# COMPLEX WAVELET TRANSFORM IN PATTERN RECOGNITION

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Abstract: The paper presents basic properties of the complex wavelet transform, the dual tree signal decomposition and features of such a decomposition in the comparison with the wavelet transform using wavelets defined either in the analytical or recurrent forms. This approach to signal and image analysis is studied in the mathematical form and verified for simulated structures as well. The final part of the paper presents the use of this method in feature extraction, pattern recognition and classification.

Keywords: Digital signal processing, time-scale signal analysis, complex wavelet transform, signal decomposition and reconstruction, computer vision

## 1. INTRODUCTION

Complex Wavelet Transform forms an important area of digital signal and image processing allowing multidimensional signal decomposition and reconstruction. Basic ideas of mathematical approach are described in many books and papers including works of Prof. Kingsbury (Kingsbury and Magarey, 1997; Kingsbury, 1998 1999 2000). The paper is divided into (i) studies of complex WT, (ii) its comparison to the classical DWT and (iii) description of its properties.

The detailed mathematical description is accompanied with processing of simulated and real biomedical MR images. Final remarks conclude the paper with the application of this transform to (i) signal or image decomposition and perfect reconstruction (ii) signal denoising and, (iii) feature extraction for pattern recognition.

## 2. DUAL TREE COMPLEX WAVELET TRANSFORM

The dual-tree CWT comprises of two parallel wavelet filter bank trees that contain carefully designed filters of different delays that minimize the aliasing effects due to downsampling (Kingsbury, 1999; Vetterli, 1992). The dual-tree CDWT of a signal $x(n)$ is implemented using two critically-sampled DWTs in parallel on the same data, as shown in Fig. 3. The transform is two times expansive because for an $N$-point signal it gives $2N$ DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. So the filters are designed in a specific way such that the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform and subband signals of the lower DWT can be interpreted as the imaginary part. When designed in this way the DT CDWT is nearly shift invariant, in contrast to the classic DWT.

### 2.1 *Translation Invariance by Parallel Filter Banks*

The orthogonal (Neumann and Steidl, 2004) two-channel filter banks with analysis low-pass filter given by the $z$-transform $H_0(z)$, analysis highpass filter $H_1(z)$ and with synthesis filters $G_0(z)$ and $G_1(z)$ is shown by the diagram below (Fig. 1) .

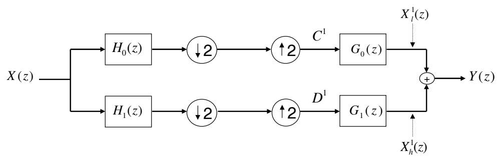


Fig. 1. DWT filter bank

For an input signal $X(z)$, the analysis part of the filter bank followed by upsampling produces the low-pass (eq. (1)) and the high-pass (eq. (2)) coefficients

$$C^1(z^2) = \frac{1}{2}\{X(z)H_0(z) + X(-z)H_0(-z)\} \quad (1)$$

$$D^1(z^2) = \frac{1}{2}\{X(z)H_1(z) + X(-z)H_1(-z)\} \quad (2)$$

respectively, and decomposes the input signal into a low frequency part $X_l^1(z)$ and a high frequency part $X_h^1(z)$, the output signal is the sum of these two components (eq. (3))

$$Y(z) = X_l^1(z) + X_h^1(z) \quad (3)$$

where

$$\begin{aligned} X_l^1(z) &= C^1(z^2)G_0(z) \\ &= \frac{1}{2}\{X(z)H_0(z)G_0(z) + X(-z)H_0(-z)G_0(z)\} \end{aligned} \quad (4)$$

$$\begin{aligned} X_h^1(z) &= D^1(z^2)G_1(z) \\ &= \frac{1}{2}\{X(z)H_1(z)G_1(z) + X(-z)H_1(-z)G_1(z)\} \end{aligned} \quad (5)$$

This decomposition is not shift invariant due to the terms in $X(-z)$ of (eq. (4)) and (eq. (5)), respectively, which are introduced by the downsampling operators. If the input signal is shifted, for example $z^{-1}X(z)$, the application of the filter bank results in the decomposition (eq. (6))

$$z^{-1}X(z) = \tilde{X}_l^1(z) + \tilde{X}_h^1(z) \quad (6)$$

For an input signal $z^{-1}X(z)$ we have

$$C^1(z^2) = \frac{1}{2}\{z^{-1}X(z)H_0(z) + (-z^{-1})X(-z)H_0(-z)\} \quad (7)$$

and

$$\tilde{X}_l^1(z) = \frac{1}{2}z^{-1}\{X(z)H_0(z)G_0(z) - X(-z)H_0(-z)G_0(z)\} \quad (8)$$

and similarly for the high-pass part, which of course is not the same as $z^{-1}X_l^1(z)$ if we substitute for $z^{-1}$ in eq. (4). From this calculation it can be seen that the shift dependence is caused by the terms containing $X(-z)$, the *aliasing terms*.

