Wavelet Transform Application in Biomedical Image Recovery and Enhancement

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ABSTRACT

The paper presents selected mathematical methods of digital image enhancement applied to magnetic resonance images of the human brain. This topic forms a general interdisciplinary area of Digital Signal Processing (DSP). The work is motivated by a need to process digital images after their acquisition. This is caused by two reasons. Firstly, the digital images can be taken in low quality and secondly the quality of images is getting lower during their transmission. The goal of this work comes out from a need to enhance biomedical images. The methods described in the paper have been designed generally, i.e. it is possible to use them according to declared limitations and recommendations for both all digital images (understood as two-dimensional signals) and one-dimensional signals. The main part of the paper presents methods of the recovery of degraded parts and resolution enhancement of digital images. The wavelet transform approach has been adopted here and used for the recovery of the corrupted image regions. Proposed wavelet transform method for image resolution enhancement forms an alternative to the linear and the Fourier transform interpolation. Results of the proposed methods are presented for simulated signals and biomedical magnetic resonance images. Resulting algorithms are closely related to signal segmentation, change points detection and prediction with the use in process control, computer vision, signal or image processing and computer intelligence.

Keywords: Discrete wavelet transform, wavelet decomposition and reconstruction, biomedical image analysis, image regions recovery, image enhancement, resolution enhancement, computer intelligence

1 INTRODUCTION

The paper is devoted to digital image enhancement, which falls within the generic multidisciplinary area of information engineering, known as digital signal processing [9].

There are many applications in which signals are converted into a digital form and then digital signal processing methods are applied. In the case of digital image processing, the digital signal is two-dimensional. This work presents some of tools for digital image enhancement: recovery of missing or corrupted parts of images and image resolution enhancement.

The **recovery of degraded parts (blocks, regions)** of the digital image forms the main part of digital image enhancement. There are deterministic and probabilistic methods described in literature [4, 12] to solve this problem. The deterministic algorithms are usually based on autoregressive modelling, matrix moving average, or bilinear interpolation. Probabilistic methods include usually Bayesian modelling. Signals containing more random components can be more completely described by their probability distributions. Therefore Bayesian probabilistic methods are important in the analysis of two-dimensional signals. Iterated Wavelet Interpolation Method (IWIM) forms a new designed method to achieve this goal. This method is based on the interpolation using wavelet functions, i.e. the input signal is decomposed by the selected wavelet function, treated in the wavelet domain, and then reconstructed back into the image with recovered corrupted or missing regions.

The **selection of the signal resolution** [5, 11] is the next fundamental problem encountered in the digital processing of both one-dimensional and two-dimensional signals. This defines the sampling period in the case of time series or the pixel size in the case of images. Changing the resolution of a signal or image allows both global and detailed views of specific one-dimensional or two-dimensional signal components. Signal and image resolution enhancement is therefore also a fundamental problem in signal analysis. There are numerous existing models and algorithms for digital image enhancement and this field of study is currently very active. The Image Resolution Enhancement using the Wavelet Transform (IREWT) is presented here as a new designed method to solve efficiently this problem.

The methods described further have been developed and verified for simulated one-dimensional and two-dimensional signals and then applied to processing of real biomedical images of the human brain obtained by the magnetic resonance method. All resulting algorithms are verified in the computational and visualization Matlab environment providing tools for remote signal processing using Matlab web server and computer network.

The paper presents selected methods and algorithms related to signal and image decomposition and reconstruction using the wavelet transform at first. Methods of the wavelet decomposition are then used for signal and image recovery of their corrupted parts using selected threshold limits and image resolution enhancement.

2 PRINCIPLES OF DISCRETE WAVELET TRANSFORM

Wavelet transforms (WT) provide an alternative to the short-time Fourier transform (STFT) for non-stationary signal analysis [3]. Both the STFT and the WT result in signal decomposition into two-dimensional function of time and frequency respectively scale. The basic difference between these two transforms is in the construction of the window function which has a constant length in the case of the STFT (including rectangular, Blackman and other window functions) while in the case of the WT wide windows are applied to low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global signal analysis can be combined in this way. Wavelet functions used for signal analysis are derived from the initial basic (mother) function forming the set of functions

$$W_{m,k}(t) = \frac{1}{\sqrt{a}}\, W\left(\frac{1}{a}\,(t-b)\right) = \frac{1}{\sqrt{2^m}}\, W\left(2^{-m}t - k\right) \qquad (1)$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k2^m$. Wavelet dilation corresponds to the spectrum compression. The most common choice includes Daubechies wavelets even though their frequency characteristics stand for approximation of band-pass filters only. On the other hand harmonic wavelets introduced in [8] can have broader application in many engineering problems owing to their very attractive spectral properties.

