

COUPLED PHYSICAL MODEL OF MICROWAVE DRYING OF WOOD

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Abstract

The work is focused on phenomenological and numerical assembling of coupled FEM model of microwave wood drying process in its early stages which analyze the stress-strain distribution and possibility of failure occurrence due to high temperature gradients. Model takes into account the temperature dependency of physical and mechanical properties with its orthotropic character. Thermal field in wood was described by conduction type of spreading. Coupling of physical fields is based on diffusive character of thermal flow. Scripted algorithm is fully parameterized and allows forming different sets of kilns, materials, and chambers.

Introduction

Drying of wood is very complex process which influences the final quality of wood in technology process. The complexity is usually reason that appropriate theoretical background can not be used in its general form. Even we are able to describe most of important process in wood during drying process, we are very often unable to set together coupled models which include interaction among most of these phenomena. Also solution of such few models which were constructed with most of coupled effects is very difficult due to large amount of unknown variables as in mathematical sense (a lot of degrees of freedom) as in physical point of view (a lot of physical properties which forms the material models for such simulations).

Microwave drying of wood is the most complex problem in the area due to combination of very different physical fields which differ on geometry scale (small changes in structure is usually neglected for high frequencies of emag fields. On the contrary small change in structure can influence the moisture and temperature field distribution), time scale (RF field is very quick opposite to water and thermal movement) and domain occupation (rapid changes of gradient for RF and usually small changes for water and thermal movement or moisture and temperature gradient changes). All such facts often put competing requirements on mathematic and algorithmic construction of appropriate model.

Topic of the work was partly solved in several research works. Pure microwave drying of wood including moisture and heat flow can be found in Antti L. (1999), Perre P. and Turner I.W. (1997), Zhao H. and Turner I.W. (2000) and Hansson et al. (2006).

Theory

In microwave kiln the source of energy is modeled with dissipative character which can be related to heated problem by density of energy q_{abs} .

$$q_{abs} = \omega \cdot \varepsilon_{eff}'' \cdot |\mathbf{E}|^2 \quad (1)$$

ω is angular velocity (s^{-1}), ε_{eff}'' is effective relative loss factor, \mathbf{E} is electric field ($V \cdot m^{-1}$).

Estimation of electric field together with magnetic field is based on solution of reduced Maxwell's equations

$$\begin{aligned}
\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \cdot \mathbf{D} &= \rho_e \\
\nabla \cdot \mathbf{B} &= 0.
\end{aligned} \tag{2}$$

\mathbf{B} is the magnetic flux density, \mathbf{D} is electric flux density, \mathbf{H} is magnetic field intensity, \mathbf{J} is current density, ρ_e is electric charge density. Due to anisotropy of wood we can itemize these variables to $\mathbf{D} = \varepsilon_r \mathbf{E}$, $\mathbf{B} = \mu_r \mathbf{H}$, $\mathbf{J} = \sigma \mathbf{E}$, where ε is permittivity, μ is permeability and σ is electric conductivity of material.

Natural requirement for continual charge is satisfied by equation of continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_e}{\partial t}. \tag{3}$$

By rearrangement the following PDE has to be solved:

$$\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}) - k_0^2 \left(\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0} \right) \mathbf{E} = \mathbf{0} \tag{4}$$

Although the microwave type of drying has volumetric character, the heating is not uniform in the whole region. This fact is caused not only by anisotropic material properties, but also by specific geometry on lower scales. The phase change of water to steam can locally and dramatically change dissipation of microwave energy. These changes can occur as hotspot(s) in small regions of material (Kriegsmann G. A. (1997)). Temperature difference is compensated by thermal energy exchange between hotspot and narrow surroundings. Exchange is mainly conductive and can be rewritten in the following form.