One possibility to obtain a shift invariant decomposition can be achieved by the addition of a filter bank to Fig. 1 with shifted analysis filters $z^{-1}H_0(z)$, $z^{-1}H_1(z)$ and synthesis filters $zG_0(z)$, $zG_1(z)$ and subsequently taking the average of the lowpass and the highpass branches of both filter banks (Fig. 2).

If we denote the first filter bank by index $a$ and the second one by index $b$ then this procedure implies the following decomposition (eq. (9))

$$X(z) = X_l^1(z) + X_h^1(z) \quad (9)$$

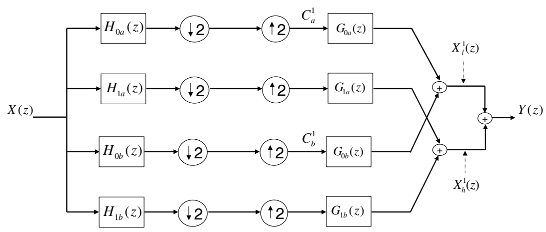


Fig. 2. One level complex dual tree

where for the lowpass channels of tree $a$ and tree $b$ we have

$$\begin{aligned} X_l^1(z) &= \frac{1}{2}\{C_a^1(z^2)G_{0a}(z) + C_b^1(z^2)G_{0b}(z)\} \\ &= \frac{1}{4}\{X(z)[H_0(z)G_0(z) + H_0(z)G_0(z)] + \\ &\quad + X(-z)[H_0(-z)G_0(z) - H_0(-z)G_0(z)]\} \\ &= \frac{1}{2}X(z)H_0(z)G_0(z) \end{aligned} \quad (10)$$

and similarly for the high-pass part. The aliasing term containing $X(-z)$ in $X_l^1$ has vanished and the decomposition becomes indeed shift invariant.

Using the same principle for the design of shift invariant filter decomposition, Kingsbury suggested in (Kingsbury, 1998) to apply a 'dual-tree' of two parallel filter banks and combine their bandpass outputs. The structure of a resulting analysis filter bank is sketched in Fig. 3, where index $a$ stands for the original filter bank and the index $b$ is for the additional one. The dual-tree complex DWT of a signal $x(n)$ is implemented using two critically-sampled DWTs in parallel on the same data.

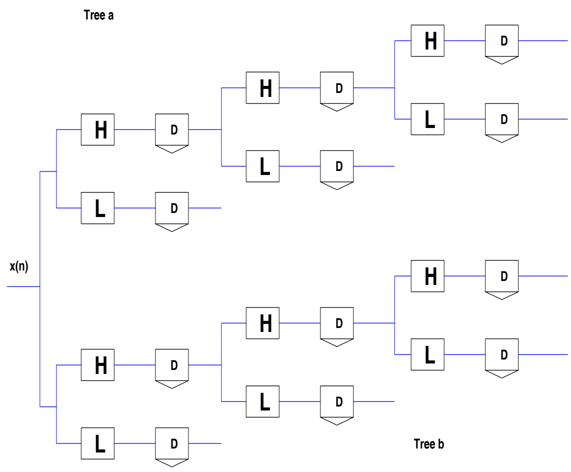


Fig. 3. Complex dual tree

In one dimension, the so-called dual-tree complex wavelet transform provides a representation of a signal $x(n)$ in terms of complex wavelets, composed of real and imaginary parts which are in turn wavelets themselves. In fact, these real and imaginary parts essentially form a quadrature pair (Romberg et al., 2002), the complex wavelet associated with the dual tree CDWT is shown in Fig. 4. To extend the transform to higher-

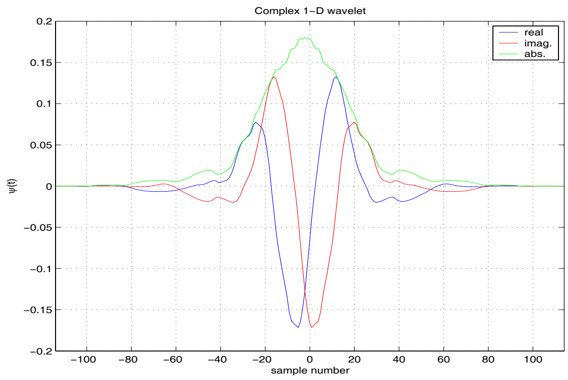


Fig. 4. Impulse response of a complex wavelet

dimensional signals, a filter bank is usually applied separably in all dimensions. To compute the 2D CWT of images these two trees are applied to the rows and then the columns of the image as in the basic DWT. This operation results in six complex high-pass subbands at each level and two complex low-pass subbands on which subsequent stages iterate in contrast to three real high-pass and one real low-pass subband for the real 2D transform. This shows that the complex transform has a coefficient redundancy of 4:1 or $2^m : 1$ in $m$ dimensions. In case of real 2D filter banks the three highpass filters have orientations of $0^0, 45^0$ and $90^0$, for the complex filters the six subband filters are oriented at $\pm 15^0, \pm 45^0, \pm 75^0$ (Fig. 5).

