j}$ denotes the processed center pixel location, i refers to the vertical direction and j refers to the horizontal direction. The median operation $med(B(:))$ sorts the pixels in the mask in ascending order in the form of a row vector. The middle value is taken as the output.

In special cases we can analyze and process sequences of image frames corrupted with different levels of noise. These frames of images can be stored in the selected three-dimensional matrix $X3(1 : M, 1 : N, 1 : K)$. The operation of a moving three-dimensional array mask is illustrated in algorithm presented in Fig. 2.

```
for i=1:M-2
   for j=1:N-2
      for k=1:K-2
        B(1:3,1:3,1:3)= ...
           X3(i-1:i+1,j-1:j+1,k-1:k+1);
        y(i,j,k)= med(B(:));
      end
   end
end
```
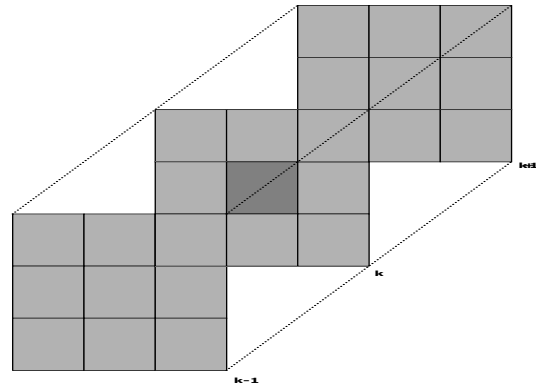

Figure 2: The three-dimensional median filter mask

3   Biomedical Images

Magnetic resonance imaging (MRI) is based on the absorption and emission of energy in the radio frequency range of the electromagnetic spectrum. MRI produces an image of the nuclear magnetic resonance (NMR) signal in a thin slice through the human body for investigation of brain, liver, kidneys and other soft tissue organs. Fig. 3 depicts the original 20 frames of the MR brain image. A rendered 3D view of the stack of images is displayed in Fig. 4. The three-dimensional array values are represented as $V(x, y, z)$ where $x = 1, \ldots, X$ represents the x-pixel co-ordinates, $y = 1, \ldots, Y$ represents the y-pixel co-ordinates and $z = 1, \ldots, Z$ are the corresponding slices.
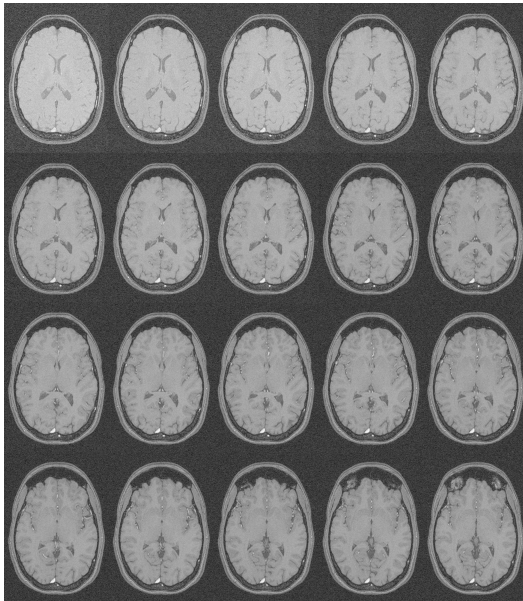