3 WAVELET DECOMPOSITION AND RECONSTRUCTION

The principle of image wavelet decomposition [13] is presented in Fig. 1 for an image matrix $[G(n,m)]_{N,M}$. Any one-dimensional signal can be considered as a special case of an image having one column only.

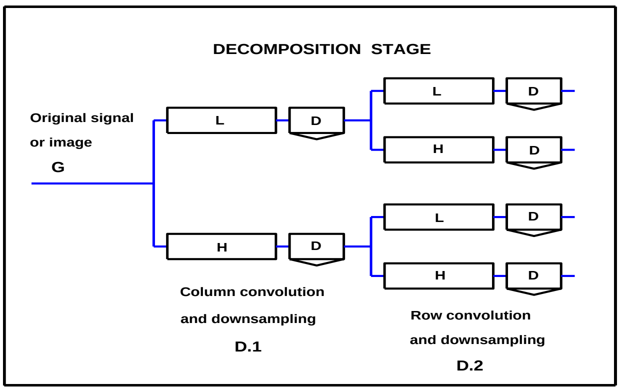

Figure 1: Principle of the 2-D wavelet decomposition followed by downsampling

The **decomposition stage** includes the processing of the image matrix by columns at first using wavelet (high-pass) and scaling (low-pass) function followed by row downsampling by factor D in stage $D.1$.

Let us denote a selected column of the image matrix $[G(n,m)]_{N,M}$ as signal $\{x(n)\}_{n=0}^{N-1} = [x(0), x(1), ..., x(N-1)]^T$. This signal can be analyzed by a half-band low-pass filter represented by the scaling function with its impulse response

$$\{l(n)\}_{n=0}^{L-1} = [l(0), l(1), \cdots, l(L-1)]^T \qquad (2)$$

and corresponding high-pass filter represented by the wavelet function based upon impulse response

$$\{h(n)\}_{n=0}^{L-1} = [h(0), h(1), \cdots, h(L-1)]^T \qquad (3)$$

The first stage presented in Fig. 1 assumes the convolution of a given signal and the appropriate filter for decomposition at first by relations

$$d_0(n) = \sum_{k=0}^{L-1} l(k)x(n-k) \qquad (4)$$

$$d_1(n) = \sum_{k=0}^{L-1} h(k)x(n-k) \qquad (5)$$

for all values of n followed by subsampling by factor D. In the following decomposition stage $D.2$ the same process is applied to rows of the image matrix followed by row downsampling. The decomposition stage results in this way in four images representing all combinations of low-pass and high-pass initial image matrix processing.

The **reconstruction stage** shown in Fig. 2 includes row upsampling by factor U at first and row convolution in stage $R.1$. The corresponding images are then summed. The final step $R.2$ assumes column upsampling and convolution with reconstruction filters followed by summation of the results again.

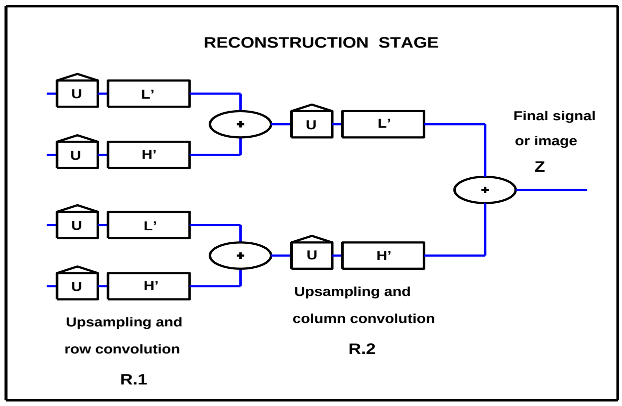

Figure 2: Principle of the backward 2-D wavelet reconstruction

In the case of one-dimensional signal processing, steps $D.2$ and $R.1$ are omitted. The whole process is called signal/image decomposition and perfect reconstruction using D=2 and U=2.