$$\rho C \frac{\partial T}{\partial t} - \left[\begin{aligned} &\frac{\partial}{\partial x} \left(k_{11} \frac{\partial T}{\partial x} + k_{12} \frac{\partial T}{\partial y} + k_{13} \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial y} \left(k_{21} \frac{\partial T}{\partial x} + k_{22} \frac{\partial T}{\partial y} + k_{23} \frac{\partial T}{\partial z} \right) + \\ &+ \frac{\partial}{\partial z} \left(k_{31} \frac{\partial T}{\partial x} + k_{32} \frac{\partial T}{\partial y} + k_{33} \frac{\partial T}{\partial z} \right) \end{aligned} \right] = q_{\text{abs}}, \tag{5}$$

ρ is density of material (kg.m^{-3}), C is heat capacity (J.K^{-1}), k_{ij} are heat conduction coefficients ($\text{kg.m.s}^{-3}.\text{K}^{-1}$), T is temperature, t is time.

In shortened notation we can write:

$$\rho C \frac{\partial T}{\partial t} - \nabla \mathbf{k} \nabla T = q_{\text{abs}}, \tag{6}$$

\mathbf{k} is matrix of heat conduction.

Also let us assume available demand on regulation of temperature gradient by convective type of thermal energy source e.g. in the following sense.

$$\rho C \frac{\partial T}{\partial t} - \nabla \mathbf{k} \nabla T = q_{\text{abs}} + k_{h_T} \cdot (T_{\text{ext}} - T), \tag{7}$$

k_{h_T} is heat transfer coefficient ($\text{Wm}^{-2}\text{K}^{-1}$), T_{ext} is temperature in the near surroundings (K).

Moisture flow is arguable modeled just in part of diffusion process, but in this study the part of moisture flow will be presented and emphasis is putted on the first part of microwave drying where coupling of emag, temperature and stress-strain field plays very important role. Full notation including the influence of moisture and free water was declared in Koñas P. (2008).

Temperature gradient caused by RF emag field results in force response, which can be described by the following way. Mechanical energy dissipation and its transformation are neglected, also memory effects due to viscous character of wood can be omitted for such early stages of simulation what is for realized emag and mechanical coupling. Generally elastic strains and viscous-elastic strain are composed from thermal strain and moisture (swelling/shrinkage) strain.

$$\begin{aligned}\boldsymbol{\varepsilon}_c^{\text{el}} &= \boldsymbol{\varepsilon}^{\text{el}} + \boldsymbol{\varepsilon}_w^{\text{el}} + \boldsymbol{\varepsilon}_T^{\text{el}} \\ \boldsymbol{\varepsilon}_c^{\text{vel}} &= \boldsymbol{\varepsilon}^{\text{vel}} + \boldsymbol{\varepsilon}_w^{\text{vel}} + \boldsymbol{\varepsilon}_T^{\text{vel}},\end{aligned}\quad (8)$$

$\boldsymbol{\varepsilon}_c^{\text{el}}$ is the whole elastic strain, $\boldsymbol{\varepsilon}^{\text{el}}$ is pure elastic strain, $\boldsymbol{\varepsilon}_w^{\text{el}}$ is elastic strain due to moisture gradient, $\boldsymbol{\varepsilon}_T^{\text{el}}$ is elastic strain due to temperature gradient, $\boldsymbol{\varepsilon}_c^{\text{vel}}$ is the whole viscous-elastic strain, $\boldsymbol{\varepsilon}^{\text{vel}}$ is pure viscous-elastic strain, $\boldsymbol{\varepsilon}_w^{\text{vel}}$ is viscous-elastic strain due to moisture gradient and $\boldsymbol{\varepsilon}_T^{\text{vel}}$ is viscous-elastic strain due to temperature gradient.

For purposes of the study the complete strain response is reduced into the form:

$$\boldsymbol{\varepsilon}_c^{\text{el}} = \boldsymbol{\varepsilon}^{\text{el}} + \boldsymbol{\varepsilon}_T^{\text{el}},$$

Withal value of this strain will be determined by appropriate coefficient that is related to temperature and time together. Mentioned components of strain can be defined by common way.

$$\boldsymbol{\varepsilon}_T^{\text{el}} \equiv (T - T_{\text{ext}}) \cdot (\beta_1 | \beta_2 | \beta_3 | \beta_4 | \beta_5 | \beta_6) = (T - T_{\text{ext}}) \cdot \boldsymbol{\beta}, \quad (9)$$

$\boldsymbol{\beta}$ is thermal expansion coefficient.

