LUMPED-PARAMETER MODEL OF ROTOR-BEARING SYSTEMS

J. Tuma, J. Skutova

VSB – Technical University of Ostrava, Faculty of Mechanical Engineering, Czech Republic

Abstract

The paper deals with the model of a flexible rotor supported on journal bearings. It is assumed high rotational speed of the rotor and therefore the gyro effects on rotor bearings occur. This model differs from other solutions by the use of complex variables for the planar coordinates. The complex variables simplify mathematical equations and their implementation in Simulink.

1 Introduction

It is known that the journal bearing with an oil film becomes instable if the rotor rotation speed crosses a certain value, which is called the Bently-Muszynska threshold [1]. To prevent the rotor instability, the active control can be employed. The arrangement of proximity probes and piezoactuators in a rotor system are shown in Fig. 1. It is assumed that bushings, which are inserted into bearing bore with clearance, are a movable part in two perpendicular directions while rotor is rotating.

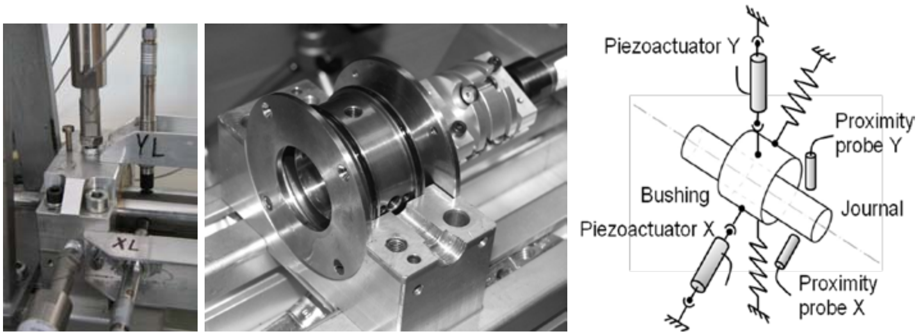

Figure 1: Arrangement of the controllable journal bearing

The research work supported by the GAČR (project no. 101/07/1345) is aimed at the design of the journal bearing active control based on the bushing position manipulation by the piezoactuators according to the proximity probe signals, which are a part of the closed loop including a controller. The effect of the feedback on the rotor stability is analyzed by [5]. Simulation of a rotor behavior requires creating a mathematical model, preferably in Matlab Simulink environment.

The paper discusses the design of a mathematical model of rigid and flexible rotors. To simplify the model equations and the block diagram in Simulink, the complex variables are used which is the main advantages of the presented approach.

2 Lumped parameter model of journal bearings

There are many ways how to model journal bearings of a rotor system

- the concept developed by Muszynska [2], consisting in replacement of an oil film continuum by a rotating system composed of a spring and damper
- the lubricant flow prediction using a FE method for Reynolds equation solution [4].

This paper prefers the Muszynska because this concept offers an effective way to understand the rotor instability problem and to model a journal vibration active control system by manipulating the bushing position by actuators [1], which are a part of the closed loop composed of proximity probes and a controller. The solution of the Reynolds equation gives more precise rotor dynamic characteristics including rotor stability.

Let the rotor angular velocity is designated by Ω in radians per a second. It is assumed that the bushing is a movable part in two perpendicular directions while rotor is rotating. As was mentioned the mathematical model proposes to use complex variables to describe motion of the rotor and bushing in the plane, which is perpendicular to the rotor axis. The coordinate system is tied to stationary bearing housing with a cylindrical hole, inside of which is inserted a movable bearing bushing. The positions of the journal and bushing centre are given by the intersection point of both the movable component axis with the mentioned complex plane. The origin of the complex plane is situated in the centre of the mentioned cylindrical bearing bore as it is shown in Fig. 2. The position of the journal centre in the complex plane is designated by a position vector **r** while the position of the bushing is designated by a position vector **u**.

