

# SOLUTION OF HIGH CURRENT CIRCUITS

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## Abstract

**The paper deals with theoretical solution of heavy currents. Theoretical calculations of single- and three-phase circuit are presented in complex quantities. Competency of program Matlab for solving heavy currents is investigated. A simulation example of synchronous generator under load is introduced. The courses of selected quantities of generator for choosen operating conditions are introduced, too.**

## Introduction

An impulse towards creation of this article comes out from my experience in education of electric machines. The education makes a good use of the simulation program Matlab (with Simulink). The user of simulation program may not take results of simulation process automatically as correct. User has to wonder about results - whether or not the result of simulation is correct, how near is the result to reality in spite of simplifications and neglectings used in the model, whether the estimated difference (error) of the result with respect to the known reality is allowed (acceptable), if the error is allowed for the subsequent simulation experiments, too, etc. Of course, a very complex and sophisticated reality (an object or a process) cannot be verified by accurate analytical calculation. Neither its simplified and approximated model can. As a rule, this only can be done by simulation performed on a model. We have to keep in mind that basically there are two challenges: 1) modeling the reality by a properly oriented and sophisticated model (in our case a mathematical one) and 2) properly performed, object (task) oriented simulation on the model. Hence, it is appropriate to verify the ability of the model and simulation program for simple models and simple simulation, so the accuracy and reliability of the result can be analytically verified, and we become confident in the accuracy of simulation results, prior to performing a complex tasks. This is the philosophy of this article.

In the theory of AC high currents phasor variable or complex form are as a rule used. [1] In the theory of electrical machines we use active or reactive component of current, power, etc. [2] These components are important in a regulation of excitation current (excitation voltage) of synchronous generator, active power of induction motor, etc. The support of Matlab, in the examination of complex and variable components, are introduced. The simple simulation models of RL circuits and the calculation accuracy are examined in the subsequent chapters.

## 1 Calculation of currents in the single-phase series RL circuit

The single-phase series RL circuit and parameters of elements are in Figure 1. Selected resistance value is  $R = 100\Omega$ , inductance  $L = 0,318$  H. The voltage source is set to peak voltage  $U = 100 \cdot \sqrt{2}$  V at a frequency  $f = 50$  Hz. In order to work in Matlab with the phasor (complex) variables, user must use the *powergui* block. This block must be set into the phasor simulation. The circuit includes ampere-meter, where output signal is set to measure complex variables. Output value of ampere-meter is divided by  $\sqrt{2}$ , to work with rms signal. The signal is decomposed then to the real (active) and imaginary (magnetisation) component of current and the amplitude of current and its phasor. The resulting values of the current components are shown in Figure 1.

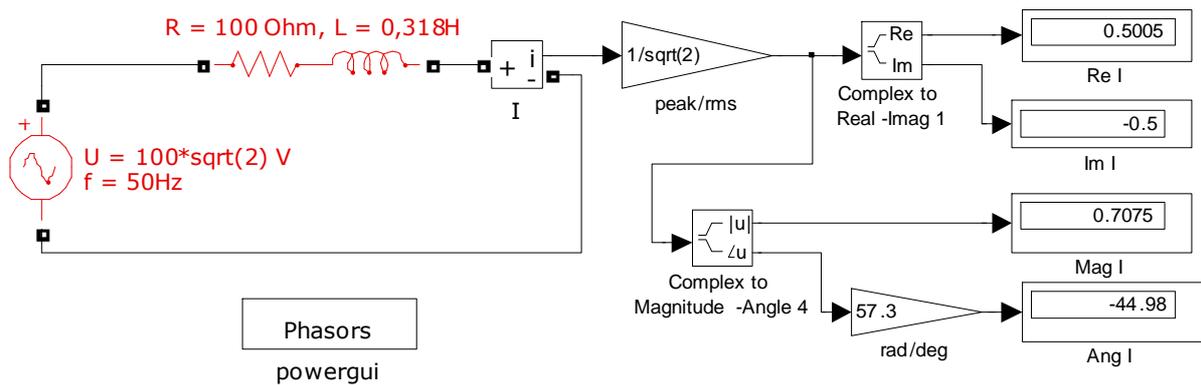


Fig.1.: single-phase series RL circuit – solution in complex form

Whether or not the values of current are correct can be controlled by calculation result. The RL load impedance is [1]

