18. NUMERICAL DIFFERENTIATION AND INTEGRATION

18.1 Interpolation

Characteristics:
1. Interpolation in one or more dimensions: linear, cubic, spline
2. Possibilities of triangulation based randomly located 3D data points

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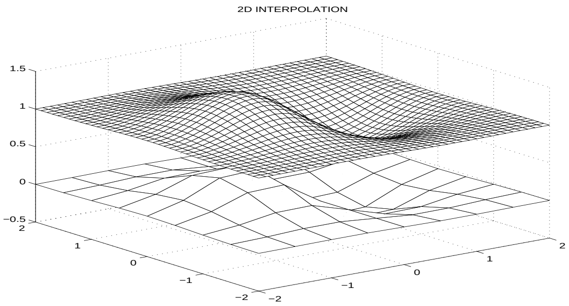
% Example 18.1: 1D and 2D data interpolation
  figure(1); x=0:10; y=sin(x); plot(x,y,'o');
    xi=0:.2:10; yilin=interp1(x,y,xi); % 1D interpolation
                yisp=interp1(x,y,xi,'spline');
    hold on; plot(xi,yilin,'g',xi,yisp,'r'); hold off
  [X,Y]=meshgrid(-2:0.5:2); Z=-X.*exp(-X.^2-Y.^2);
  figure(2); mesh(X,Y,Z);
    [XI,YI]=meshgrid(-2:0.1:2);
    ZI=interp2(X,Y,Z,XI,YI,'spline'); % 1D interpolation
    hold on; mesh(XI,YI,ZI+1); hold off
% Example 18.2: data gridding
  x=4*(rand(100,1)-0.5); y=4*(rand(100,1)-0.5);
  z=-x.*exp(-x.^2-y.^2)+0.5;
    [XI,YI]=meshgrid(-2:0.1:2); ZI=griddata(x,y,z,XI,YI);
  figure(3); mesh(XI,YI,ZI); grid on;
    hold on; stem3(x,y,z,'o'); hold off
```

COMMANDS

INTERP1
INTERP2
GRIDDATA

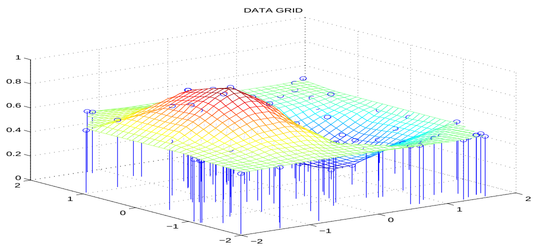
DIFF
LIMIT
INT
EZPLOT

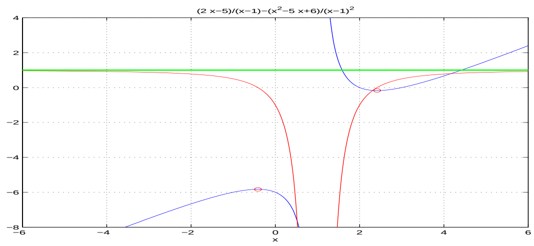


18.2 Numeric and Symbolic Derivative

Basic definitions:
1. Numerical estimate of a derivative: $y'(k) \approx (y(k+1) - y(k))/h$
2. Symbolic definition $f'(x) = \lim_{h\to 0}(f(x+h) - f(x))/h$

```
%%% Example 18.3: Numerical derivative
  a=0; b=pi; N=100; h=(b-a)/N;
  x=[a:h:b]; y=sin(x); yd=diff(y)/(pi/100);
%%% Example 18.4: Symbolic analysis of a given function
  syms x; f=(x^2-5*x+6)/(x-1); df=diff(f);
     pretty(f); pretty(df); pretty(simple(df))
  ezplot(f); hold on
    h=ezplot(df); set(h,'Color','r');
    hold off; axis([-6 6 -8 4]); grid on
% Evaluation and plot of the function derivative limits
  AP=limit(df,x,+inf); AM=limit(df,x,-inf);
  line([-6 6],double([AM AP]),'LineWidth',2,'Color','g')
% Evaluation and plot of extrems of the given function
  x0=double(solve(df)); extrem=[x0 subs(f,{x},{x0})]
  hold on; plot(extrem(:,1),extrem(:,2),'or'); hold off
```





18.3 Numeric and Symbolic Integration

Basic definitions:
1. Integration by a trapesoidal rule: $Q = \int_a^b f(x)\,dx \approx h/2 * (f(a) + 2\sum_{i=1}^{N-1} f(a+i\,h) + f(b))$

```
%%% Example 18.5: Numerical integration by the trapesoidal rule
  a=0; b=pi; N=100; h=(b-a)/N; x=[a:h:b]; y=sin(x);
  Q=h/2*(sin(a)+sin(b)+2*sum(sin(a+[1:N-1]*h)))
%%% Example 18.6: Symbolic integration
  syms x;  f=(x+1)/(x^2+5*x+6); ezplot(f); hold on
  fint=int(f); h=ezplot(fint); set(h,'Color','r');
     grid on; axis([-4 4 -4 4]); hold off
```

EXAMPLES 18

18.1 Using symbolical methods find extrems and limits of the given function

18.2 Derive estimates of the diference approximations the first and second derivative

18.3 Evaluate the definite integral of the given function using various numerical methods with a chosen step and compare results with the symbolical solution