(0, 0) – coordinates of the cylindrical bore center
r = ($x(t)$, $y(t)$) – coordinates of the journal (rotor) center
u = ($u_x(t)$, $uy(t)$) – coordinates of the bushing center

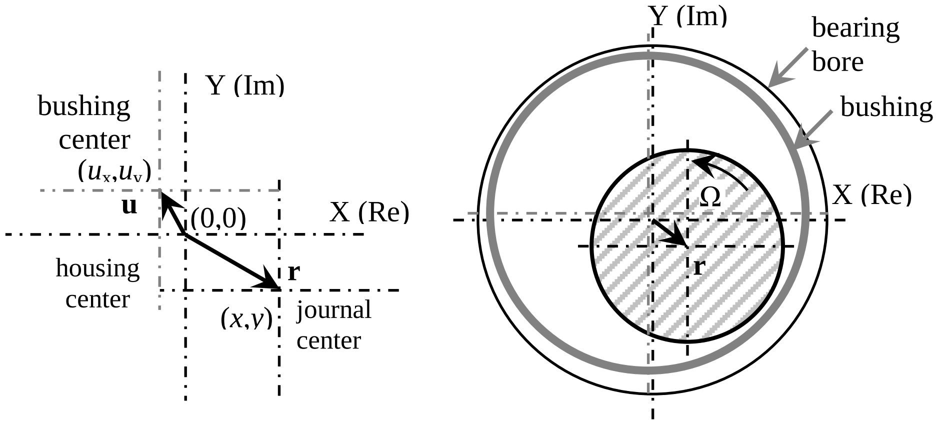

Figure 2. Coordinate system

The internal spring, damping and tangential forces are acting on the rotor. The external forces refer to forces that are applied to the rotor, such as unbalance, impacts and preloads in the form of constant radial forces. All these external forces are considered as an input for the mathematical model. The fluid pressure wedge is the actual source of the fluid film stiffness in a journal bearing and maintains the rotor in equilibrium. As Muszynska has stated these bearing forces can be modeled by a spring and damper system, which is rotating at the angular velocity $\lambda\Omega$ (see Fig. 3), where λ is a dimensionless parameter, which is slightly less than 0.5.

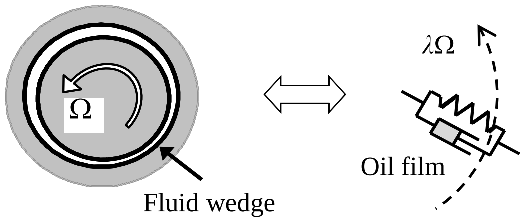

Figure 3: Model of oil film

The parameter λ is denominated by Muszynska as the fluid averaged circumferential velocity ratio. It is assumed that the rotating journal drags the fluid in a space between two cylinders into motion and acts as a pump. It is easy to understand that the fluid circular velocity is varying across the gap as a consequence of the fluid viscosity. The validity of Muszynska's assumption can be verified experimentally. It is known that an oscillation (an onset of instability) of the rotor starts when the rotor rotational speed exceeds a certain value and stops when RPM decreases under the other one. It can be shown by experiment, that when the rotor system is excited by a non-synchronous perturbation force with respect to the rotor rotational speed the resonance appears at the frequency, which is approximately equal to $\lambda\Omega$. The simulation is prepared to prove the same properties of the mathematical model, which is based on the substitution of the continuous oil film by the spring and damper system.

Fluid forces acting on the rotor in coordinates rotating at the same angular frequency as the spring and damper system are determined by the position of the journal centre relating to the bushing centre and therefore are given by the formula (Tondl, 1991)

$$\mathbf{F}_{rot} = K\left(\mathbf{r}_{rot} - \mathbf{u}_{rot}\right) + D\left(\dot{\mathbf{r}}_{rot} - \dot{\mathbf{u}}_{rot}\right), \tag{1}$$

where the parameters, K and D, specify proportionality of stiffness and damping to the relative position of the journal centre displacement vector $\mathbf{r}_{rot} - \mathbf{u}_{rot}$ and velocity vector $\dot{\mathbf{r}}_{rot} - \dot{\mathbf{u}}_{rot}$, respectively. To model the rotor system, the fluid forces have to be expressed in the stationary coordinate system, in which the rotor centre-line displacement and velocity vectors are designated by $\mathbf{r} - \mathbf{u}$ and $\dot{\mathbf{r}} - \dot{\mathbf{u}}$, respectively.

To model the rotor system, the fluid forces have to be expressed in the stationary coordinate system, in which the rotor centre-line displacement and velocity vectors are designated by $\mathbf{r}$ and $\dot{\mathbf{r}}$, respectively. Conversion the complex rotating vector $\mathbf{r}_{rot}$ to the stationary coordinate system can be done by multiplication this vector by $\exp(j\lambda\Omega t)$, which is the same as multiplying the vector in the stationary coordinates by $\exp(-j\lambda\Omega t)$, see Fig. 4.