$$\mathbf{Z} = R + j\omega L = 100 + j(2 \cdot \pi \cdot 50) \cdot 0,318 = 100 + j100 \quad (1)$$

The current from Ohm's law applies:

$$\mathbf{I} = \frac{U}{\mathbf{Z}} = \frac{100}{100 + j100} = \frac{100}{100 + j100} \cdot \frac{100 - j100}{100 - j100} = \frac{10^4 - j10^4}{2 \cdot 10^4} = 0,5 - j0,5 = 0,707 \angle -45^\circ \quad (2)$$

Comparing the results according to equation (2) and a figure 1, we can see, that simulation exactly corresponds to the theoretical calculation. Active and reactive component of current have the same value  $I = 0,5A$ , the magnitude of current  $I = 0,707A$  and translation vector is  $-45^\circ$ . In the single-phase RL circuit are for check calculation another value of load required. The results of the simulation and of the theoretical check calculation are equal again.

## 2 Calculation of currents and voltages in the three-phase systems

The calculation of currents and voltages in three-phase systems was carried out according to the example given in [1]. The three-phase symmetrical system having line to-line and phase to neutral voltage  $3 \times 380/220V$ ,  $f = 50Hz$ , works into an impedance star. The star is composed from three equal impedances. In the first, second and third case the impedance is of pure resistive, inductive and capacitive character. The tasks calculates the current in impedance, voltage over the impedance and voltage between nodal point of impedance and neutral wire. A simplified scheme of the three-phase system is in Figure 2.

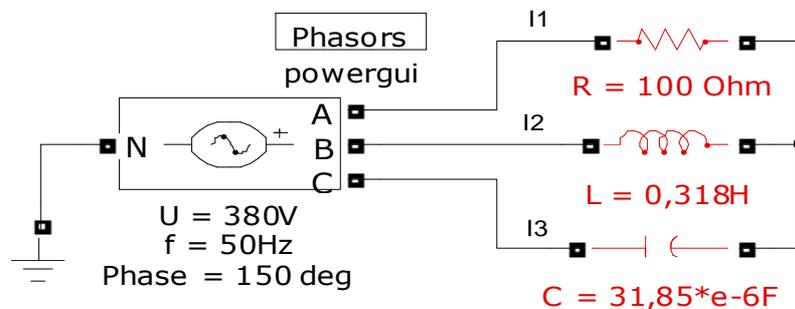


Fig.2.: Three-phase system with symmetrical impedances

The assumes for theoretical calculation are: the vector of line to-line voltage  $U_{12}$  is oriented to the positive real axis of the complex associated plane, the vector of line to-line voltage  $U_{23}$  is displaced by  $-120^\circ$  towards  $U_{12}$  and vector of line to-line voltage  $U_{31}$  voltage is displaced by  $+120^\circ$  towards  $U_{12}$ . Mathematical term is according to relation (3a-3c).

$$U_{12} = 380 \angle 0^\circ \quad (3a)$$

$$U_{23} = 380 \angle -120^\circ \quad (3b)$$

$$U_{31} = 380 \angle 120^\circ \quad (3c)$$

The impedances of circuit are according to the relation (4a-4c):

$$Z_1 = R = 100$$

$$Z_2 = j\omega L = j \cdot 2 \cdot \pi \cdot 50 \cdot 0,318H = j100 \quad (4a-4c)$$

$$Z_3 = -j \frac{1}{\omega C} = -j \frac{1}{2 \cdot \pi \cdot 50 \cdot 31,85 \cdot 10^{-6} F} = -j100$$