4 DIGITAL IMAGES REGIONS RECOVERY

Image regions recovery represents basic problems in image processing with many different applications including engineering, reconstruction of missing data during their transmission and enhancement of biomedical structures as well [1, 7, 16]. This problem occurs also in filling-in blocks of missing or corrupted data. The following method is based on the two-dimensional discrete wavelet transform approach. Iterated interpolation [6] based upon the wavelet transform forms the new method designed here. This method is verified for simulated data and then applied to processing of real magnetic resonance images. Sum of Squared Errors (SSE), Peak Signal-to-Noise Ratio (PSNR), and subjective aesthetic notion are the criteria of the consistency between the original image and image after the recovery.

We can view a sequence of lost samples as the result of a particular noise process acting on the original signal. However, unlike the traditional case, this noise process is not uncorrelated with the original signal. The designed method comes out from the signal wavelet denoising, which tries to keep transform coefficients of high PSNR while zeroing out coefficients having lower PSNR. Our primary assumption in this algorithm is that the transformation used to generate the wavelet transform coefficients mostly ensures that if vector $\boldsymbol{c}$ is hard-thresholded to zero with $\delta \sim \sigma_e$, then with high probability $|\hat{c}| << |e|$, i.e., hard-thresholding of **c** removes more noise than signal by the following relation

$$\overline{c}(k) = \begin{cases} \mathrm{c}(k) & \text{if } |c(k)| > \delta \\ 0 & \text{if } |c(k)| \leq \delta \end{cases} \tag{6}$$

where δ is a threshold limit, $\overline{c}$ is a vector of thresholded coefficients, c is a vector of wavelet coefficients of the signal containing an additive noise e, $\hat{c}$ is a vector of wavelet coefficients of the signal without noise, and σ_e is a variance of noise.

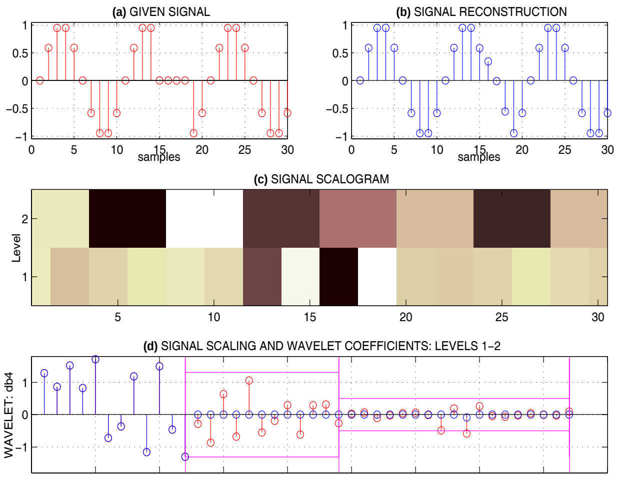

Figure 3: The first iteration of the simulated 1-D signal (sinwave) recovery process presenting **(a)** given corrupted signal, **(b)** corrupted signal decomposition into two levels, **(c)** recovered signal (after the 1st iteration), and **(d)** thresholding of wavelet coefficients

This algorithm makes changes just to the lost sequence of samples by the wavelet transform coefficients hard-thresholded to zero. When the value of the lost samples is changed, we can continue to evaluate these samples again. Input signal for the wavelet decomposition, hard-thresholding, and backward wavelet reconstruction, is a result of the previous iteration. The algorithm is repeated until the SSE value between the recovered and the original signal is acceptably low or required $PSNR$ value is achieved.

