

Approximate the given set of values $\{\mathbf{x}, \mathbf{y}\} = \{x_i, y_i\}_{i=1}^N$ by a chosen function type. The solution includes:

- the mathematical description of the method used
- the constants of the approximate function using the own program for the solution of the system of linear algebraic equations (numeric and symbolic solution) and eventually using the function POLYFIT
- the graphical illustration of the given and approximated values and the determination of the sum of the square errors
- the graphical 3D illustration of the sum of the square errors for the two chosen constants of the approximation function in a selected surroundings of these constants

Solving this task you can use some of the following functions POLYFIT, POLYVAL, PLOT, STEM, HOLD, FOR-END, SUM, INV, MESH, CONTOUR and also SOLVE, EZPLOT.

M1.1	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=1	M1.21	$f(x) = c_1 + c_2x^2$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [1.1 2.9 2.9 4.3 3.5]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [1 1.4 1 1.4 2.8]'
M1.2	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=1	M1.22	$f(x) = c_1 + c_2x^3$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [1.5 2.7 1.4 2.6 2.9]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0.9 1.2 0.8 0.9 2.2]'
M1.3	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=1	M1.23	$f(x) = c_1x + c_2x^2$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [1.7 3.1 3.7 4.3 5.1]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [0.4 0.4 1.1 3.7 4.5]'
M1.4	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=1	M1.24	$f(x) = c_1 + c_2/x$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [0.5 2.7 1.8 5.0 3.0]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [10.4 3.6 2.2 1.6 0.4]'
M1.5	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=2	M1.25	$f(x) = c_1 + c_2e^{-x}$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0.9 1.2 3.9 2.7 1.7]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [2.7 2.5 1.8 1.6 1.5]'
M1.6	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=2	M1.26	$f(x) = c_1e^{-0.5x} + c_2e^{-2x}$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [2.1 1.2 1.1 2.3 3.9]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [2.4 1.9 0.8 0.2 0.6]'
M1.7	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=2	M1.27	$f(x) = c_1 + c_2x^2$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [1.3 2.2 3.6 2.7 2.6]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0.7 0.6 1.3 1.9 2.5]'
M1.8	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=2	M1.28	$f(x) = c_1 + c_2x^3$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [1.7 2.3 4.4 3.0 2.5]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0.3 0.6 1.0 1.8 2.6]'
M1.9	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=3	M1.29	$f(x) = c_1x + c_2x^2$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0.2 1.5 1 2 3]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [0.4 1.0 1.8 4.1 5.5]'
M1.10	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=3	M1.30	$f(x) = c_1 + c_2/x$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.3 \ 0.4 \ 0.6 \ 0.9]'$, [0 2.2 1 0.6 3.7]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [10.5 3.6 2.7 1.3 1.3]'
M1.11	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=3	M1.31	$f(x) = c_1 + c_2e^{-x}$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [0.5 1.5 2.5 10.9 10.9]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [2.8 2.1 2.0 1.2 1.2]'
M1.12	$f(x) = c_1x^M + c_2x^{M-1} + \dots + c_{M+1}$, M=3	M1.32	$f(x) = c_1e^{-0.5x} + c_2e^{-2x}$
	$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [2.1 1.1 1.8 10.2 13.4]'		$\mathbf{x}, \mathbf{y} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$, [2.9 1.9 1.0 0.4 0.3]'

Approximate the given set of values $\{x_i, y_i\}_{i=1}^N$ by a chosen nonlinear function using the gradient method. Carry out numerical and if possible also symbolic solution, illustrate the sum of the square errors for the two chosen constants of the approximate function and display the way of searching the extreme. At the same time bring in the mathematical description of the used method and the commented algorithm of the solution.

M1.41	$f(x) = e^{c_1x} + c_2$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [1.1 \ 0.8 \ 0.5 \ 0.6 \ 0.4]'$
M1.42	$f(x) = e^{c_1x} + c_2$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [1.3 \ 0.7 \ 0.5 \ 0.6 \ 0.5]'$
M1.43	$f(x) = c_1e^{c_2x}$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.3 \ 0 \ 0.8 \ 2.1 \ 3.9]'$
M1.44	$f(x) = c_1e^{c_2x}$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.2 \ 0.3 \ 0.5 \ 2.5 \ 3.8]'$
M1.45	$f(x) = c_1e^{c_2x} + c_3$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [1.3 \ 0.9 \ 0.6 \ 0.4 \ 0.5]'$
M1.46	$f(x) = c_1e^{c_2x} + c_3$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [1.2 \ 0.8 \ 0.6 \ 0.4 \ 0.4]'$
M1.47	$f(x) = c_1/(1 + c_2x)$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.8 \ 0.5 \ 0.3 \ 0.2 \ 0.1]'$
M1.48	$f(x) = c_1/(1 + c_2x)$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.8 \ 0.4 \ 0.3 \ 0.2 \ 0.2]'$
M1.49	$f(x) = c_1/(c_2 + c_3x)$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.7 \ 0.6 \ 0.3 \ 0.2 \ 0.1]'$
M1.49	$f(x) = c_1/(c_2 + c_3x)$	$\mathbf{x} = [0.1 \ 0.4 \ 0.9 \ 1.6 \ 1.8]'$	$\mathbf{y} = [0.8 \ 0.4 \ 0.3 \ 0.2 \ 0.2]'$