The currents in different impedances, for the chosen axes system (orientation of phasor voltages), are calculated by following relations. The denominator in the relations is the same for all terms. The derivation of the denominator the reader finds in [1].

$$I_1 = \frac{U_{31}Z_2 - U_{12}Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} = \frac{380\angle 120^\circ \cdot j100 - 380\angle 0^\circ \cdot (-j100)}{10^4} = -3,29 + j1,9 = 3,8\angle 150^\circ \quad (5a)$$

$$I_2 = \frac{U_{12}Z_3 - U_{23}Z_2}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} = \frac{380\angle 0^\circ \cdot (-j100) - 380\angle -120^\circ \cdot j100}{10^4} = 1,9 - j0,51 = 1,97\angle -15^\circ \quad (5b)$$

$$I_3 = \frac{U_{23}Z_1 - U_{31}Z_2}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} = \frac{380\angle -120^\circ \cdot 100 - 380\angle 120^\circ \cdot j100}{10^4} = 1,39 - j1,39 = 1,97\angle -45^\circ \quad (5c)$$

The voltage over impedances are obtained by the following relations:

$$U_{z1} = Z_1 I_1 = -329 + j190 = 380\angle 150^\circ \quad (6a)$$

$$U_{z2} = Z_2 I_2 = 51 + j190 = 197\angle 75^\circ \quad (6b)$$

$$U_{z3} = Z_3 I_3 = -139 - j139 = 197\angle -135^\circ \quad (6c)$$

The voltage between nodal point of impedance and neutral wire is obtained by the following relations:

$$U_0 = \frac{1}{3}(U_{z1} + U_{z2} + U_{z3}) = -139 + j80,3 = 160,7\angle 150^\circ \quad (7)$$

To calculate currents of the simulation, ampere-meters are used in each and every stage, as shown in Figure 1. The voltage values over impedances and between nodal point and neutral wire are obtained by voltmeter. All measuring elements are set inside their blocks for the measurement of complex variables. Output signals of all measuring units are divided  $\sqrt{2}$  to obtain the rms signal. The signal is decomposed then to the real (active) and imaginary (magnetisation) component of current (voltage) and the amplitude of current (voltage) and phasor.

The most significant impact in this simulation is presented by block *Three-Phase Programmable Voltage Source*. The input parameters of this block are set as follows: effective (rms) value of line to-line voltage  $U = 380\text{V}$ , frequency  $f = 50\text{ Hz}$  and phase displacement. It was investigated by the simulation, that the accuracy of the analytical calculation compared with simulation value, is depended on the phase displacement.

In the block *Three-Phase Programmable Voltage Source* the phase displacement was set on  $0^\circ$ . According to first assumption there should be angle shift of voltage system according to the relations (3a-3c). Equivalent results by the simulation, corresponding by the relations (5a-7), are not obtained. The simulation results identical to the result of calculations (5a - 7) are obtained by set of phase displacement on  $150^\circ$ . See the first column in table 1. The absolute values remain constant in any settings of phase displacement, however, the phasor of current (voltage), hence real and imaginary component of current (voltage) are changed. In order to avoid possible error due to single-example-check, further calculations were performed, with a different arrangement of circuit elements and parameters values. It was confirmed, that the setting of phase displacement determines, whether or not the simulation is equal to analytical calculation.

In the table 1 the selected results of simulation are presented. The real and imaginary component, the amplitude and the angle of current in the resistive load, the voltage over resistive load and voltage between nodal point of impedance and neutral wire are presented. The amplitudes of parameters are always the same. The simulated parameters in the other two impedances are not introduced, to keep the text short.