Fig. 3 presents the 1st iteration of the described signal recovery algorithm. Result is shown in Fig. 4 presenting that 20 iterations were sufficient for acceptable SSE plotted in Fig. 5. The best

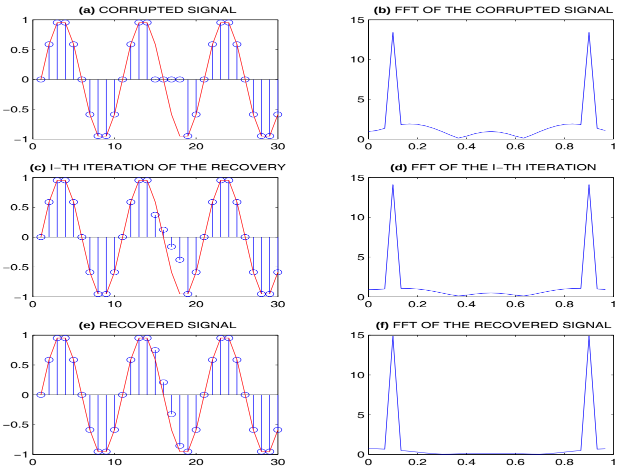

Figure 4: The 1-D simulated signal recovery process presenting **(a)** corrupted signal, an ideal shape of sine wave, **(b)** its FFT, **(c)** recovered signal after the first iteration of the recovery algorithm, **(d)** its FFT, **(e)** recovered signal (after 20 iterations), and **(f)** its FFT

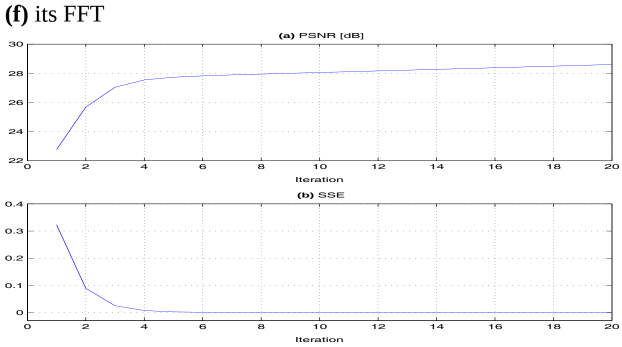

Figure 5: Results of the iterative process presenting **(a)** the PSNR values and **(b)** the SSE for the whole iteration process of the 1-D signal recovery

result has been obtained by the Daubechies wavelet function of the 4th order used for the decomposition into two levels and reconstruction stage by the wavelet decomposition and reconstruction schemas shown in Figs. 1, 2.

Now it is possible to apply the proposed algorithm to the real MR image of a human brain. Fig. 6 presents the wavelet decomposition of the original corrupted MR image (see Fig. 6(a)) into one decomposition level (Fig. 6(b)) using the Daubechies wavelet function of the 8th order. Fig. 6(d) shows the wavelet coefficients and modified, i.e. thresholded wavelet coefficients. Recovered MR image (after the first iteration) can be seen in Fig. 6(c).

The final recovered MR image with low acceptable SSE and high $PSNR$ has been obtained after 350 iterations of the iterative wavelet interpolation algorithm. Fig. 7(a) presents the original corrupted image and Fig. 7(b) recovered image. Evolution of the $PSNR$ and SSE values during the whole recovery process is shown in Fig. 7(c),(d).

The best result of the MR image recovery has been obtained using the Daubechies wavelet function of the 8th order (see Table 1) for the wavelet decomposition into one level. Coefficients in the wavelet domain have been modified by the hard-thresholding method and reconstructed back to the real image.

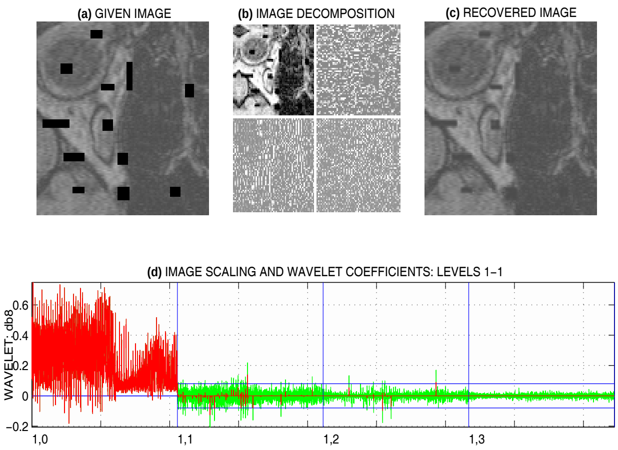

Figure 6: The first iteration of the real MR image recovery process presenting **(a)** given corrupted image, **(b)** image decomposition into one level, **(c)** backward wavelet reconstruction, and **(d)** wavelet, scaling coefficients

