HEAT CONDUCTION IN RADIOACTIVE WASTE REPOSITORY

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Abstract

The paper presents some numerical tests for the temperature development around nuclear waste packages stored in vaults. We deal with two simplified models. In the first of them, a single long cylinder of the diameter 1 m is considered. It is filled with radioactive waste and surrounded by rock. The second model presents a long cuboidal package stored in a vault. The computation was carried out in Matlab using its tools for sparse matrices and iterative methods.

1 Radioactive waste and heat conduction.

The radioactive waste and spent nuclear fuel repository management belongs to the actual issues of contemporary physics and engineering sciences. The extreme complexity of the task leads to considering and solving the separated aspects of the problem the results of which have to be taken into account in the design of storage and disposal facilities. In the presented paper, we consider the heat production as a consequence of high-level radioactivity.

The paper presents numerical tests of the temperature development around waste packages stored in vaults which are built in the depth of about 300 m. The model of the repository is based on the well known Yucca Mountain repository project and on the data presented by some European projects [1, 2]. We deal with two models. In the first of them, a single long steel cylinder of the diameter 1 m is considered. It is filled with radioactive waste and surrounded by rock. The second model describes a long cuboidal package stored in a vault. The numerical formulation reduces to the one-dimensional task in the first case, while it leads to a two-dimensional problem in the second case.

We consider the equation for the heat conduction [7]

$$\nabla k \nabla u + q = \rho c \frac{\partial u}{\partial t}, \tag{1}$$

where k is the thermal conductivity [W/mK], q is the volumetric heat release [W/m^3], ρ is the density [kg/m^3] and c is the specific heat capacity [J/kgK]. The equation has to be fulfilled in some domain Ω the dimensions of which can be tens up to hundreds of meters. The temperature $u(x, y, z, t)$ depends on three space variables and on the time. The function $u(., ., ., 0)$ is given and it gives the initial condition

$$u(x, y, z, t) = u_0(x, y, z) \tag{2}$$

for all $t = 0$. The Dirichlet boundary condition

$$u(x, y, z, t) = 10 \tag{3}$$

is considered in each point of the boundary of the domain Ω, i.e. the constant temperature 10 °C sufficiently far from the heat source is considered. As follows from the character of the problem, the constant in the Dirichlet boundary condition changes additively the solution in the whole domain. The heat produced by the radioactive material corresponds to the approximate volumetric heat release $1036.10^{-0.056t}$ W/m^3, where t is time measured in years. The characteristics of the radioactive packages, rock and air used in our tests can be found in Table 1.

Table 1: Constants used for heat conduction

	k [W/mK]	c [J/kgK]	ρ [kg/m^3]
Waste packages	3.1	509	5400
Rock	3.0	760	2670
Air	0.03	1000	1

2 One-dimensional model.

In order to estimate the main characteristics of the heat distribution, we can simplify the problem in the following way. We consider a very long cylindrical package stored directly in the rock. The problem can be then viewed as a one-dimensional problem: the temperature in the package and around is a function of the distance from the axis of the cylinder. Reformulating the laplacian in the cylindrical coordinates, we get the equation

$$k\Big(\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x}\Big) + q = \rho c \frac{\partial u}{\partial t}$$

which has to be valid for $x \in \langle 0, L\rangle$, $t \in \langle 0, \infty)$, and the boundary conditions

$$\frac{\partial u}{\partial x} = 0$$

for $x = 0$ and $t \in \langle 0, \infty)$, and

$$u = 10$$

for $x = L$ and $t \in \langle 0, \infty)$. The center of the symmetry (the axis of the cylinder) corresponds to the point $x = 0$. The diameter of the package is 0.5 m. The functions k, ρ and c are piecewise constant in $\langle 0, L\rangle$, while $q = 1036 \,.\, 10^{-0.056t}$ in $\langle 0, 0.5\rangle$ and it is zero in the rest of the domain. The quantity t is measured in years.

The finite difference method was used for the numerical solution of this task. The domain $\langle 0, L\rangle$ was divided into n subintervals bounded by numbers

$$x_0 = 0 < x_1 < x_2 < \ldots < x_n = L.$$

The second order difference scheme for the first and for the second derivatives was applied in each node. Then two additional nodes behind each end of the domain have to be considered. To avoid the evaluating the term $1/x$ for $x = 0$, the following equation can be used

$$\lim_{x\to 0} f'' + \frac{1}{x} f' = 2f''$$

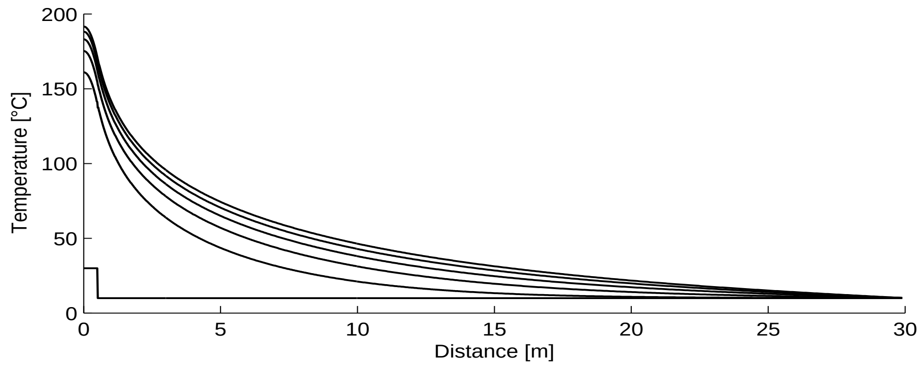

Figure 1: Temperature distribution around the cylindrical canister computed for 0, 1, 2, 3, 4 and 5 years after installation.