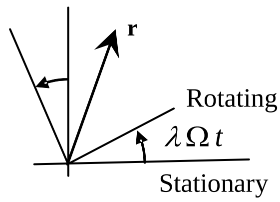

Figure 4. Transformation of rotating coordinates to stationary coordinates

The relationship between the mentioned vectors in rotating and stationary coordinates are given by the formulas

$$\begin{aligned} \mathbf{r}_{rot} &= \mathbf{r}\exp(-j\lambda\Omega t) & \mathbf{u}_{rot} &= \mathbf{u}\exp(-j\lambda\Omega t) \\ \dot{\mathbf{r}}_{rot} &= (\dot{\mathbf{r}} - j\lambda\Omega\,\mathbf{r})\exp(-j\lambda\Omega t) & \dot{\mathbf{u}}_{rot} &= (\dot{\mathbf{u}} - j\lambda\Omega\,\mathbf{u})\exp(-j\lambda\Omega t) \end{aligned} \tag{2}$$

Substitution into the fluid force equation results in the following formula

$$\mathbf{F} = K\left(\mathbf{r} - \mathbf{u}\right) + D\left(\dot{\mathbf{r}} - \dot{\mathbf{u}}\right) - jD\lambda\Omega\left(\mathbf{r} - \mathbf{u}\right), \tag{3}$$

where the complex term $jD\lambda\Omega\left(\mathbf{r} - \mathbf{u}\right)$ has the meaning of the force acting in the perpendicular direction to the vector $\mathbf{r} - \mathbf{u}$. As the rotor angular velocity increases, this force can become very big and can cause rotor instability.

The parameters K and D, specifying oil film stiffness and damping, are a function of the journal centerline position vector, namely the oil film thickness. The position vector corresponds to the eccentricity of the journal in the bushing. It was proved that the closer position of the journal to the bearing wall and simultaneously the thinner oil film, the greater value of both these parameters. Some authors, such as Muszynska [3], assume that it is possible to approximate these functions by formulas

$$K = K_0 \Big/ \left(1 - \left(|\mathbf{r}|/e\right)^2\right)^3, \quad D = D_0 \Big/ \left(1 - \left(|\mathbf{r}|/e\right)^2\right)^2, \quad \lambda = \lambda_0 \left(1 - \left(|\mathbf{r}|/e\right)^2\right)^{1/5} \tag{4}$$

where e is a journal bearing clearance. The authors of this paper analyzed the other formula structure as well [5].

3 Lumped parameter model of rigid rotors

Due to the fact that the rotor is considered as a rigid body, the ends of the position vectors lie on a straight line, which is identical with the rotor axis, see Fig. 5. Angles φ_{Re} and φ_{Im} designate the inclinations of the rotor axis from the bearing housing axis, which is forming an intersection of two perpendicular planes serving for projection of the rotor axis. The plane, which is coinciding with the real axis, is horizontal while the other plane, which is coinciding with the imaginary axis, is vertical. The angle φ_{Re} specifies the inclination of the rotor axis projection into the horizontal plane from the bearing housing axis while the angle φ_{Im} specifies the inclination of the rotor axis projection into the vertical plane from the bearing housing axis.

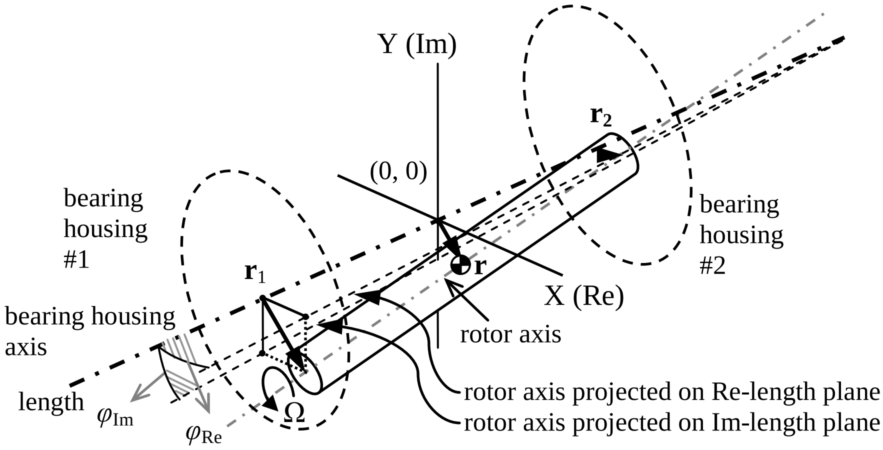

Figure 5. Rotor inclinations

There are two bearings supports of the rotating rotor. Let **r** be a position vector of the rotor center of gravity and l_1 and l_2 is the distances of the center of gravity from the journal bearings. The force **F** has to be indexed according to the journal bearings