Stress-strain relation is reduced to very simple relation of Hook's law.

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}_c^{\text{el}}, \quad (10)$$

$\boldsymbol{\varepsilon}_c^{\text{el}}$ is immediate elastic strain (structural), \mathbf{D} is matrix of elasticity

Materials and methods

Mechanical/physical properties and its dependency to temperature/moisture can be defined by common way mentioned e.g. in Bodig et al. (1982). Relations correspond with dominant linear part of behavior driven by diffusive character of response. Difficulties arise from orthotropic definition of material characteristics. Usually no coupling between directions is defined according to material properties, just independent and similar relation is defined. In the work only dependency of Young module and Shear module of elasticity is proposed to be significantly related to temperature. Poisson ratio variability according to temperature is omitted.

Thus, Young and Coulomb modules of elasticity are linear equation.

$$\begin{aligned}E_{T_i} &= k_{b_i^T} (T - T_{\text{ext}}) + E_{r_i} \\ G_{T_j} &= k_{b_j^T} (T - T_{\text{ext}}) + G_{r_j},\end{aligned}\quad (11)$$

For $i = 1..3, j = 4..6$.

Matrix of elastic coefficients can be separated into constant and dependent parts.

Let us declare matrix \mathbf{D}_X .

$$\mathbf{D}_X = \begin{bmatrix} \frac{(\mu_{z,y}\mu_{y,z} - 1)X_1}{k_D} & -\frac{(\mu_{x,y} + \mu_{z,y}\mu_{x,z})X_2}{k_D} & -\frac{(\mu_{x,z} + \mu_{x,y}\mu_{y,z})X_3}{k_D} & 0 & 0 & 0 \\ \frac{(\mu_{y,x} + \mu_{z,x}\mu_{y,z})X_1}{k_D} & \frac{(\mu_{z,x}\mu_{x,z} - 1)X_2}{k_D} & -\frac{(\mu_{y,z} + \mu_{y,x}\mu_{x,z})X_3}{k_D} & 0 & 0 & 0 \\ \frac{(\mu_{z,x} + \mu_{y,x}\mu_{z,y})X_1}{k_D} & -\frac{(\mu_{z,y} + \mu_{z,x}\mu_{x,y})X_2}{k_D} & \frac{(\mu_{y,x}\mu_{x,y} - 1)X_3}{k_D} & 0 & 0 & 0 \\ k_D & k_D & k_D & X_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_6 \end{bmatrix} \quad (12)$$

In this declaration we used k_D as constant with the following meaning $k_D = \mu_{z,y}\mu_{y,z} + \mu_{y,x}\mu_{x,y} + \mu_{y,x}\mu_{z,y}\mu_{x,z} + \mu_{z,x}\mu_{x,y}\mu_{y,z} + \mu_{z,x}\mu_{x,z} - 1$ and vector $\mathbf{X} = (X_1, X_2, X_3, X_4, X_5, X_6)$ for simple substitution during separation.

Now we can define matrix of elasticity \mathbf{D} .

$$\mathbf{D} = \mathbf{D}_X|_{X=EG} + (\mathbf{T} - \mathbf{T}_{ext}) \cdot \mathbf{D}_X|_{X=\mathbf{K}_{bT}} \quad (13)$$

$$\mathbf{EG} = \mathbf{E}_r \circ \mathbf{G}_r \equiv (\mathbf{E}_x, \mathbf{E}_y, \mathbf{E}_z, \mathbf{G}_{xy}, \mathbf{G}_{yz}, \mathbf{G}_{xz})$$

Withal

$$\mathbf{K}_{bT} \equiv (\mathbf{k}_{b_1^T}, \mathbf{k}_{b_2^T}, \mathbf{k}_{b_3^T}, \mathbf{k}_{b_4^T}, \mathbf{k}_{b_5^T}, \mathbf{k}_{b_6^T}),$$