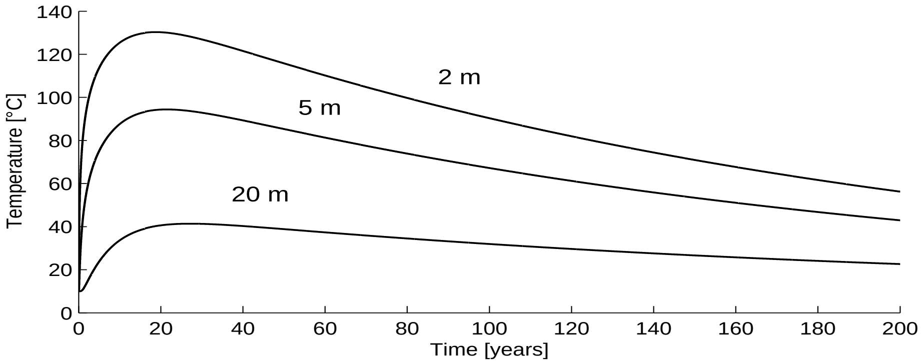

Figure 2: Temperature progression around the cylindrical canister. The distances from the axis of the cylinder are 2 m, 5 m and 20 m.

when supposing $f \in C^2$ and $f'(0) = 0$.

The resulting system of algebraical equations involve a tridiagonal matrix. The optimal solution algorithm is then the Gauss elimination method reachable in Matlab as *mldivide* function.

In order to achieve a faster method, we can use nonequidistant partitioning of the domain. The original interval $\langle 0, L\rangle$ is divided into subintervals $I_1, I_2, \dots, I_m$ such that the k-th of them contains the nodes

$$x_{k,1}, x_{k,2}, \dots, x_{k,n_k},$$

such that

$$x_{k-1,n_k} = x_{k,1}$$

for $k = 1, 2, \dots, m$ and the distances between any neighbors are $h_0 2^k$ for some constant $h_0 > 0$, i.e. they are twice larger that the distances between nodes in the previous interval I_{k-1}. The finite difference scheme can be easily adapted for such a case. However, the matrix obtained is no more tridiagonal thus the advantages of such an approach are problem dependent.

As a result of this simple one-dimensional problem we obtain the rough estimate of the dimensions of the region which should be considered when computing some more specific problem. Figures 1 and 2 show the temperature distribution in the depository dependent on time. It turns out that the package of the diameter about one meter influences the region of the diameter about tens of meters in the first several years after storing them in the repository.

3 Two-dimensional model.

We suppose a very long cuboidal canister stored in a vault in a rock. The canister contains radioactive material and its cross section is a square with its edge of 1.5 m long. The corridor 3 m high and 5 m wide and the package is stored nonsymmetric in it. Such a model can be represented by the equation in two space dimensions.

The numerical solution can be obtained after applying the finite element method to the weak formulation of the problem. This is to find the function u such that $u(., t) \in V$ for all $t \in \langle 0, T\rangle$ and

$$\Big(\frac{\partial u}{\partial t}, v\Big) + a(u, v) = b(v) \tag{4}$$

and

$$(u(., 0) - u_0, v) = 0 \tag{5}$$

for all $v \in V$, where V is some finitedimensional subspace in $H_0^1(\Omega)$ and $(.,.)$ is the inner product in $L_2(\Omega)$. The bilinear form a is given by

$$a(u,v) = \int_\Omega \frac{k}{\rho c}\Big(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\Big)\,\mathrm{d}x\,\mathrm{d}y$$

and the linear functional b is defined by

$$b(v) = \int_\Omega \frac{q}{\rho c} v\,\mathrm{d}x\,\mathrm{d}y$$

Using the Crank-Nicolson computational scheme for the time variable in (4), we obtain the set of equations

$$\Big((u^k - u^{k-1}), v\Big) + \frac{d^k}{2}\Big(a(u^k, v) + a(u^{k-1}, v)\Big) = \frac{d^k}{2}\Big(b^k(v) + b^{k-1}(v)\Big)$$

for $k = 1, 2, \ldots$, where $d^k = t^k - t^{k-1}$ are the time steps and u^k are the approximations of u in the discrete times t^k. The initial approximations u^0 is given by (5) and the Dirichlet boundary condition (3) is applied.