Table 1. Comparison of selected variables at different preselect phase displacement

		Preselect phase displacement of voltage in the source			
		<b><math>150^\circ</math></b>	$-30^\circ$	$0^\circ$	$90^\circ$
$I_1$	Real	<b>-3,29</b>	3,29	3,8	0
	Imag	<b>1,9</b>	-1,9	0	3,8
	Magnitude	<b>3,8</b>	3,8	3,8	3,8
	Angle	<b><math>150^\circ</math></b>	$-30^\circ$	$0^\circ$	$90^\circ$
$U_{z1}$	Real	<b>-329</b>	329	380	0
	Imag	<b>190</b>	-190	0	380
	Magnitude	<b>380</b>	380	380	380
	Angle	<b><math>150^\circ</math></b>	$-30^\circ$	$0^\circ$	$90^\circ$
$U_0$	Real	<b>-139</b>	139	160	0
	Imag	<b>80</b>	-80	0	160
	Magnitude	<b>160</b>	160	160	160
	Angle	<b><math>150^\circ</math></b>	$-30^\circ$	$0^\circ$	$90^\circ$

The simulation results lead to the conclusion: In the praxis, it is not always possible to determine accurately the phase voltage displacement of the power source. The absolute value of the measured quantity is guaranteed. It will be the same for any phase displacement of settings. It is not guaranteed, that the real (active) and imaginary (reactive) components of currents or voltages are the same as performed by simulation, as would be results from an analytical calculations of practical measurements. The autor agrees, that a defect may be on his side, as (with high probability) the block *Three-Phase Programmable Voltage Source* may work with the three-phase voltage system, oriented in the other axes (angles), than assumed in relations (3a-3c).

### 3 Three-phase generator loaded with electrical resistance

In this chapter a synchronous generator is used as a voltage source. The generator is loaded by a pure active resistance, first. The output winding of generator is connected in wye without neutral point output. The scheme of load connecting to the terminals of the generator is given on the figure 3. The scheme is simplified. Other measuring blocks are not introduced. The model of generator and its parameters are taken from demo samples of Matlab. For this article sake, a short description is sufficient:

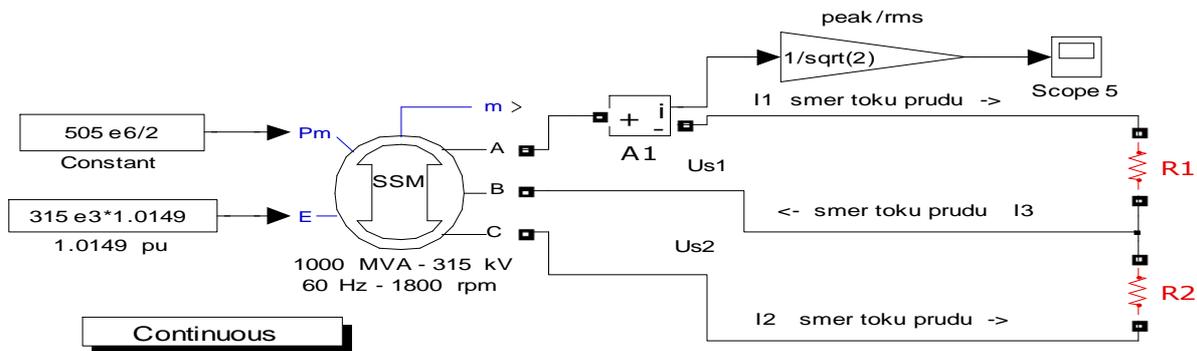


Fig. 3.: Scheme of load connecting to the terminals of the generator

To verify the correct work of the simulation in the program Matlab, the relations by [3] are used. The generator is loaded by a pure active resistance, first. The task calculates currents in individual phases. According by [3] for the currents apply:

$$I_1 = \frac{U_{zdrúžen\acute{e}_{ef}}}{R_1} = \frac{U_{f\acute{a}z_{ef}} \cdot \sqrt{3}}{R_1} \quad (8a)$$

$$I_2 = \frac{U_{zdrúžen\acute{e}_{ef}}}{R_2} = \frac{U_{f\acute{a}z_{ef}} \cdot \sqrt{3}}{R_2} \quad (8b)$$

$$I_3 = \sqrt{I_1^2 + I_2^2 + 2 \cdot I_1 \cdot I_2 \cdot \cos 60^\circ} \quad (8c)$$