\mathbf{EG} is vector which originates from spreading of \mathbf{E}_r by \mathbf{G}_r . For simplification of substituted components we will declare individual parts of elasticity matrix.

$$\mathbf{D}_{EG} = \mathbf{D}_X|_{X=EG}, \mathbf{D}_{\mathbf{K}_{bT}} = \mathbf{D}_X|_{X=\mathbf{K}_{bT}}. \quad (14)$$

Applying of Eq. (13), Eq. (14) and Eq. (8) on Eq. (10) we can obtain the following relationship.

$$\boldsymbol{\sigma} = (\mathbf{D}_{EG} + (\mathbf{T} - \mathbf{T}_{ext}) \cdot \mathbf{D}_{\mathbf{K}_{bT}}) \cdot (\boldsymbol{\varepsilon}^{el} + (\mathbf{T} - \mathbf{T}_{ext}) \cdot \boldsymbol{\beta}) \quad (15)$$

Eq. (15) is combined with stress equilibrium conditions including transient effects:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\partial \sigma_{ij}}{\partial x_j} = \mathbf{F}_i, \quad (16)$$

\mathbf{u} is vector of displacements $\mathbf{u} \equiv (u, v, w)$, F_i are components of volume forces

Withal strains are declared by common way.

$$\boldsymbol{\varepsilon}^{el} \equiv \left(\frac{\partial u}{\partial x} \Big| \frac{\partial v}{\partial y} \Big| \frac{\partial w}{\partial z} \Big| \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \Big| \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \Big| \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right), \quad (17)$$

The final relationship for stress-strain components according to unknown variable displacement \mathbf{u} can be formed in this grouped form.

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \left(\nabla \mathbf{c}_{EG} + (\mathbf{T} - \mathbf{T}_{ext}) \nabla \mathbf{c}_{\mathbf{K}_{bT}} \right) \nabla \mathbf{u} + \mathbf{C}_T \cdot \mathbf{T} + \mathbf{C}_{T^2} \cdot \mathbf{T}^2 + \mathbf{C} = \mathbf{F}. \quad (18)$$

Definition of \mathbf{C} coefficients can be revealed by simple rearranging.

$$\begin{aligned} \mathbf{C}_T &= \left(\mathbf{D}_{EG} \cdot \boldsymbol{\beta} - 2\mathbf{D}_{K_bT} T_{\text{ext}} \boldsymbol{\beta} \right); \mathbf{C}_{T^2} = \mathbf{D}_{K_bT} \boldsymbol{\beta} \\ \mathbf{C} &= \left(T_{\text{ext}} \mathbf{D}_{K_bT} - \mathbf{D}_{EG} \right) \cdot T_{\text{ext}} \boldsymbol{\beta}. \end{aligned} \quad (19)$$

Matrixes of diffusion constants (\mathbf{c}) from Eq. (18) can be defined by similar way as matrix of elastic constants (\mathbf{D}). Let us define matrix \mathbf{c}_X .

$$\mathbf{c}_X = \begin{bmatrix} D_{X11} & 0 & 0 & 0 & D_{X12} & 0 & 0 & 0 & D_{X13} \\ 0 & \frac{1}{2}D_{X44} & 0 & \frac{1}{2}D_{X44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}D_{X66} & 0 & 0 & 0 & \frac{1}{2}D_{X66} & 0 & 0 \\ 0 & \frac{1}{2}D_{X44} & 0 & \frac{1}{2}D_{X44} & 0 & 0 & 0 & 0 & 0 \\ D_{X21} & 0 & 0 & 0 & D_{X22} & 0 & 0 & 0 & D_{X23} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}D_{X55} & 0 & \frac{1}{2}D_{X55} & 0 \\ 0 & 0 & \frac{1}{2}D_{X66} & 0 & 0 & 0 & \frac{1}{2}D_{X66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}D_{X55} & 0 & \frac{1}{2}D_{X55} & 0 \\ D_{X31} & 0 & 0 & 0 & D_{X32} & 0 & 0 & 0 & D_{X33} \end{bmatrix}$$

Individual matrixes can be declared by substitution of X .

