

MODELING OF MAGNETIC FIELD DISTRIBUTION IN MAGNETOELASTIC FORCE SENSOR

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Abstract

Nowadays the development of the numerical methods and abilities of the computer art enabled solution of the coupled problems - the problems that deal with the processes produced by interaction of some physical fields. The magnetoelastic pressure force sensor is one of such examples. The paper is aimed to the proposal for modeling of the magnetic field distribution in the ferromagnetic core of the magnetoelastic sensor with respect to the variation of the permeability due to mechanical stress.

1 Principles of Magnetoelastic Sensor and its Electromagnetic and Stress Field Model

The magnetic and mechanical properties of ferromagnetic materials are tightly related. The application of mechanical stress brings a magnetization change of the ferromagnetic material (Villari effect) and, vice versa, presence of a magnetic field causes a deformation of the shape of the body made of a ferromagnetic material (Joule effect). The magnetoelastic sensor operates on the Villari effect principle. Villari effect is based on the change of permeability of a ferromagnetic body exposed to mechanical force, which results in mechanical stress within the body. The permeability increment $\Delta\mu$ proportional to the mechanical stress σ , is given by relation [1]

$$\Delta\mu = \frac{2\lambda_{ms}}{B_s^2} \mu^2 \sigma \quad (1)$$

where λ_{ms} is the magnetostriction coefficient for $B = B_s$, B_s being the magnetic flux density at saturation, and μ being the magnetic permeability.

The pressure force magnetoelastic sensor core consists of fifty ferromagnetic transformer sheets, thickness of each being 0.5 mm. Each sheet contains four holes of the radius of 1 mm. The conductors of primary ($N_1 = 10\text{turns}/0.35\text{mm}$) and secondary windings ($N_2 = 8\text{turns}/0.25\text{mm}$) are led through these holes collaterally. The sensor is assigned for measuring of nominal pressure force 120 kN that is equivalent to 100M Pa. A simplifying representation of the magnetoelastic sensor core is depicted in Figure 1 (left).

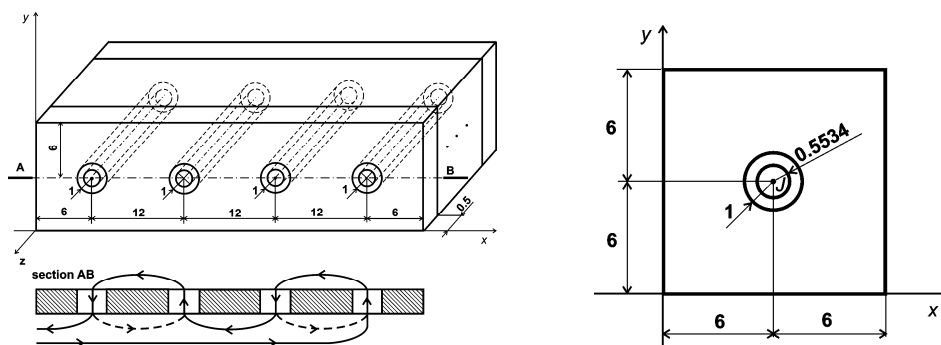


Figure 1: A simplified form of the magnetoelastic sensor core and the integrative sensor element

If Cartesian coordinate system is applied to a simplified form of sensor core, the current density vector has a nonzero component only in the z -direction. This component produces magnetic field whose vectors have nonzero components only in the directions x and y . Moreover, the dimensions of the sheet are designed in such a way that the sheet can be divided into four squared shapes, each with a circle hole in the middle. One of such shapes is called the integrative sensor element (Figure 1 - right).

With respect to the symmetry of the sheet it is sufficient to determine the magnetic field distribution in the integrative sensor element only. The primary winding is replaced by a conductor (with radius 0.5534 mm) the cross-section of which is the same as the cross-section of the primary windings conductors.

The sensor is supplied by a harmonic current of density \mathbf{J} and frequency f . The displacement currents are small when set against the source current density and, therefore, they can be neglected on the right-hand side of the first Maxwell equation. The current flows parallel to the z -axis, which means that $\mathbf{J} = \mathbf{k} J_z$, (\mathbf{k} being the unit vector in the z -direction), and the magnetic vector potential \mathbf{A} has also only one nonzero component ($\mathbf{A} = \mathbf{k} A_z$). Electrical conductivity of laminated sensor core is also small, so that in the first approach eddy current density can be neglected. The magnetic field in the integrative sensor element can be described by a scalar elliptic partial differential equation [2]

$$-\text{div}\left(\frac{1}{\mu} \text{grad } A_z\right) = J_z \quad (2)$$

The magnetic vector potential has no specific technical meaning, but the spatial distribution of the magnetic vector potential defines spatial distribution of the magnetic flux density unambiguously. The values of magnetic flux density were computed from the magnetic vector potential values. For the case of planar field the magnetic flux density \mathbf{B} can be written in the form

$$\mathbf{B} = \mathbf{i} \frac{\partial A_z}{\partial y} + \mathbf{j} \left(-\frac{\partial A_z}{\partial x}\right) \quad (3)$$

where \mathbf{i} and \mathbf{j} being the unit vectors in the directions of the x -axis and y -axis respectively.