$$\begin{aligned} \mathbf{F}_1 &= K(\mathbf{r}_1 - \mathbf{u}_1) + D(\dot{\mathbf{r}}_1 - \dot{\mathbf{u}}_1) - jD\lambda\Omega\,(\mathbf{r}_1 - \mathbf{u}_1) \\ \mathbf{F}_2 &= K(\mathbf{r}_2 - \mathbf{u}_2) + D(\dot{\mathbf{r}}_2 - \dot{\mathbf{u}}_2) - jD\lambda\Omega\,(\mathbf{r}_2 - \mathbf{u}_2), \end{aligned} \tag{5}$$

Let the angels φ_{Re} and φ_{Im} be combined into the complex variable $\mathbf{\Phi} = \varphi_{\mathrm{Re}} + j\varphi_{\mathrm{Im}}$, called a complex angle. The position vectors of the rotor ends in both the journal bearings are as follows

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{r} + l_1(\sin(\varphi_{\mathrm{Re}}) + j\sin(\varphi_{\mathrm{Im}})) \approx \mathbf{r} + l_1\mathbf{\Phi} = \mathrm{X1} + j\,\mathrm{Y1} \\ \mathbf{r}_2 &= \mathbf{r} - l_2(\sin(\varphi_{\mathrm{Re}}) + j\sin(\varphi_{\mathrm{Im}})) \approx \mathbf{r} - l_2\mathbf{\Phi} = \mathrm{X2} + j\,\mathrm{Y2}, \end{aligned} \tag{6}$$

The first derivation of the variables r1 and r2 with respect to time results

$$\begin{aligned} \dot{\mathbf{r}}_1 &\approx \dot{\mathbf{r}} + l_1(\dot{\varphi}_{\mathrm{Re}} + j\dot{\varphi}_{\mathrm{Im}}) = \dot{\mathbf{r}} + l_1\dot{\mathbf{\Phi}} \\ \dot{\mathbf{r}}_2 &\approx \dot{\mathbf{r}} - l_2(\dot{\varphi}_{\mathrm{Re}} + j\dot{\varphi}_{\mathrm{Im}}) = \dot{\mathbf{r}} - l_2\dot{\mathbf{\Phi}}, \end{aligned} \tag{7}$$

The equation of motion in stationary coordinates for the translational motion results from the Newton’s second law. The form of equation is as follows

$$M\,\ddot{\mathbf{r}} = -M\,\mathbf{g} + m_u r_u \Omega^2 \exp(j(\Omega t + \delta)) + \mathbf{F}_1 + \mathbf{F}_2\,, \tag{8}$$

where M is the total rotor mass and **g** is acceleration of gravity. The unbalance force, which is produced by unbalance mass m_u mounted at a radius r_u, acts in the radial direction and has a phase δ at time $t = 0$.

The equation of motion in stationary coordinates for the rotational motion results from the moment equilibrium of forces about the gravity center. The rotor rotating at the high rotation speed can be considered as a gyroscope [6]

$$A\ddot{\mathbf{\Phi}} = l_2\mathbf{F}_2 - l_1\mathbf{F}_1 + jC\Omega\dot{\mathbf{\Phi}}\,, \tag{9}$$

where A is a moment of inertia of the rotor about its axis and C is a moment of inertia of the same rotor about the axis, which is perpendicular to the rotor axis.

4 Simulation study of the model behavior during run-up

As it was stated before the numeric solution of the journal equation of motion is obtained by using Matlab-Simulink. The block diagram of the rotor system is shown in Fig. 6 and 7.

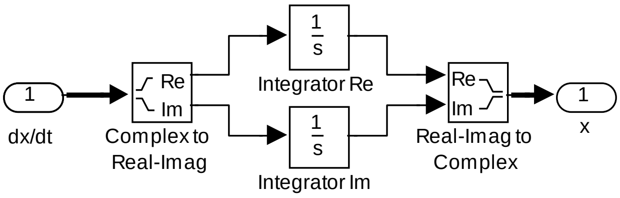

Figure 6. Integration of complex signals

Some signals in the block diagram are complex variables. Integration of the complex signals with respect to time must be done for the real and imaginary component separately. Firstly, a complex variable is to be decomposed into the real and imaginary parts, and then the result of integration will be combined back into the complex variable.