$$\mathbf{c}_{EG} = \mathbf{c}_X|_{X=EG}, \mathbf{c}_{K_bT} = \mathbf{c}_X|_{X=K_bT}$$

Heat capacity of spruce wood at constant pressure can be rewritten by Perelygin L.M. (1965) it this form:

$$1.571 + 0.00277 \cdot T \text{ [kJ.kg}^{-1} \cdot \text{K}^{-1}] \quad (20)$$

Results

Above declared mathematical notation was rewritten into FEM environment of COMSOL 3.4 by language of COMSOL Script utility. Model is almost fully parameterized as in geometrical variables as in physical properties which were used. Microwave drying kiln was simplified into simple experimental chamber $40 \times 40 \times 20$ [cm] and filled up by spruce wood lumbers $30 \times 4 \times 1.2$ [cm] (Fig. [1]). Waveguide is rectangular $10 \times 16 \times 3.6$ [cm] excited on the rear outer side.

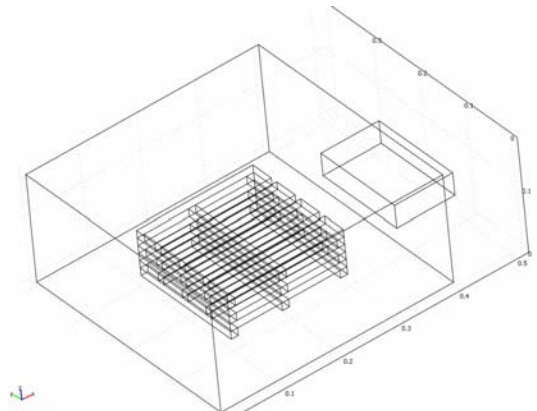


Fig. 1 Geometry of experimental drying kiln filled up by wood lumbers and with waveguide

In simulation the following physical constants were used for abstraction of spruce wood in mechanical task.

k_b^T [1..6]	0.4	-	Constant for Young and shear modules transformation according to temperature difference
T_{ref}	21	°C	Reference temperature
E_{refX}	13650	MPa	Reference Young module of elasticity in T_{ref} for X=(L)ongitudinal anatomical direction
E_{refY}	289	MPa	Reference Young module of elasticity in T_{ref} for Y=(T)angential anatomical direction
E_{refZ}	789	MPa	Reference Young module of elasticity in T_{ref} for Z=(R)adial anatomical direction
G_{refXY}	474	MPa	Reference Shear module of elasticity in T_{ref} for XY=LT plane
G_{refYZ}	53	MPa	Reference Shear module of elasticity in T_{ref} for YZ=TR plane
G_{refXZ}	573	MPa	Reference Shear module of elasticity in T_{ref} for XZ=LR plane
μ_{xy}	0.01	-	Poisson number for XY=LT plane
μ_{yz}	0.56	-	Poisson number for YZ=TR plane
μ_{xz}	0.23	-	Poisson number for XZ=LR plane
ρ	392	kg.m ⁻³	Density of spruce wood

Tab. 1. Physical and mechanical properties dried wood (spruce) used in simulation. Constants were taken over the Bodig et al. (1982).

Physical properties for conductive heat transfer in form of thermal conductivity of spruce were defined as orthotropic with only diagonal members.

$k_{x=L}$	0.288	W/(m.K)	Coefficients of thermal conductivity
$k_{x=L}$	0.125	W/(m.K)	
$k_{x=L}$	0.147	W/(m.K)	
$\beta_{1=L}$	0.0065	-	Coefficients of temperature expansion
$\beta_{2=T}$	0.043	-	
$\beta_{3=R}$	0.098	-	
$\beta_{4..6}$	0	-	
ϵ_r	100-i·20	-	Relative permittivity
μ_r	0.8	-	Relative permeability
σ	10^{-4}	S.m ⁻¹	Electric conductivity

Boundary values were specified on rectangular waveguide as a power source with 30 kW, 2.5 GHz. Waveguide is excited by transverse electric (TE) wave in TE₁₀ mode with no electric field component in direction of propagation. Excitation electric field is given by this definition:

$$\mathbf{E}_0 = (0, 0, \cos(\pi(y - 40[\text{cm}])/16[\text{cm}])) \text{ [V/m]} \quad (21)$$

Walls of chamber were assumed as perfect electric conductor. No symmetry due to kiln structure was implemented. Model was solved by direct solver (PARDISO) with quadric elements on very fine mesh. Solution was realized on 2x2 CPU AMD64 computing server and time of computing was 1 day 5 hours and 33 minutes. Model consists of 347150 triangular elements and 924780 degrees of freedom.