The current  $I_3$  is calculated according to the cosine law. The angle  $60^\circ$  in relation (8c) is the phase displacement between the line to-line voltage  $U_{s1}$ ,  $U_{s2}$ , between phase A-B and phase B-C. The size of phase displacement depends on phasor diagram. The phase displacement between line to-line voltage  $U_{s1}$ ,  $U_{s2}$  is  $60^\circ$ , if the phase displacement of balanced phase voltages is  $120^\circ$ . [3]

Line to-line voltage of generator is given as  $U_s = 315\text{kV}$ . The current in the individual phases, for any selected values of resistors  $R_1$  a  $R_2$ , are analytical calculated by the relation (8a-8c). The values of currents, obtained by calculation and simulation, for selected values of resistors, are compared in the table 2. In the figures 4a, 4b is a detail of current course  $I_1$  a  $I_3$  obtained by simulation, where the values of resistors are  $R_1 = R_2 = 7500\Omega$ . By substitution of values voltage generator and values resistors in the relation (8a-8c) applies:

$$I_1 = I_2 = \frac{315 \cdot 10^3 \text{V}}{7500\Omega} = 42\text{A}$$

$$I_3 = \sqrt{42^2 + 42^2 + 2 \cdot 42 \cdot 42 \cdot \cos 60^\circ} = 73\text{A}$$

Table 2. Comparison of current values, obtained by calculation and simulation, for selected values of resistors

			Selected parameters of resistors			
			$R_1 = R_2 = 7,5\text{k}\Omega$	$R_1 = R_2 = 1\text{k}\Omega$	$R_1 = 5\text{k}\Omega$ $R_2 = 15\text{k}\Omega$	$R_1 = 0,1\text{k}\Omega$ $R_2 = 7,5\text{k}\Omega$
amplitude of effective value currents	$I_1$ [A] (vo fáze A)	Calculation	42	315	63	3150
		Simulation	42,7	323	64	2880
	$I_2$ [A] (vo fáze C)	Calculation	42	315	21	42
		Simulation	42,5	312	21	35
	$I_3$ [A] (vo fáze B)	Calculation	73	546	76	3171
		Simulation	73,7	549	77	2888