To avoid the problem with the choice of initial conditions, we assume that for $t = 0$, the rotor does not rotate and therefore $x(0) = 0$ and lies on the bottom of the bushing. The stiffness of oil film depends on the eccentricity of the bearing journal axis position, what is the value of y (0), it is necessary to solve nonlinear equation

$$k(y(0))y(0) = -Mg/2 \tag{10}$$

with the use of the Matlab function fsolve.

To test the model response, the following values of the parameters were employed. The bushing position $\mathbf{u}_2 = 0$, while the variable position $\mathbf{u}_1$ corresponds to the rotation at the speed of 40 rad/s with the amplitude of 20 μm. The moments of inertia corresponds a short rigid hollow rotor of the outer diameter of 30 mm and the inner diameter of 20 mm, which was made of steel. The rotational speed runs up from 0 to 400 rad/s during 10 s. The simulation results are shown in Fig. 7. The rotation of the bushing in the journal bearing #1 does not influence the position of the rotor in the journal bearing #2.

M =2.38 kg;	rotor mass
lam_0 = 0.475;	fluid averaged circumferential velocity ratio (lambda)
K_0 = 4000 N/m;	oil film stiffness
D_0 = 2000 Ns/m;	oil film damping coefficient
e = 0.0001 m;	clearance in the journal bearing (100 μm)
MMR = 0.00001 kg m	product of the unbalance mass m mounted at a radius r_u.
L1 = 0.1 m;	distance of the rotor gravity center from the journal bearing #1
L2 = 0.1 m;	distance of the rotor gravity center from the journal bearing #2
A = 0.0006 kg m2;	a moment of inertia of the rotor about its axis
C = 0.008 kg m2;	moment of inertia of the same rotor about the axis, which is perpendicular to the rotor axis.

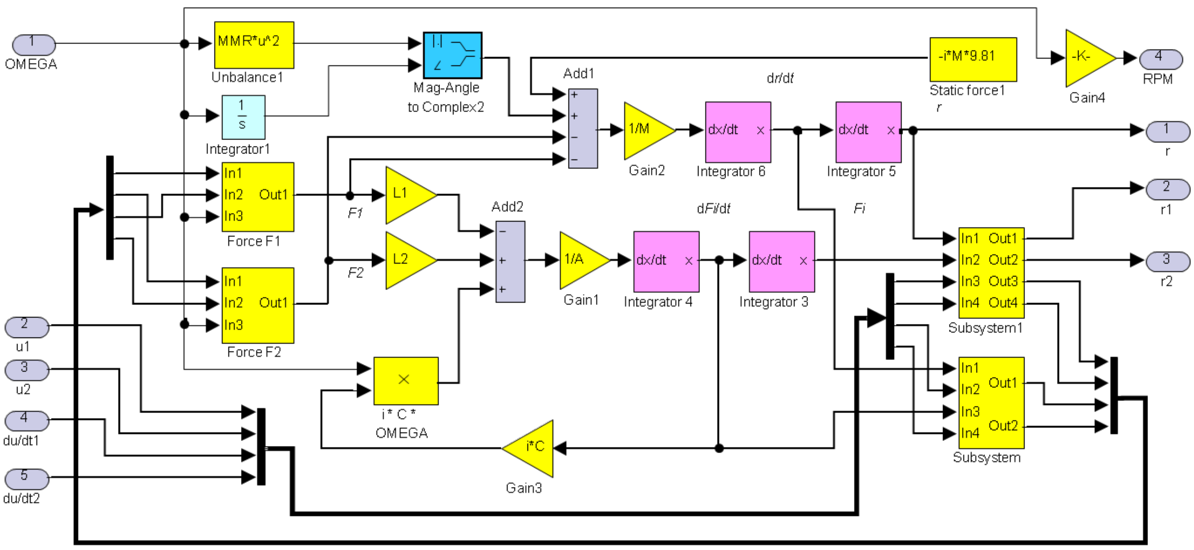

Figure 7. Block diagram of the rotor system supported on two journal bearings

The rotational speed runs up from 0 to 400 rad/s during 10 s. The simulation results are shown in Fig. 8. The rotation of the bushing in the journal bearing #1 does not influence the position of the journal in the journal bearing #2.