The results of simulation correspond with expected values according to theoretical response of the system in comparison with results of Antti L. (1999) and Zhao H. and Turner I.W. (2000). Model is convergent in all studied physical fields. Distribution of resistive heating (Fig. 2a), Temperature (Fig. 2b), Von Misses equivalent (Fig. 3) and displacement (Fig. 4) are continuous

without significant singularities. Model is solvable in reasonable time and full parameterization allows forming of large amount of models with highly variable geometry and parametrically defined material properties of dried wood material.

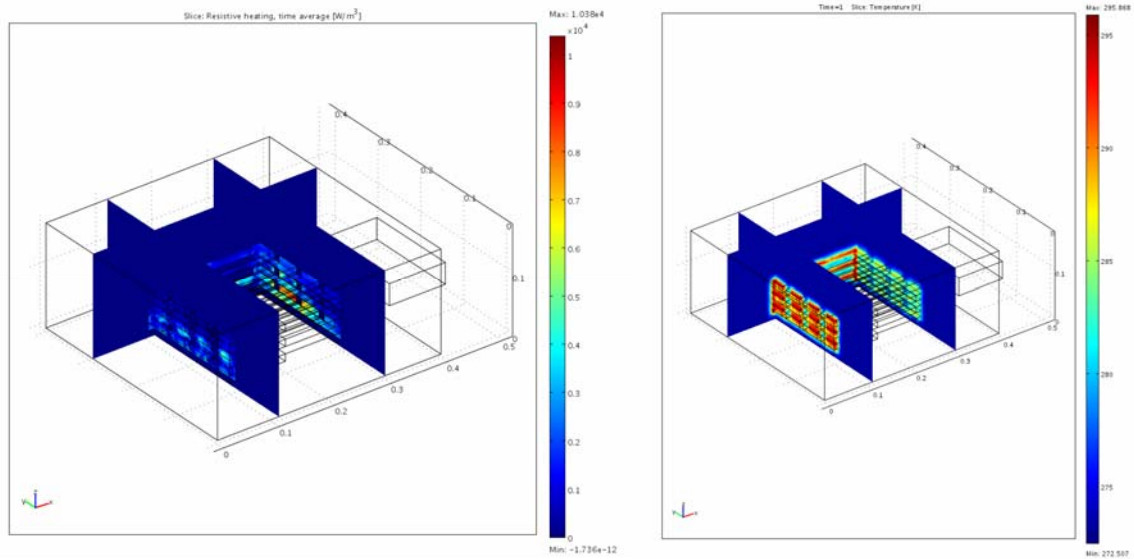


Fig. 2 a) Resistive heating due to microwave penetration, **b)** Temperature distribution in spruce lumbers