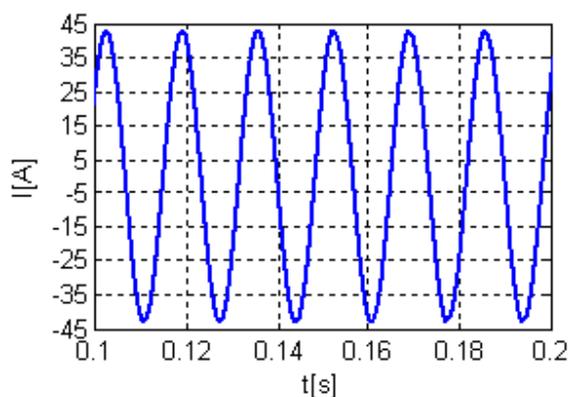


Fig. 4a): course of current in the phase A

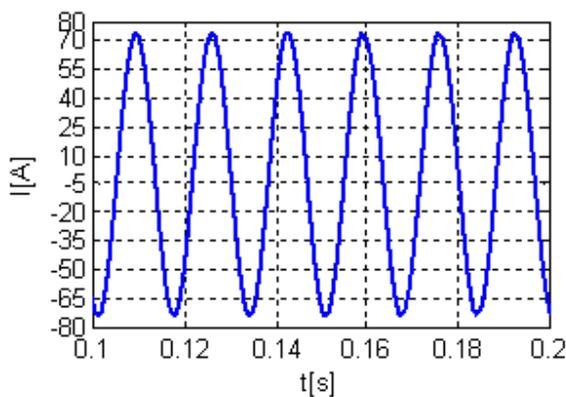


Fig. 4b): course of current in the phase B

The currents in table 2, obtained by the simulation and by the analytical calculation, are the same, if the values between resistances are not too (multiple) different. In the last column of table 2 is the difference between resistance 75-multiple. The value of resistance  $R_1 = 100\Omega$  between phases A-B is near to the ratio of the two-phase short circuit generator. In this case the voltage in the phases A, B decreases, because the excitation regulator is not used. The using of excitation regulator ensures the assumed value voltage in the phases  $U_s = 315\text{kV}$ . The values of currents in the last column but still correspond to relations (8a-8c), only a individual voltage are asymmetric in the phases. The current  $I_1$  in the phase A is obtained by simulation for resistance  $R_1 = 100\Omega$  by equation (8a), by the decreased voltage of generator  $U_s = 288\text{kV}$ .

#### 4 Courses of voltage and current of generator by the load changes

To follow the previous chapter, the change of generator load in this case is carried out by the figure 5. Switches shunt the resistors in each phase. The shunt results in a change of resistance values in the phase. Simulation of load change is performed for two selected cases.

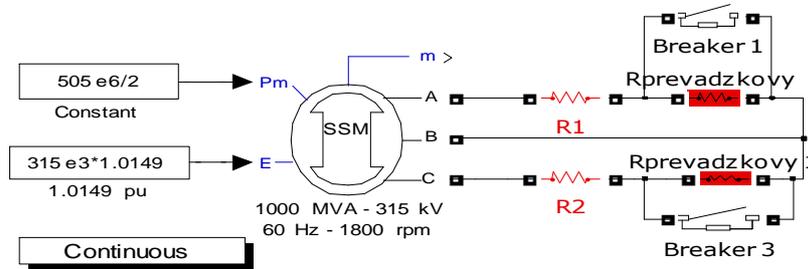


Fig. 5: Simplified scheme to simulate a load change of a generator

The first case is the simulation of change load resistance  $R = 7500\Omega$ . The generator is loaded in two phases by the resistance  $R = 7500\Omega$ , first. At the time  $t = 1s$  the resistance in phase A is shorted. The resulting value resistance in phase A after shutting is  $R_1 = 100\Omega$ . This change corresponds the change of settings in the first column to last column in the table 2. The course of current by change of load is shows in the figure 6a. The rms value of currents corresponds by table 2. In the figure 6b) asymmetrical line-to-line voltage of generator is shown after the load change.