Figure 3: Sequence of MR image slices

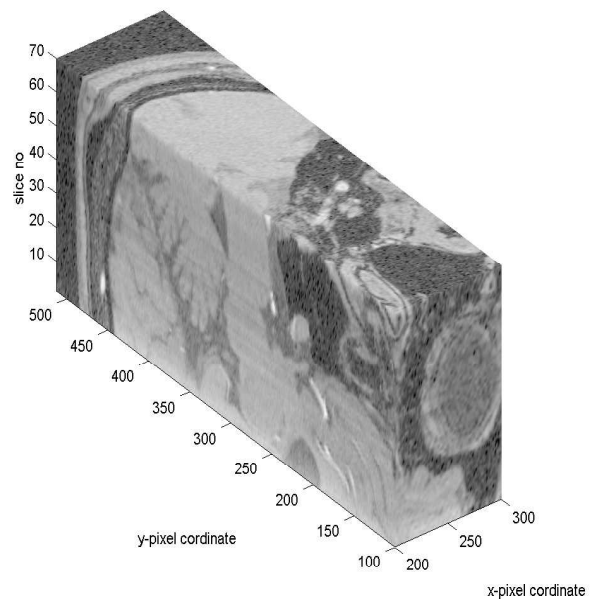
Figure 4: The 3D model visualization

4 Experimental Results

The application of the two-dimensional median filter and the three-dimensional filter is tested on a selected slice of MR image corrupted with different levels of impulsive noise and on three sequences of image slices each corrupted with different noise levels.

Since noise removal techniques are designed to enhance the image quality, we evaluate their performance regarding two criteria. Criterion 1 considers the quality of the resulting de-noised image based on its visual impression (MAE). Criterion 2 considers the quantity of the removed noise (MSE).

The *mean square error* (MSE) is defined by relation

$$MSE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (f(m,n) - \hat{f}(m,n))^2 \tag{1}$$

where $f(m,n)$ and $\hat{f}(m,n)$ represent the original image and the filtered image respectively. The *mean absolute error* (MAE) is likewise defined by expression

$$MAE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |(f(m,n) - \hat{f}(m,n))| \tag{2}$$

Fig. 5(a) shows the original image and three noisy images each of them corrupted with different noise levels (p - denotes the percentage number of pixels degraded). The results obtained by using the classical median filter and 3D median filter are presented in Fig. 5(e) and Fig. 5(f) respectively.

(a) Original slice no. 2

(b) Noisy slice no. 2 – p=0.06

(c) Noisy slice no. 2 – p=0.04

(d) Noisy slice no. 2 – p=0.02

(e) Denoised 2nd slice – 3D Median filter

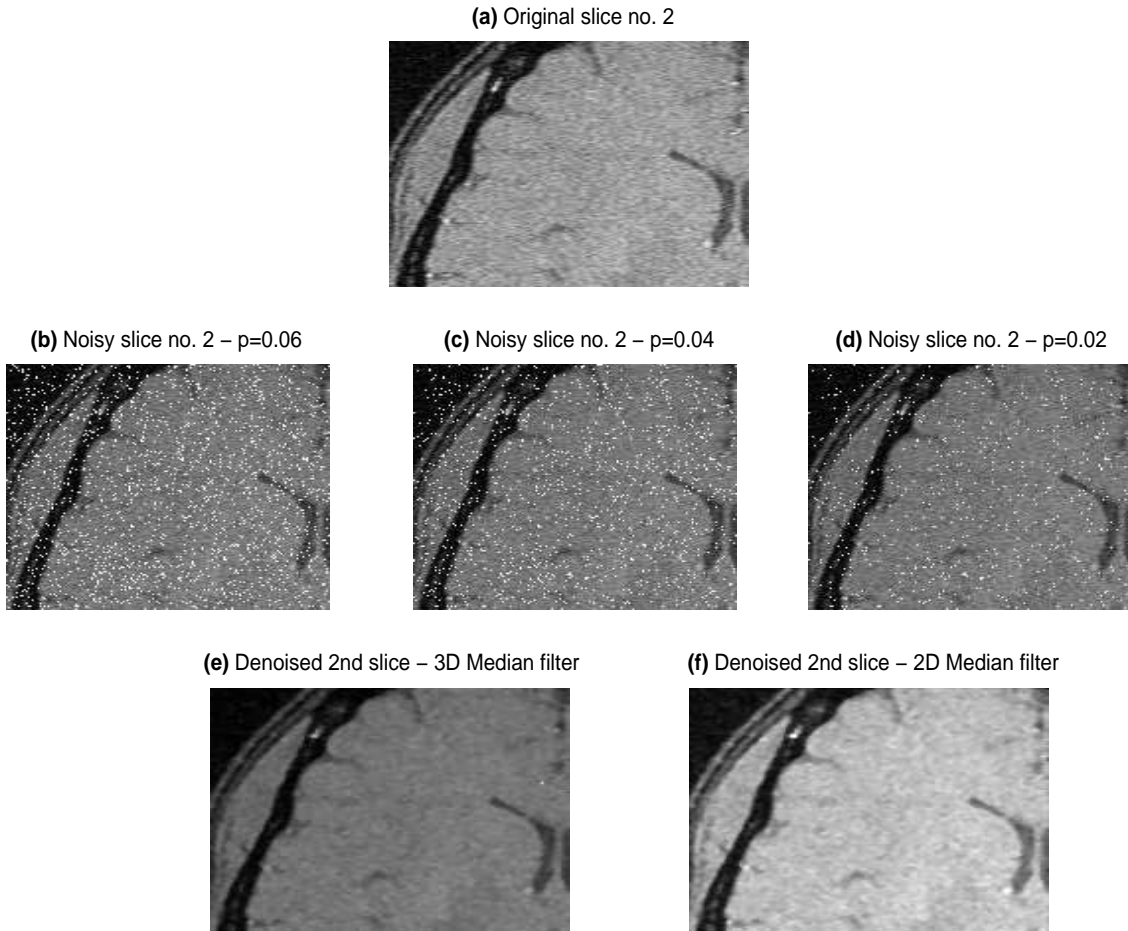**(f)** Denoised 2nd slice – 2D Median filter