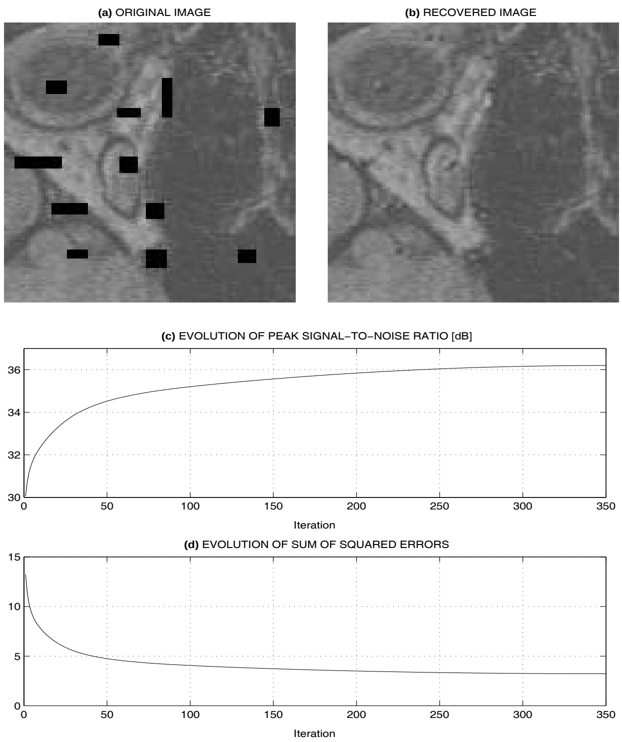

Figure 7: Recovery of the real MR image using the iterated wavelet interpolation method presenting **(a)** given corrupted image, **(b)** recovered image (after 350 iterations), **(c)** evolution of the Peak Signal-to-Noise Ratio (PSNR) value, and **(d)** evolution of the Sum of Squared Errors (SSE) value during the iteration process

This method is sufficient in the case of a limited number of interpolated pixels. If the corrupted block is large (it usually means more than 100 pixels), it is necessary to divide this region into more layers and to recover them step by step. Then the recovery algorithm starts by grouping the interpolated pixels (pixels in the lost block) into layers as shown in Fig. 8.

Layers are recovered in stages with each layer recovered by mainly using the information from the proceeding layers, that is, layer 0 = image is used to recover layer 1, layers 0 and 1 are used to recover layer 2 etc. The layer grouping in Fig. 8 is of course one possibility, and many different groupings can be chosen depending on the size and shape of the lost blocks. Beyond the grouping into layers and associated recovery of layers in stages, the main steps of the algorithm amount to evaluating several complete transforms over the target layer, selective hard-thresholding of wavelet transform coefficients, inverse transforming to generate intermediate results, and finally clipping to obtain the recovered layer. Starting with an initial threshold δ_0, these steps are carried out iteratively where at each iteration the threshold is evaluated again and the layers are recovered to finer detail using the new threshold. Prior to the first iteration, pixels in the lost block are assigned initial values, usually, the mean value computed from the surroundings of the outer boundary of layer 1.

	Decomposition method (wavelet function)	**PSNR1 [dB]**	**PSNR2 [dB]**
1	*Haar*		30.143
2	*Daubechies of the 2nd order*		33.118
3	*Daubechies of the 4th order*		34.715
4	*Daubechies of the 8th order*	29.078	36.200
5	*Symmlet of the 2nd order*		33.118
6	*Symmlet of the 4th order*		35.657
7	*Symmlet of the 8th order*		35.978

Table 1: Peak Signal-to-Noise Ratio (PSNR) of the real MR image containing corrupted regions ($PSNR1$) and the same MR image after the recovery process ($PSNR2$) reconstructed by the selected wavelet functions after 350 iterations

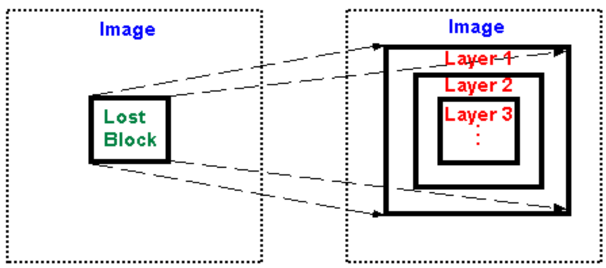

Figure 8: Layers of pixels in the recovery algorithm

5 RESOLUTION ENHANCEMENT OF DIGITAL IMAGES

The principle of signal and image wavelet decomposition and reconstruction for signal resolution enhancement [2, 15] is presented in Fig. 9 for an image matrix $[G(n,m)]_{N,M}$. The general principle of the image wavelet decomposition and reconstruction has been already described in Chapter 3.