The determination of magnetic field in the sensor core is formulated as a boundary-value problem in terms of vector magnetic potential. In case that there is no force acting on sensor the presented boundary-value problem is nonlinear, but isotropic. In case that there exists force acting on the sensor, the presented problem represents nonlinear anisotropic boundary-value problem because the relation between the magnetic flux density and magnetic field strength is described by the tensor of permeability.

The magnetic field is solved in the domain, consisting of three subdomains (Figure 1 right), and in each one of them the equation (2) is valid. The first subdomain is the copper current wire, in which $\mu = \mu_0$ and J is the current density in conductors of the primary winding. The second subdomain is the air gap between the wire and ferromagnetic core and $\mu = \mu_0$ and $J = 0$ A/mm². The third subdomain is the ferromagnetic sensor core and $J = 0$ A/mm² and the permeability depends on pressure force applied to the sensor.

In case that no pressure force is applied to the sensor, the permeability of the sensor core depends on the magnetic flux density $\mu = \mu(B)$. In case that the pressure force is applied to sensor, the

permeability is the tensor $\begin{pmatrix} \mu_{xx} & 0 \\ 0 & \mu_{yy} \end{pmatrix}$ with components μ_{xx}, μ_{yy} , for which can be written

$$\mu_{xx}(B, \sigma_x) = \mu(B) + \Delta\mu_{xx}(B, \sigma_x) \quad (4)$$

$$\mu_{yy}(B, \sigma_y) = \mu(B) + \Delta\mu_{yy}(B, \sigma_y) \quad (5)$$

where $\Delta\mu_{xx}, \Delta\mu_{yy}$ are the permeability variations in the x -direction and y -direction according to (1), while σ_x, σ_y are the normal components of the mechanical stress.

The plane stress field in the polar coordinate system is unambiguously determined by two perpendicular components σ_r, σ_φ and shear component $\tau_{r\varphi}$ [3], that can be computed from the Airy function U by the following formulas:

$$\sigma_r = \frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2} + \frac{1}{r} \frac{\partial U}{\partial r} \quad (6)$$

$$\sigma_\varphi = \frac{\partial^2 U}{\partial r^2} \quad (7)$$

$$\tau_{r\varphi} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial U}{\partial \varphi} \right) \quad (8)$$

For an infinitely planar strip with circular hole, if the strip is exposed a constant continuous pressure stress p in x -direction, the Airy function takes a the form [3]:

$$U(r, \varphi) = -p \left[\frac{r^2}{4} - \frac{a^2}{2} \ln r - \frac{(r^2 - a^2)^2}{4r^2} \cos 2\varphi \right] \quad (9)$$

where a is a diameter of the circular hole, r, φ are the polar coordinates.

Combining the relations (6), (7), (8) and (9), expressions for radial stress σ_r , peripheral stress σ_φ and shear stress $\tau_{r\varphi}$ can be written:

$$\sigma_r(r, \varphi) = -\frac{p}{2} \left[1 - \frac{a^2}{r^2} + \left(1 + \frac{3a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \cos 2\varphi \right] \quad (10)$$

$$\sigma_\varphi(r, \varphi) = -\frac{p}{2} \left[1 + \frac{a^2}{r^2} - \left(1 + \frac{3a^4}{r^4} \right) \cos 2\varphi \right] \quad (11)$$

$$\tau_{r\varphi}(r, \varphi) = -\frac{p}{2} \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\varphi \quad (12)$$

The relations (10)-(12) are accepted for an infinitely plane with a circular hole and integrative sensor element is not a body of such type. But if the distances between the edge of body and the centre of the circular hole as well as between the centres of neighbouring holes are at least six times longer than the radius of the circle, and the distance between the body the edge of the body on which the pressure acts is at least three times longer than the radius of circle, then these relations can be used for the determination of stress in a two-dimensional surface containing a circular hole enclosed with a boundary [4].

2 Results

After solving a problem, the obtained results (at the nodal points of the triangular mesh consisting of 15777 nodes and 31232 triangles) were plotted.