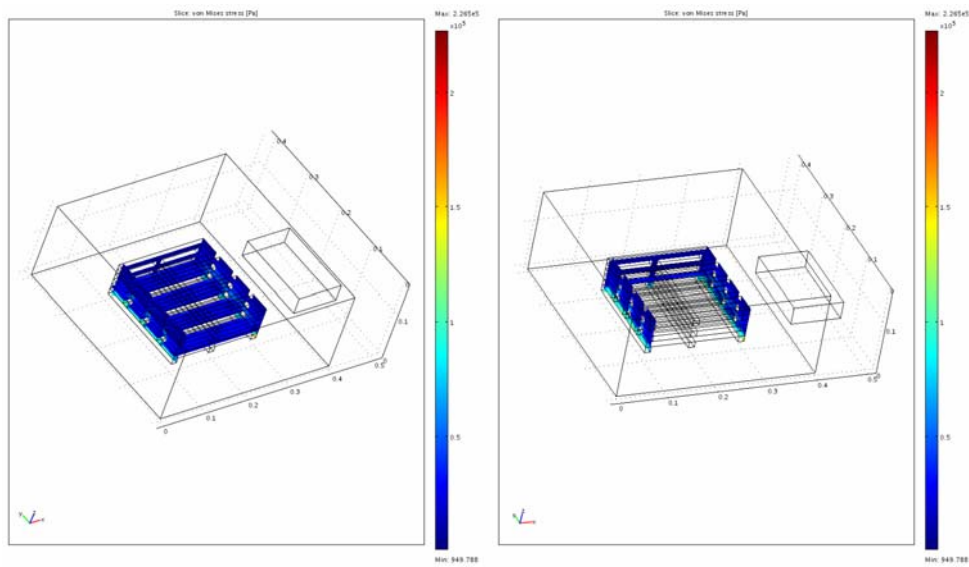


Fig. 3 Von Mises equivalent of stress distribution a) in the kiln, **b)** on surface of spruce lumbers

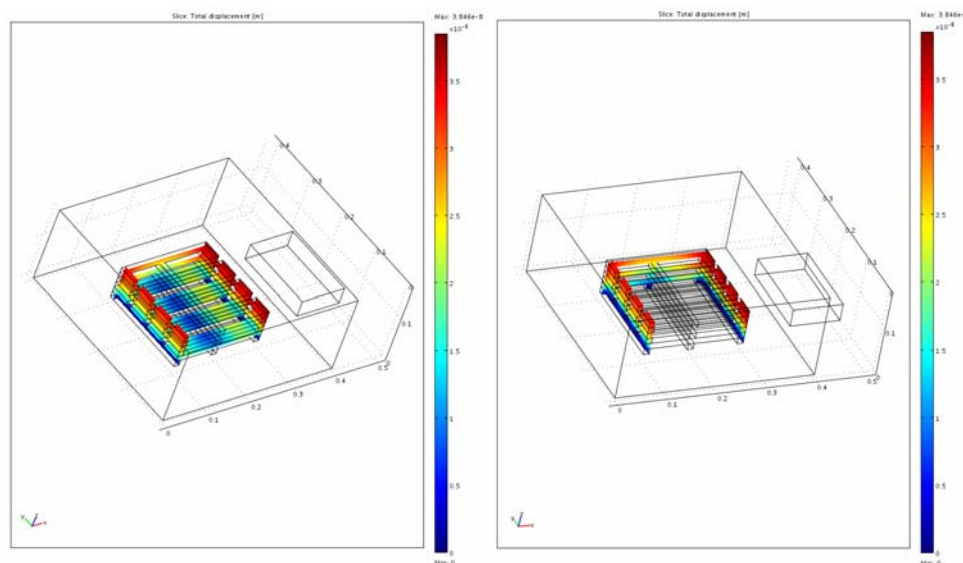


Fig. 4 Displacement of the **a)** whole spruce lumbers, and **b)** on surface of spruce lumbers

Discussion and conclusion

Even that model corresponds with theoretical response it should be noted that real technology process of microwave process is more complicated mainly according to multivivid excitation, more waveguides, positioning of waveguides with complex interaction. Also detailed substructuring of wood can

Regardless, the model allows finding hotspots in microwave drying process for variable position of waveguide and excitation parameters. Finally very important identification of failure occurrence due to high temperature gradients can be done. Such approach offers refinement/change of input parameters for microwave drying process of wood.

Acknowledgements

The Research project GP106/06/P363 Homogenization of material properties of wood for tasks from mechanics and thermodynamics (Czech Science Foundation) and Institutional research plan MSM6215648902 – Forest and Wood: the support of functionally integrated forest management and use of wood as a renewable raw material (2005-2010, Ministry of Education, Youth and Sport, Czech Republic) supported this work. Author also thanks K. Nikl for his useful remarks.

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