The whole process can be used for:

- Signal or image wavelet decomposition and perfect reconstruction using D=2 and U=2
- Signal or image resolution enhancement in the case of D=1 and U=2

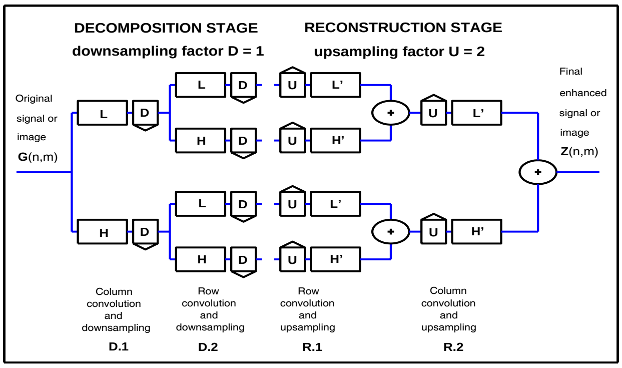

Figure 9: The principle of signal and image resolution enhancement by the DWT

The process of signal resolution enhancement is presented in Fig. 10 for a real biomedical image of the human brain. The initial image matrix shown in Fig. 10(a) as a 3-D plot is decomposed into wavelet coefficients shown in Fig. 10(c) providing enhanced image matrix shown in Fig. 10(b).

Table 2 presents results of the DWT use for image resolution enhancement with different wavelet functions chosen from the given set available in the wavelet transform library in comparison with the result of the DFT algorithm. It is obvious from Table 2 that the DFT method provides good results from the objective criteria point of view (MSE value), but subjective view to the resulting images gives the important conclusion about smooth texture and non-sharp edges in the image. The DWT methods solve the problems of the resolution enhancement better from the MSE value point of view (most of the selected wavelet functions), furthermore the observer or MR images specialist can see the preservation of the image texture and sharp edges.

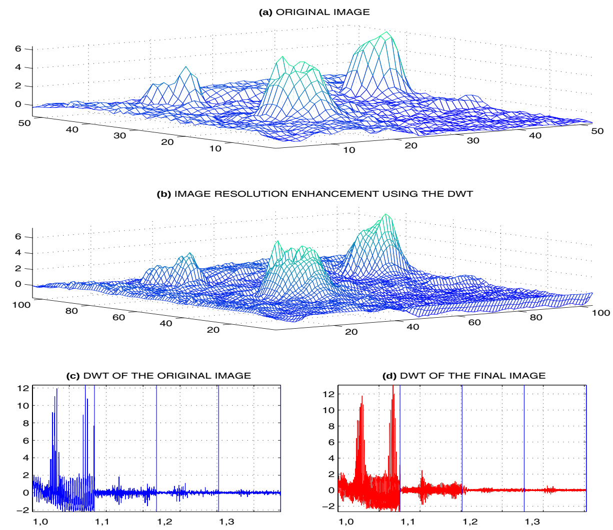

Figure 10: Application of the DWT for MR image resolution enhancement with results presented as a 3-D plot

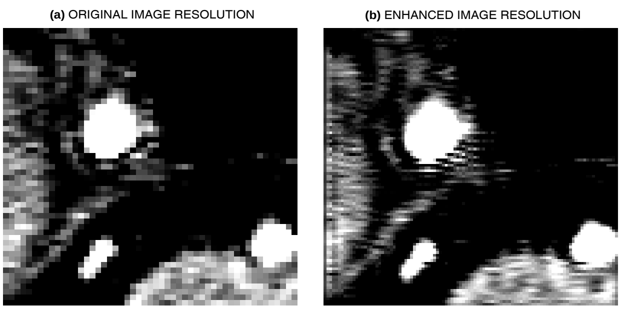

Figure 11: Application of the DWT for MR image resolution enhancement

The best results have been obtained by the DWT algorithm using the Symmlet wavelet function of the 4th order applied to the decomposition into one level. This result is presented in Fig. 11.