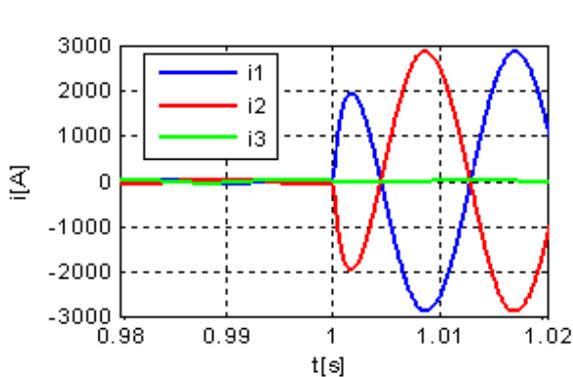


Fig. 6a): The current before and after load change

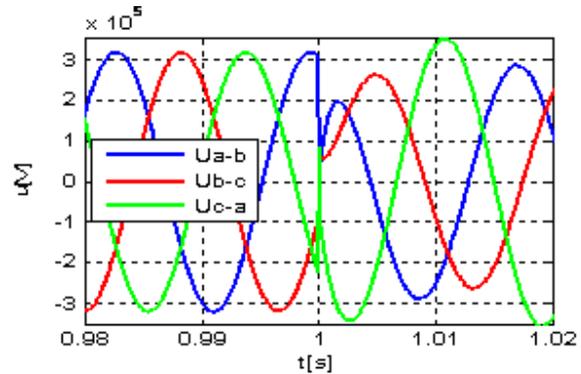


Fig. 6b): Line to-line voltage before and after load change

The second case is a simulation of load change of resistance  $R = 7500\Omega$  to resistance  $R = 1\Omega$  in phases A and C. The change occurs simultaneously at the time  $t = 1s$  on both phases. Such a change of resistance value is in fact a three-phase short-circuit of generator. In the figure 7a) the characteristic exponential decrease of short-circuit current in different phases is shown. [4] Figure 7b) shows the voltage of the generator. The generator voltage decreases after occurrence of short-circuit at time  $t = 1s$  to small values.

The author was interested in an analytical process too, to compare of short-circuit currents according to the theoretical relationships described in [4], or using the example provided in [5]. However, the model of synchronous machine does not give the necessary parameters for the equations describing the course of short-circuit currents. Therefore the intention could not be realized.

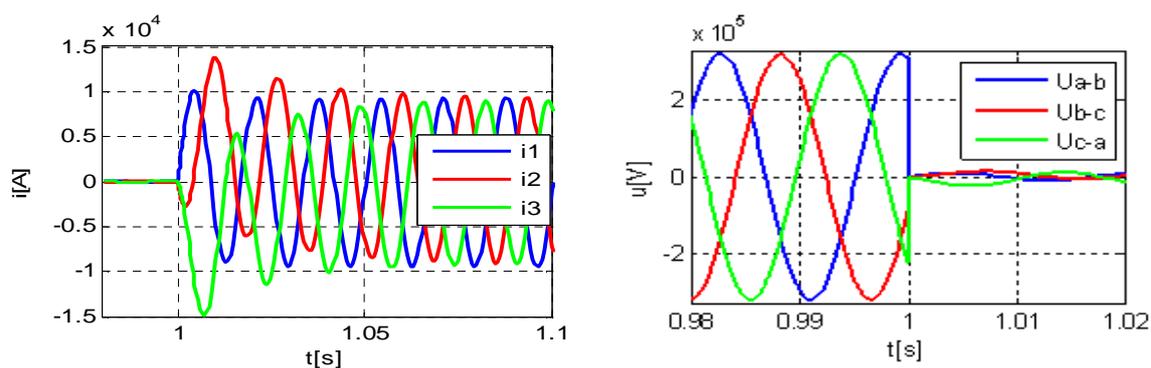


Fig. 7a): Current in the phases at three phase short-circuit Fig. 7b): Line to-line voltage before and after load change

## Conclusion

In the paper, partial results of simulation are introduced. The results allow to check the accuracy of working in the program Matlab (with Simulink). This allows to work at the level of more complicated simulations. Even if it is not always possible to verify the analytical results of the simulation, the main idea is ability to explain the physical basis of the achieved results.

By using of simulation it is possible to identify other operational characteristics of synchronous machines, author deals about it in other published work. [6] Using information from chapter 4 of this article it is possible to reveal the internal or external loading characteristics of the synchronous machine. [4]

The main idea of the individual chapters is the analytical calculation, supported by simulation. The first and second chapter presents the simulation in complex (phasor) variables. The intention was, to apply the knowledge of complex variables to chapter 3 and 4, using the inductive load. The intention was, the results of simulations of real (active) and imaginary (reactive) components of current or voltage of generator or load, to support by analytical calculation [2].

The ambiguity of solutions and problems in dealing with complex variables are picked up in author notes in chapter 2. When applying a complex solution for tasks with generator, problems increased even more. The presented problems of task will be further analyzed. Author would appreciate a discussion on the topic given here. In the theory of electrical machines this topic is indeed still open and interesting.

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