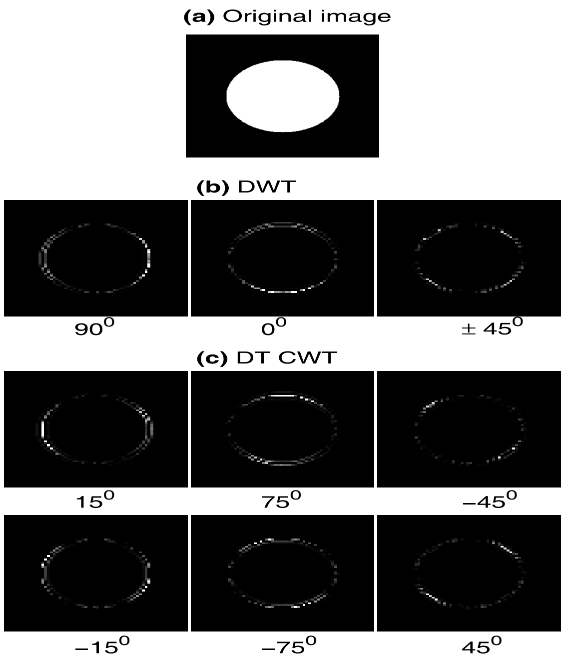


Fig. 5. Complex filter response showing the orientations of the complex wavelets

The CDWT decomposes an image into a pyramid of complex subimages, with each level containing six oriented subimages resulting from evenly spaced directional filtering and subsampling, such directional filters are not obtainable by a separable DWT using a real filter pair but complex coefficients makes this selectivity possible.

## 3. RESULTS

The shift invariance (Fig. 6) and directionality (Fig. 5) of the CWT may be applied in many areas of image processing like denoising, feature extraction, object segmentation and image classification. In the this section, we present a wavelet-based texture descriptor for grayscale images.

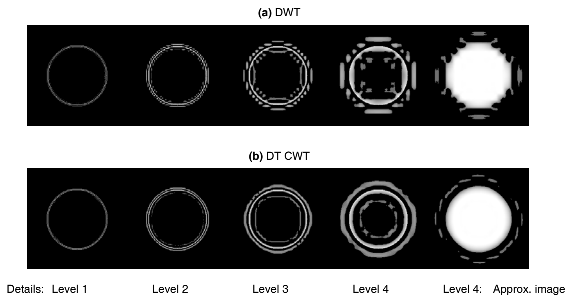


Fig. 6. Shift invariance of DT CWT compared to DWT

To describe image features various methods are used including their description in the frequency domain. In this paper we have proposed the use of image wavelet decomposition (Debnath, 2003; Rioul and Vetterli, 1991), using wavelet coefficients at selected levels to describe image features. For a classical wavelet transform method with a one level decomposition, the given textured image is decomposed into four subimages of low-low, low-high, high-low and high-high subbands.

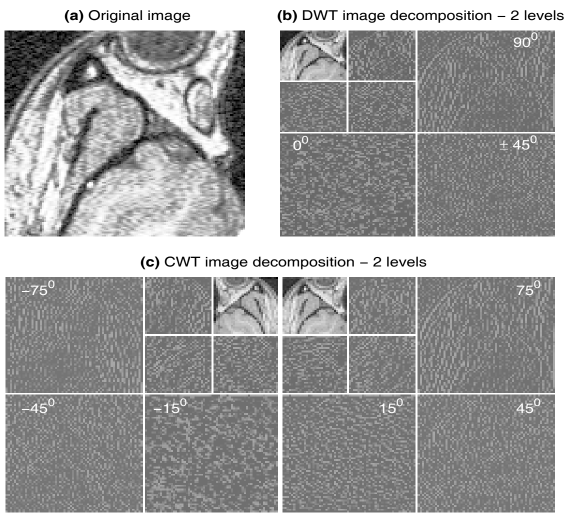


Fig. 7. (a) Original image, (b) its 2-level DWT decomposition and (c) 2-level CWT decomposition

To reduce the size of the input vector provided to the neural network, the mean and standard deviation is calculated for each subband. Equations

(11) and (12) describe the process of getting the features. $X(i,j)$ is a wavelet coefficient.

$$c_{mean} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{i=1}^{N}|X(i,j)| \tag{11}$$

$$c_{std} = \frac{1}{MN}\sqrt{\sum_{i=1}^{M}\sum_{i=1}^{N}(|X(i,j)| - c_{mean})^2} \tag{12}$$

The wavelet features are obtained by using the above two equations which implement the mean and standard deviations of the magnitude of the wavelet coefficients. It is assumed that the local texture regions are spatially homogeneous, and the mean and the standard deviation of the magnitude of the transform coefficients are used to represent the region for classification purposes. Feature vectors will form neural network inputs.

## 4. CONCLUSION

In the paper the studies and application of CWT in pattern recognition have been presented. The DT CWT is shift invariant and forms directionally selective diagonal filters. These properties are important for many applications in image processing including denoising, deblurring, segmentation and classification. In this paper we have illustrated the example of the application of complex wavelets for the classification of MR images.

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