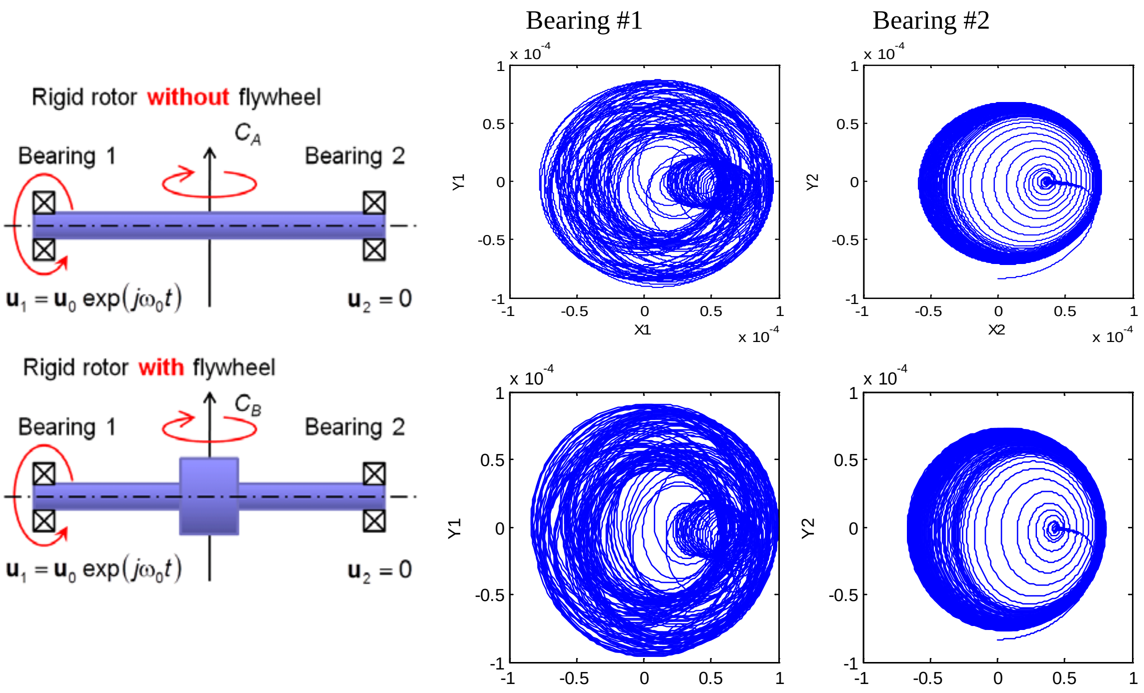

Figure 8. Effect of circulation of the bearing bushing #1 on movement of the journal in the bearing #2

5 Results

Test stand for investigation of possibilities to influence rotor behavior through external excitation of sliding journal bearings was designed and partly tested. The lumped parameter models of the rigid and flexible rotor system including gyroscopic effect were created and simulation study of rotor behavior during run-up was carried out. As it was find out for the rigid rotor, the displacement of the bushing #1 does not affect the displacement of the bushing #2.

References

[1] J. Šimek – J. Tůma – R. Svoboda: *Test stand for investigation of external excitation influence on behaviour of rotor supported in sliding journal bearings*. Colloquium Dynamics of Machines 2008, Prague 2008, p.169-174

[2] A. Muszynska *Whirl and Whip – Rotor / Bearing Stability Problems.* Journal of Sound and Vibration (1986) 110 (3), pp 443-462.

[3] A. Muszynska *Rotordynamics*, Taylor & Francis Group, New York 2005, ISBN 0-8247-2399-6.

[4] Svoboda, R.: Theoretical analysis of influence of bearing bushing external excitation on dynamic properties of the rotor, Technical report TECHLAB No. 07-301, 2007 (in Czech)

[5] J. Tůma – A. Bilošová – J. Šimek – R. Svoboda *A simulation study of the rotor vibration.* Colloquium Dynamics of machines 2008. Prague, 2008, p. 1-8.

[6] Kožešník, J : Základy teorie přístrojů, SNTL Praha 1987. (in Czech)

Acknowledgments

The presented results have been obtained during the solving o research project SP201118/2011 supported by the Ministry of Education, Youth and Sport of Czech Republic.

Prof. Ing. Jiří Tůma, CSc.
VSB – Technical University of Ostrava, Faculty of Mechanical Engineering, Department of Control systems and Instrumentation, tř. 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic,
e-mail: jiri.tuma@vsb.cz

Ing. Jolana Škutová, Ph.D.
VSB – Technical University of Ostrava, Faculty of Mechanical Engineering, Department of Control systems and Instrumentation, tř. 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic,
e-mail: jolana.skutova@vsb.cz