Figure 5: Comparison between the 2D and 3D median filtering using the single MR layer

Table 1: COMPARISON OF THE STANDARD MF AND THE 3D MEDIAN FILTER

	Mean square error (MSE)	*Mean Absolute Error (MAE)*
Noisy image	0.0162	0.0253
2D median filter	0.0014	0.0014
3D median filter	0.0013	0.0022

The MSE and MAE performance of the filters are shown in Tab. 1. From the table it can be seen the 3D median filter performs relatively well compared to the classical median filter.

In Figs. 6(a),(b),(c) are three original image slices and their corresponding noisy image slices (Fig. 6 (c),(d),(e)). The denoised images resulting from the 3x3 constant window median filtering of the center slice and the application of the 3D median filter are represented in Figs. 6(g),(h).

(a) Original Ist slice   **(b)** Original 2nd slice   **(c)** Original 3rd slice

(d) Noisy slice no. 1 – p=0.06   **(e)** Noisy slice no. 2 – p=0.04   **(f)** Noisy slice no. 3 – p=0.02

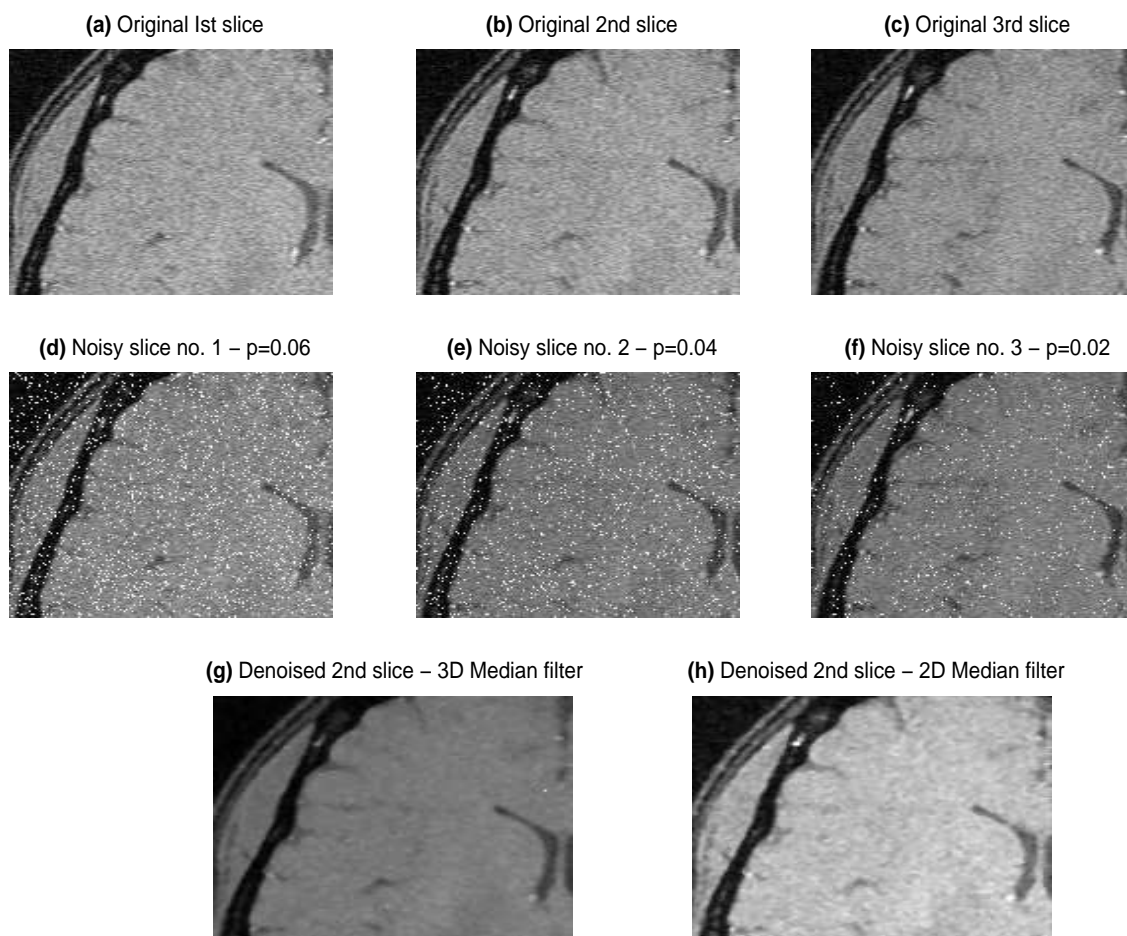**(g)** Denoised 2nd slice – 3D Median filter   **(h)** Denoised 2nd slice – 2D Median filter

Figure 6: Comparison between the 2D and 3D median filtering applied to 3 MR image layers

The MSE and MAE performance of the classical median filtering and the proposed 3D median algorithm are compared in Tab. 2. From the table it can be seen that when there is a shift of pixels with respect to the slice being processed the 3D median filter produces poor results compared to that of the classical median filter.