	Resolution Enhancement Method	MSE
1	*DFT Algorithm*	0.16452
2	*Haar Wavelet*	0.31350
3	*Daubechies Wavelet of the 2nd order*	0.21126
4	*Daubechies Wavelet of the 4th order*	0.13509
5	*Daubechies Wavelet of the 8th order*	0.14900
6	*Symmlet Wavelet of the 2nd order*	0.21126
7	*Symmlet Wavelet of the 4th order*	0.03406
8	*Symmlet Wavelet of the 8th order*	0.04882

Table 2: Mean square errors (MSE) between the reference MR image of the brain and the same image downsampled by two and then enhanced to double size using the designed algorithms

The whole algorithm of the image resolution enhancement can be written step by step as follows:

1. **Selection** of a sufficient wavelet function
2. **Selection** of the number of decomposition levels
3. **Image wavelet decomposition** by the schema in Fig. 9 using Eqs. (4), (5) where the downsampling factor D=1
4. **Backward wavelet reconstruction** using the upsampling factor U=2

6 RESULTS AND CONCLUSIONS

The result of the whole process of the MR image enhancement consisting of

(i) image denoising

(ii) image regions recovery

(iii) image resolution enhancement

is presented in Fig. 12.

Problem of the digital **images denoising** has been already solved and published in [10].

The usage of the wavelet decomposition and reconstruction for **images regions recovery** includes methods with many optional parameters and variances. It is possible to use the discrete wavelet transform in combination with whatever known method chosen by the properties of the processed image. This sort of algorithms has a wide range of its usage. Iterated wavelet interpolation method forms the most successful method to solve this problem. The best result of the MR image recovery using this method has been obtained using the Daubechies wavelet function of the 8th order for the wavelet decomposition into one level. Coefficients in the wavelet domain have been modified by the hard-

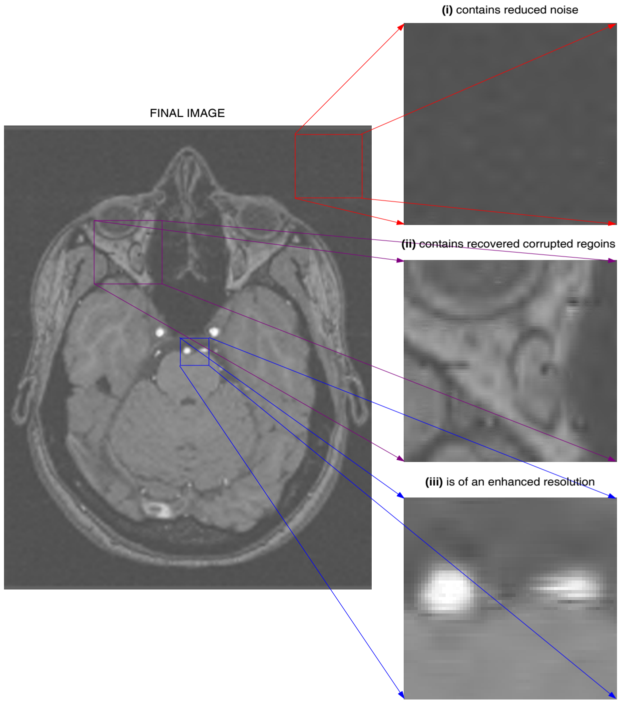

Figure 12: Real MR image after the enhancement process **(i)** containing a reduced noise component, **(ii)** containing recovered corrupted or missing regions, **(iii)** having an enhanced resolution

thresholding method and reconstructed back to the real image. The utilization of the iterated wavelet interpolation method is quite wide giving very good results especially in combination with layer grouping of the corrupted image block.