The discretization for the space variable can be done via the finite element method. We use the bilinear finite elements with rectangular supports [5]. Similarly to the one-dimensional case, the mesh can be sparser outside the region with the most significant temperature changes.

In spite of the one-dimensional numerical test, the computing the two-dimensional model is much more time consuming. For solving the sets of linear algebraic equation (see e.g. [3, 6] for the full description) we have used the conjugent gradient method (function *cgs* in Matlab) without any preconditioning. The initial approximation in each step was chosen to be equal to the solution in the previous time step. This caused the reduction of the number of the inner iterations steps from about 31 to 24.

The predicted temperature distribution along the radioactive waste repository is indicated in Figures 3 and 4. The situation after one month and after six months after storing the canisters into the vault is displayed.

4 Discussion.

The introduced paper can be viewed as a starting computational configuration for further and more detailed and involved analysis of the temperature development in the high-radioactive waste repository facilities, i.e. including heat convection and radiation and changing the elasticity properties of the material.

The values of the constants used in the heat conduction equation may vary according the specific conditions. For example, the value $k = 2$ W/mK was estimated by Case and Wagner [4].

The need of more powerful computational methods and technique is self-evident. In [3, 6] the parallel methods for computing parabolic differential equations with application to the long term events like radioactive waste reposition can be found.

References

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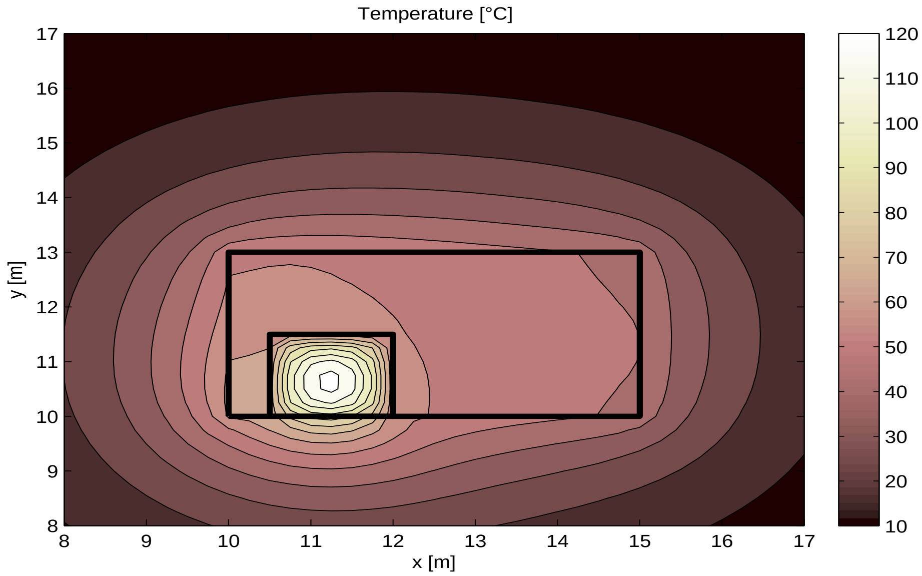

Figure 3: Temperature distribution around the canister in a vault one month after the installation. The temperature levels are scaled by 8 degrees.

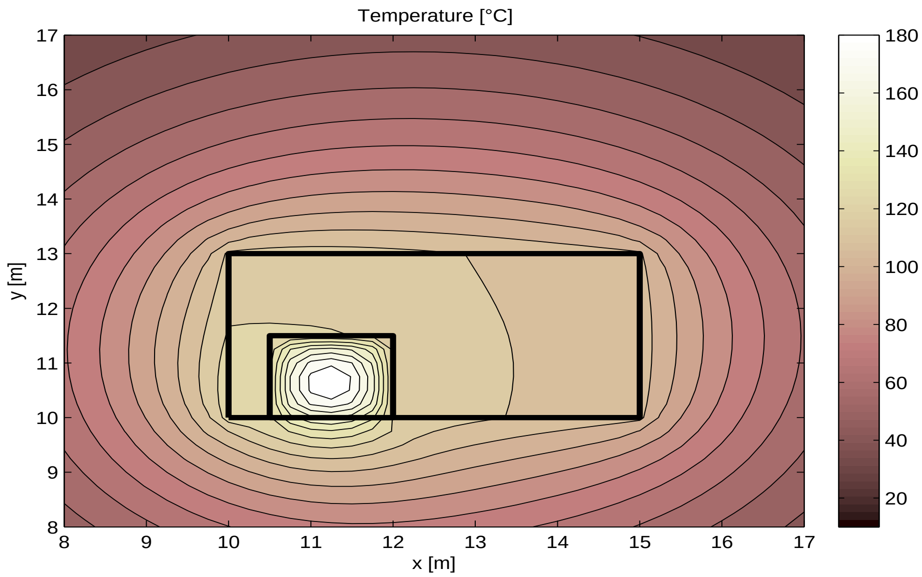

Figure 4: Temperature distribution around the canister in a vault six months after the installation. The temperature levels are scaled by 8 degrees.

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