For a noisy 3D volume the 3D median algorithm (Fig. 7) can be applied to a stack of image slices. However by using this algorithm the first and last slices are not processed, this means that these remaining slices can be processed using the 2D median filter.

Table 2: COMPARISON OF THE STANDARD MF AND THE 3D MEDIAN FILTER

	Mean square error (MSE)	Mean Absolute Error (MAE)
Noisy image	0.0162	0.0253
2D median filter	0.0014	0.0014
3D median filter	0.0021	0.0187

```
function [y]=med3(x)
%------------------------------
%3-D MEDIAN FILTERING
%-------------------------------
%y-3D output image (x,y,z)
%x--input image(noisy image)

[j,k,l]=size(x);

for jj=1:l-2  %slice
   for m=1:j-2;  %x-direction
       for n=1:k-2;  %y-direction
           aa=[x(m:m+2,n:n+2,[jj:jj+2])];
           c(m,n)=median(aa(:));
       end
  end
     y(:,:,jj) =c;
end
```

Figure 7: The three-dimensional median filtering algorithm

5   Conclusions

Non-linear filters were applied to enhance MR images corrupted with impulsive noise. In this paper we proposed both the algorithms of 2D and 3D median filtering, even though their algorithms are simple and easy to implement they give good efficiency in suppressing the impulsive noise. However it must be noted that a 3D median filter produces a smoother image appearance compared to the 2D median filter when processing noisy image sequence slices.

Acknowledgments

References

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