The last problem of digital images enhancement is in the **resolution enhancement**. The DFT method was applied at first. This method is known from literature [8, 14] as the zero-padding method for 1-D signals. Here the algorithm has been extended into two dimensions. As a new method the image resolution enhancement based upon the DWT has been adopted here. This method has been very successful from the objective point od view (measured by the MSE value) as well as from the subjective point of view (aesthetical notion) of the sharp edges preservation. The best result of the resolution enhancement by the DWT algorithm has been obtained by the Symmlet wavelet function of the 4th order applied to the decomposition into one level. There is no limitation of this method for any sort of digital images.

The new approach in DSP adopted in this paper is in the wide application of the wavelet transform in digital image processing. Wavelet transform becomes a potential very efficient mathematical tool in the whole field of digital signal processing. It is just necessary to find its right application, it can not substitute traditional or other new tools of digital signal processing. Although the utilization of the wavelet transform is not very easy, methods based on this tool give very useful and interesting results in many engineering areas.

Further research will be devoted to the appropriate estimation of threshold limits and to the precise choice of wavelet functions according to analysed signal/image. Special attention will be also paid to the 3-D interpolation of the MR scans of the human brain and processing of the resulting 3-D model of the brain.

Selected results and algorithms can be obtained from the web page of the Prague DSP research group (http://dsp.vscht.cz) providing also access to the Matlab web server allowing remote data analysis and processing.

References

[1] R. Baraniuk. Optimal Tree Approximation Using Wavelets. In *SPIE Technical Conference on Wavelet Applications in Signal Processing*, Denver, CO,U.S.A., July 1999.

[2] R. Baraniuk, R. DeVore, G. Kyriazis, and X. Yu. Near Best Tree Approximation. *Advances in Computational Mathematics*, 2002.

[3] L. Debnath. *Wavelets and Signal Processing*. Birkhauser Boston, Boston, U.S.A., 2003.

[4] A. Duci, A. J. Yezzi, S. K. Mitter, and S. Soatto. Region Matching with Missing Parts. In *7th European Conference on Computer Vision-Part III, pages: 48-64, London, U.K.* Springer-Verlag, 2002.

[5] W. Etter. Restoration of Discrete-Time Signal Segments by Interpolation Based on the Left-Sided and Right-Sided Autoregressive Parameters. *IEEE Trans. Signal Processing*, 44(5):1124–1135, May 1996.

[6] Onur G. Guleryuz. Iterated Denoising for Image Recovery. In *Data Compression Conference (DCC '02), Snao Bird, Utah*. IEEE, 2002.

[7] U. Ndili, R. Nowak, and M. Figueiredo. Coding Theoretic Approach to Image Segmentation. In *IEEE International Conference on Image Processing*, October 2001.

[8] D. E. Newland. *An Introduction to Random Vibrations, Spectral and Wavelet Analysis*. Longman Scientific & Technical, Essex, U.K., third edition, 1994.

[9] I. Pitas. *Digital Image Processing Algorithms and Applications*. John Wiley & Sons, Inc., New York, 2000.

[10] A. Procházka, I. Šindelářová, and J. Ptáček. Image Denoising and Restoration Using Wavelet Transform. In *European Control Conference, Cambridge*. IEE London, 2003.

[11] J. Ptáček, I. Šindelářová, A. Procházka, and J. Smith. Wavelet Transforms In Signal And Image Resolution Enhancement. In *16th International Conference Algoritmy 2002, Podbanske*. The Slovak Technical University, 2002.

[12] S. D. Rane, G. Sapiro, and M. Bertalmio. Structure and Texture Filling-In of Missing Image Blocks in Wireless Transmission and Compression Applications. *IEEE Transactions on Image Processing*, 12(3):296–303, March 2003.

[13] J. Romberg, H. Choi, and R. Baraniuk. Tree Structured Image Modeling. In *IEEE Southwest Symposium on Image Analysis and Interpretation*, Austin, TX, U.S.A., April 2000.

[14] Saeed V. Vaseghi. *Advanced Digital Signal Processing and Noise Reduction*. Wiley & Teubner, West Sussex, U.K., second edition, 2000.

[15] M. Wakin. Image Compression using Multiscale Geometric Edge Models. Master's thesis, Rice University, U.S.A., May 2002.

[16] R. Willett and R. Nowak. Platelets: A Multiscale Approach for Recovering Edges and Surfaces in Photon-Limited Medical Imaging. Technical report, Rice University, U.S.A., July 2003.