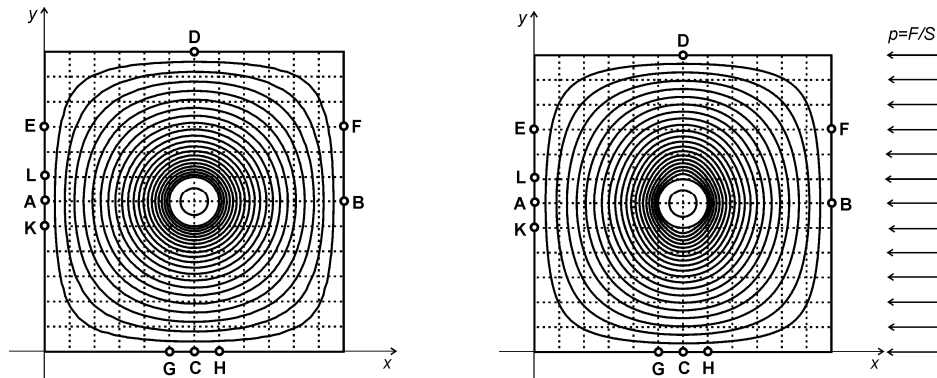


Figure 2: The equipotential lines of the vector magnetic potential without application of the pressure force (left) and with application of the pressure force 99 MPa (right)

The results are the equipotential lines of the magnetic vector potential in the integrative sensor element for sensor without applied pressure force (Figure 2 left) and with applied pressure force 99 MPa (Figure 2 right). The latter plot clearly shows the deformation of the equipotential lines mainly near the circular hole.

The magnetic vector potential has no specific technical meaning. For that reason, the values of magnetic flux density over the rectangular grid (created with the step 0.01 mm in direction of x and y coordinates) were computed from the magnetic vector potential values on the triangular mesh. The dependance of distribution of the magnetic flux density on pressure force applied to the sensor along the cross-sections AB and CD is depicted in Figure 3 (current density in the primary winding is 13.512 A/mm^2 , its frequency being 400 Hz).

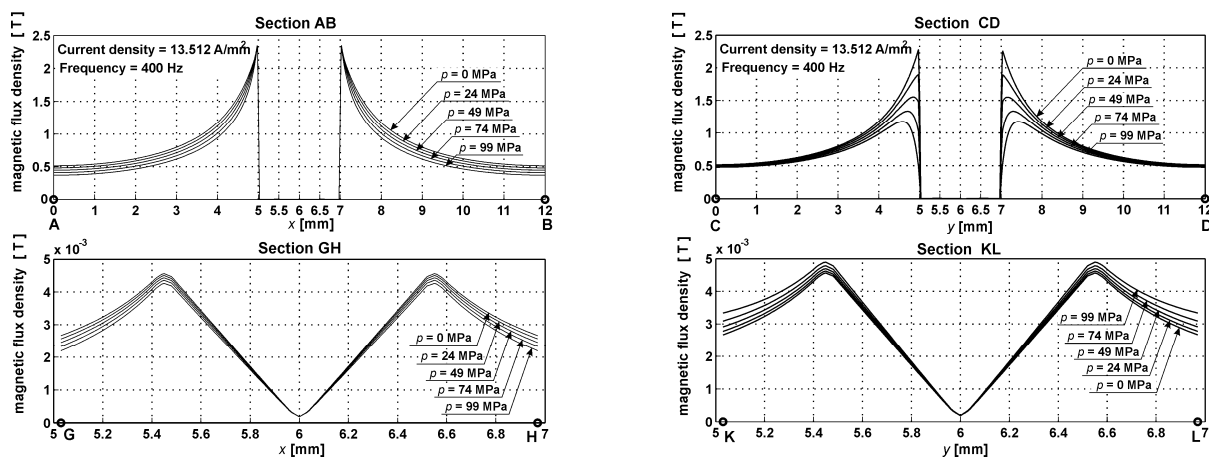


Figure 3: Distribution of magnetic flux density at the cross-section AB (left) and at the cross-section CD (right) in the dependance on pressure force applied to the sensor

The distribution of the magnetic flux density at the cross-section AB (Figure 3 left) is the same as at the cross-section CD (Figure 3 right) provided that there is no pressure force applied to the sensor, because there is no magnetic field deformation proportional to the corresponding pressure force. In case that the pressure force is applied to the sensor, the magnetic field is deformed and the magnetic flux density with increasing pressure force decreases considerably along the cross-section CD (Figure 3 right), especially along the borders of the subdomain between the air gap and ferromagnetic region.

References

- [1] Zehnula, K.: *Sensors of Non-electric Quantities*. SNTL, Prague, 1977.
- [2] Mayer, D., Ulrych, B.: *Fundamentals of Electric and Magnetic Fields Numerical Calculations*. SNTL, Prague, 1988.
- [3] Trebuňa, F., Jurica, V., Šimčák, F.: *Elasticity and strength II*. Edition of scientific and special literature. Košice, 2000.
- [4] Savin, G.: *Stress Field Distribution around Holes*. Nauka dumka, Kiev, 1968.
- [5] Fujisaki, K., Satoh, S.: *Numerical Calculations of Electromagnetic Fields in Silicon Steel Under Mechanical Stress*. IEEE Transactions on Magnetics, 2004, vol. 40, No.4, pp. 1820-1825.
- [6] Besbes, M., Ren, Z., Razek, A.: *A Generalized Finite Element Model of Magnetostriction Phenomena*. IEEE Transactions on Magnetics, 2001, vol. 37, No.5, pp. 3324-3328.
- [7] MATLAB - User's Guide, MathWorks 2009.

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